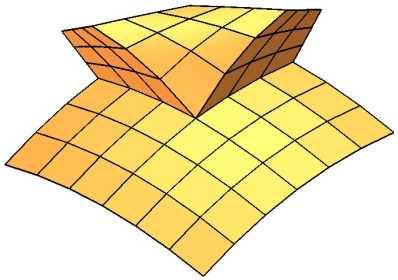
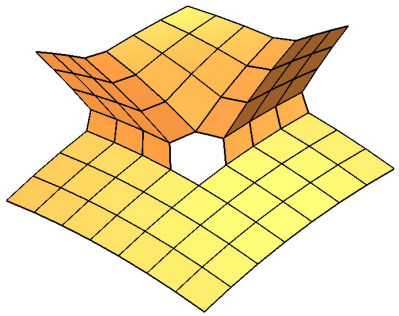
Multi-sided completion of C^2 bi-3 and C^1 bi-2 splines: A unifying approach

Kęstutis Karčiauskas
Vilnius University

Jörg Peters
University of Florida



irregular point



irregular face

NIH, Darpa

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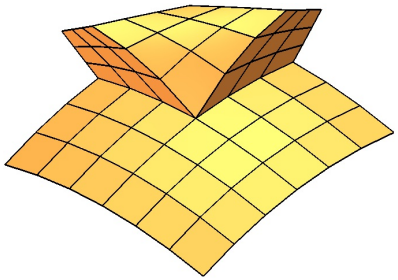

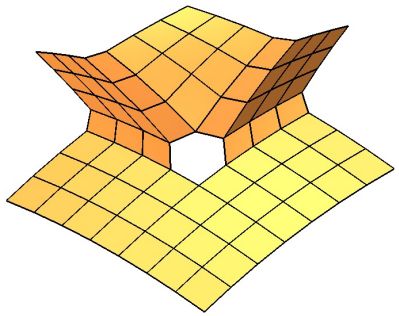

Quad mesh = spline control net?

NIH, Darpa

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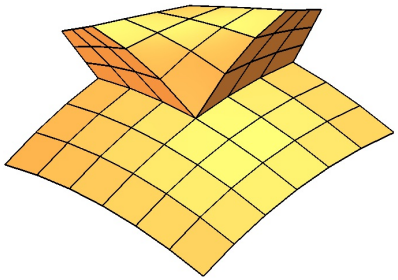
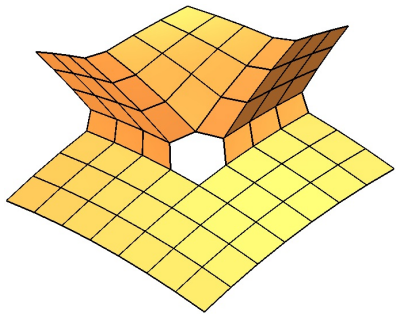


Catmull-Clark:
recursion
poor shape



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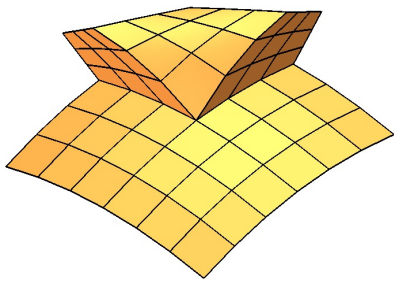
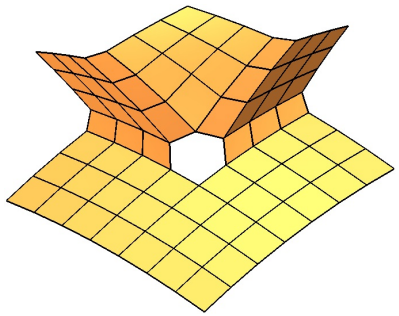


Catmull-Clark:
recursion
poor shape



Doo-Sabin:
recursion
worse shape

NIH, Darpa

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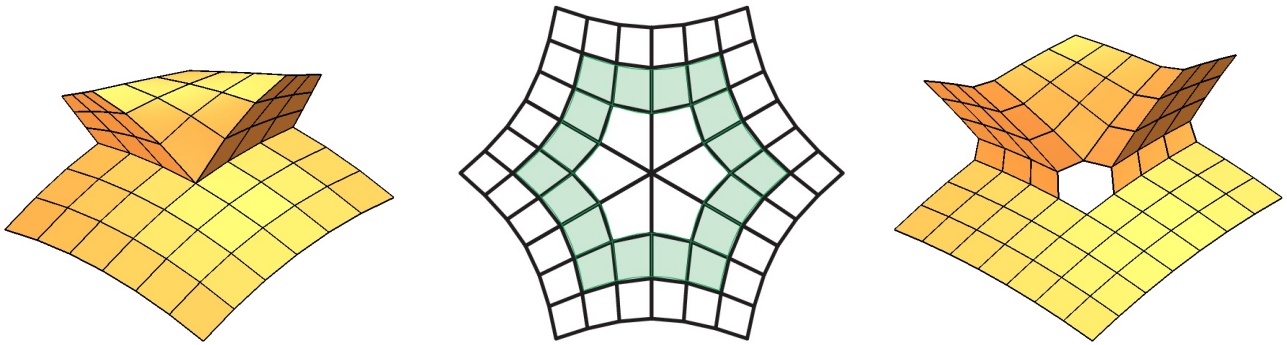

C^2 bi-3

NIH, Darpa

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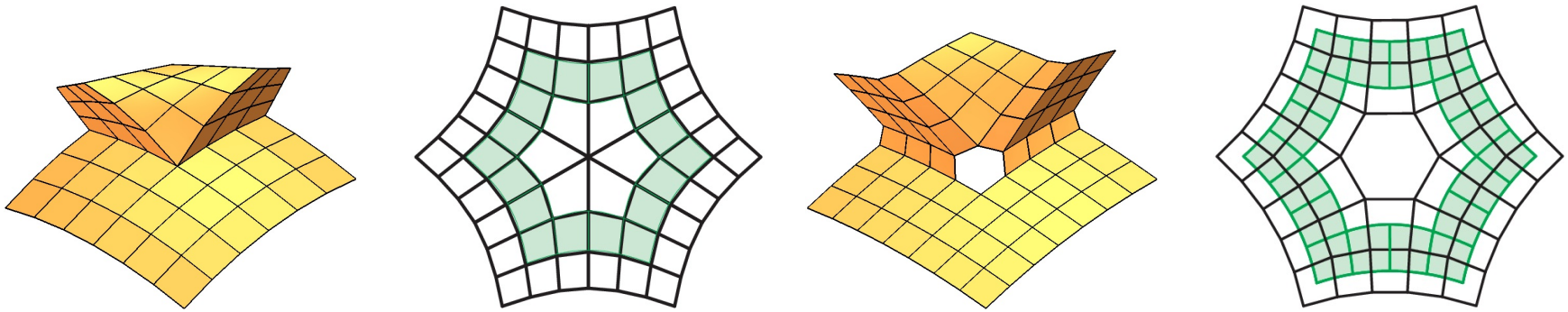

C^2 bi-3 C^1 bi-2

NIH, Darpa

Multi-sided completion of C^2 bi-3 and C^1 bi-2 splines: A unifying approach

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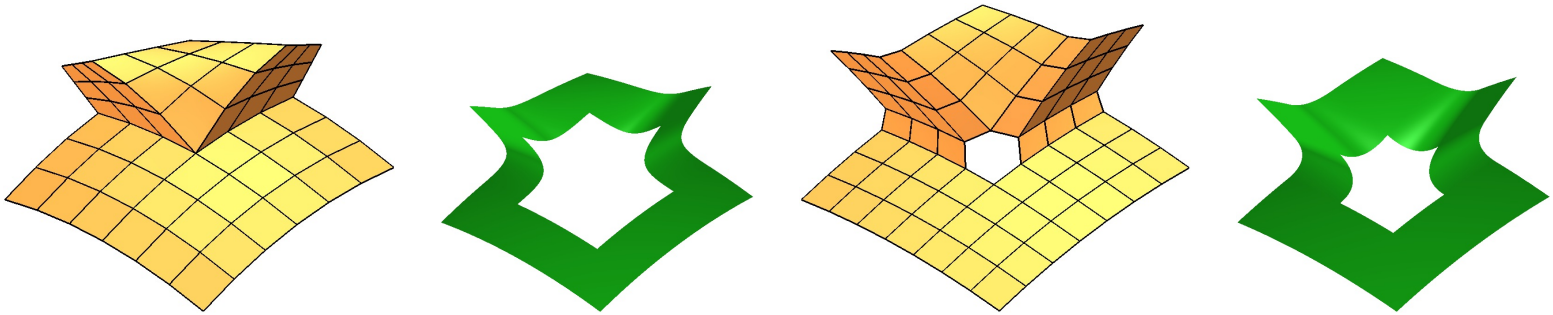
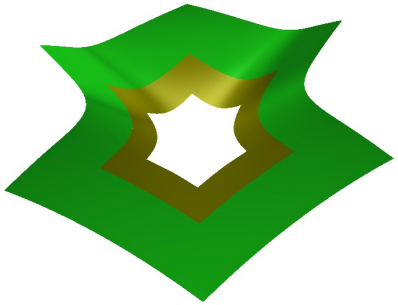


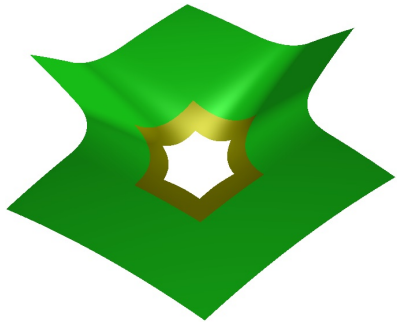
C^2 bi-3 C^1 bi-2

NIH, Darpa

Idea: transition layer + unified cap



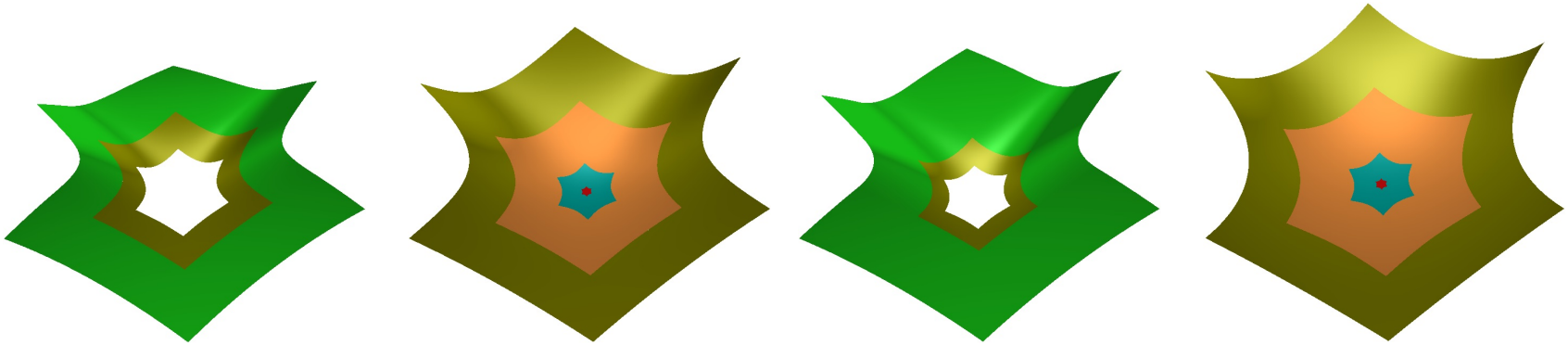
C^2 bi-3



C^1 bi-2

fill challenge: good shape, refinability

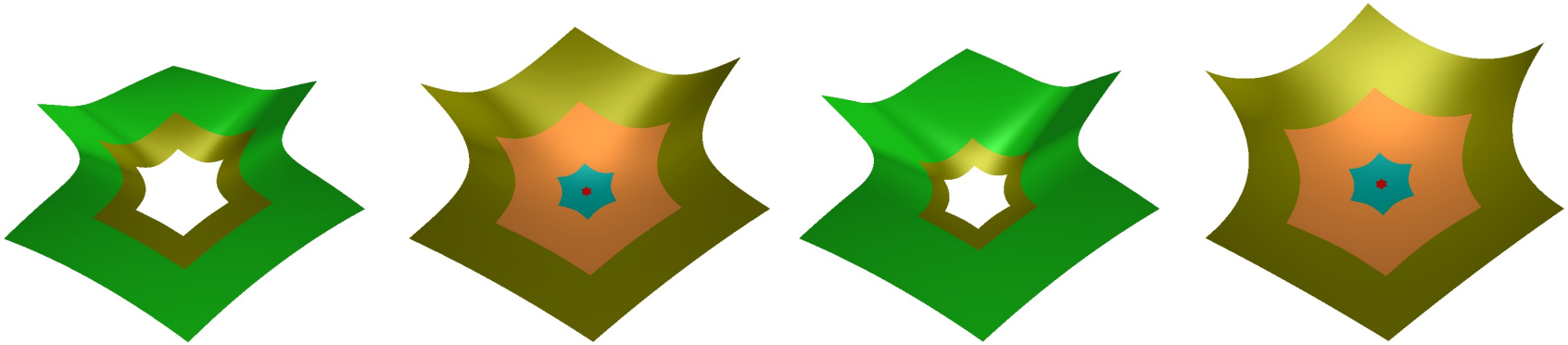
Idea: transition layer + unified cap



C^2 bi-3 C^1 bi-2

fill challenge: good shape, refinability

Idea: transition layer + unified cap



C^2 bi-3

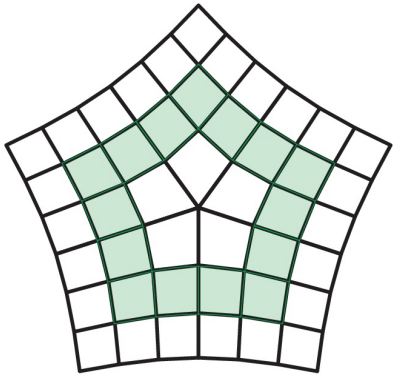
C^1 bi-2

interrogation: highlight lines
fill challenge: good shape, refinability

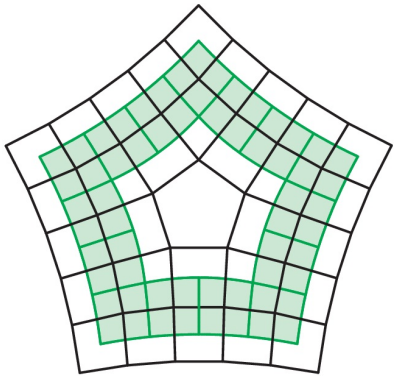
Outline

1. Transition Ring + Tensor-border
2. Unified (hybrid) cap = bi-4 rings + tiny cap
3. Examples & Comparison

Net & tensor-border

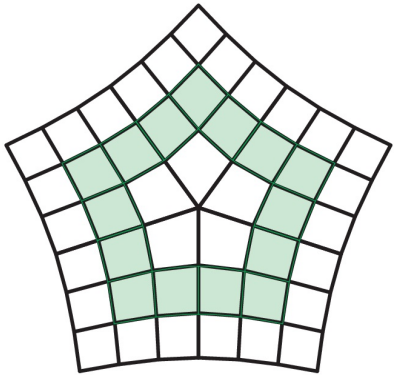


bi-3 ring

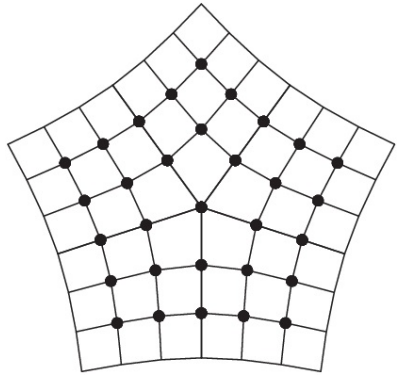


bi-2 ring

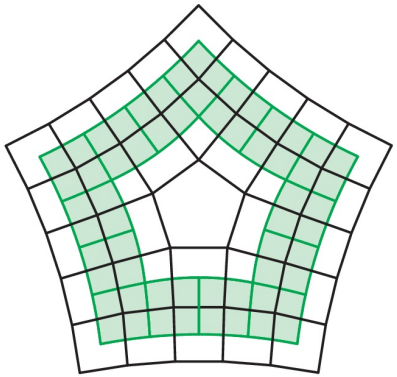
Net & tensor-border



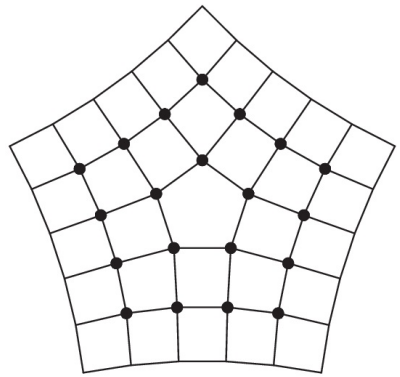
bi-3 ring



- : CC-net

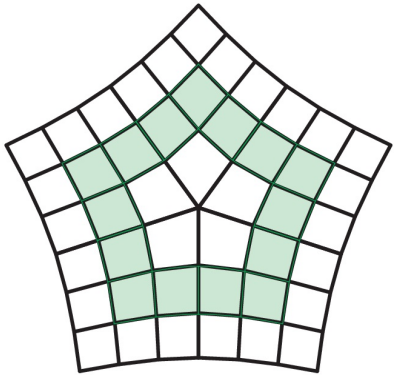


bi-2 ring

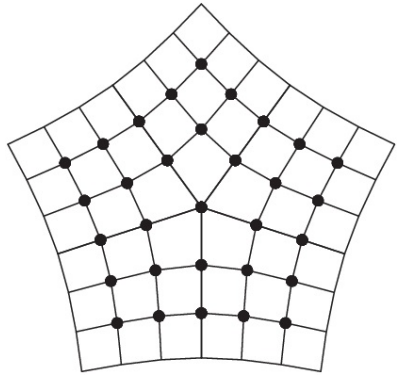


- : DS-net

Net & tensor-border

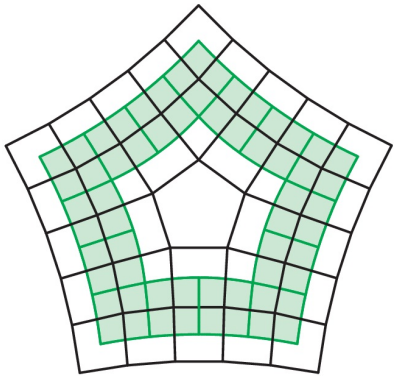


bi-3 ring

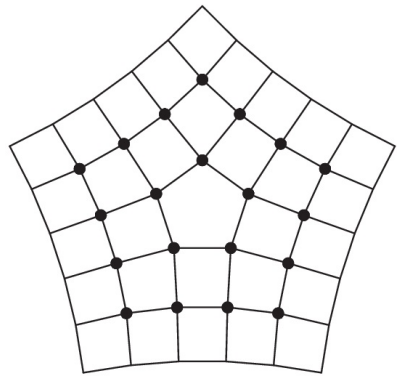


• : CC-net

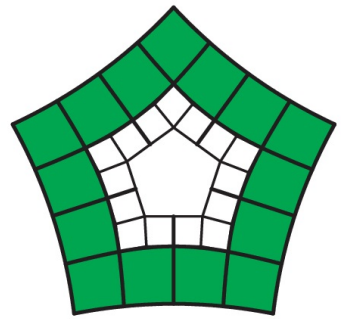
bi-3 ring + tensor-border



bi-2 ring

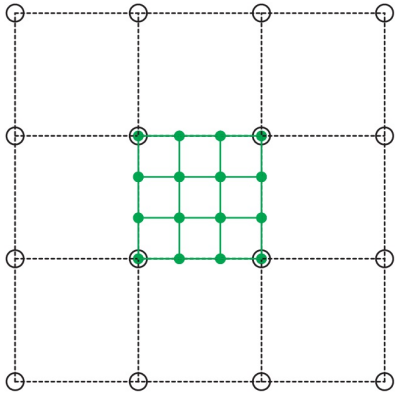


• : DS-net

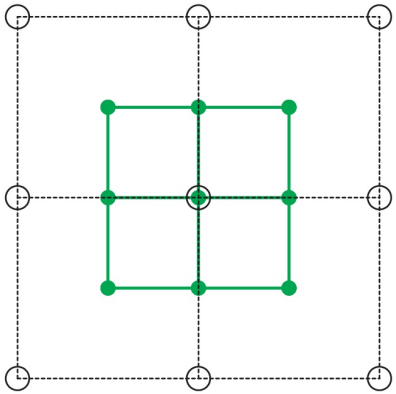


bi-2 ring + tensor-border

B-spline and BB-form (Bernstein-Bézier)

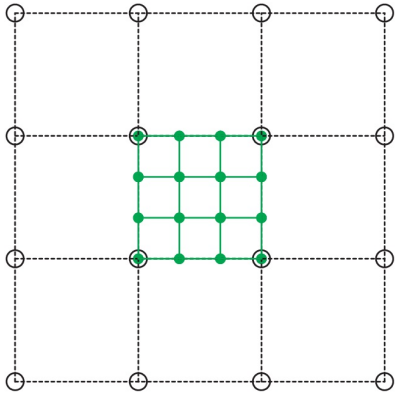


C^2 bi-3

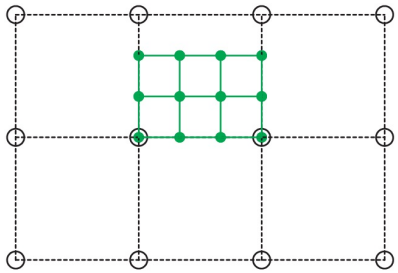


C^1 bi-2

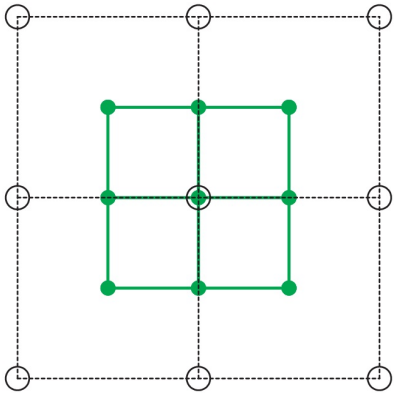
B-spline and BB-form (Bernstein-Bézier)



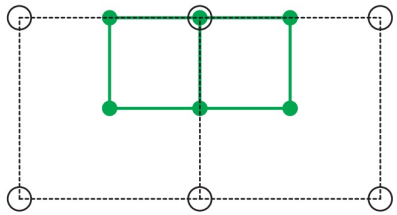
C^2 bi-3



tensor-border

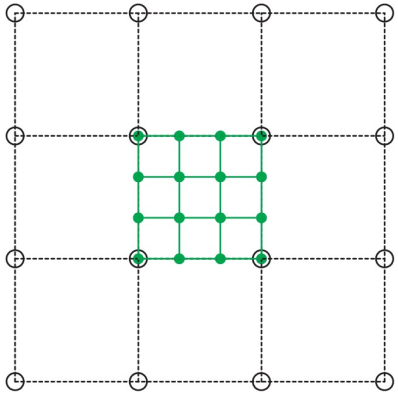


C^1 bi-2

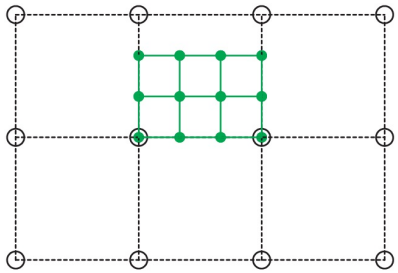


tensor-border

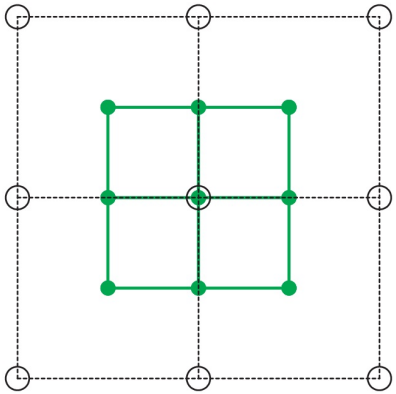
B-spline and BB-form (Bernstein-Bézier)



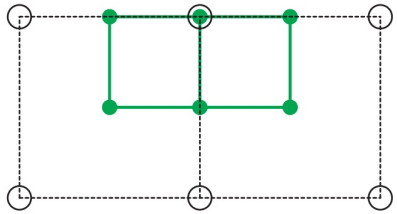
C^2 bi-3

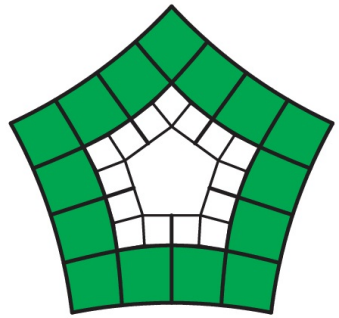


tensor-border



C^1 bi-2

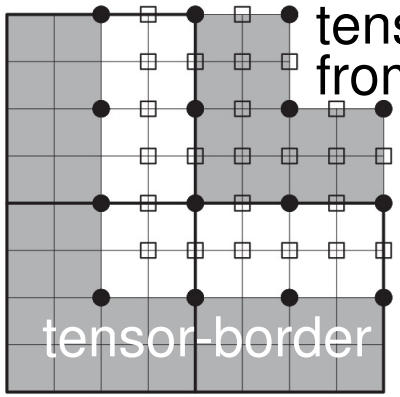




tensor-border

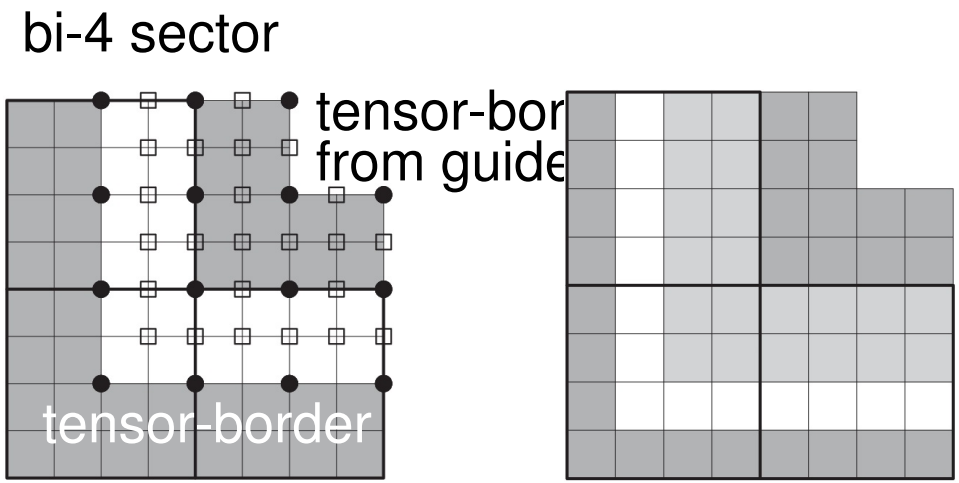
Contracting bi-4 rings

bi-4 sector



transition: from bi-3

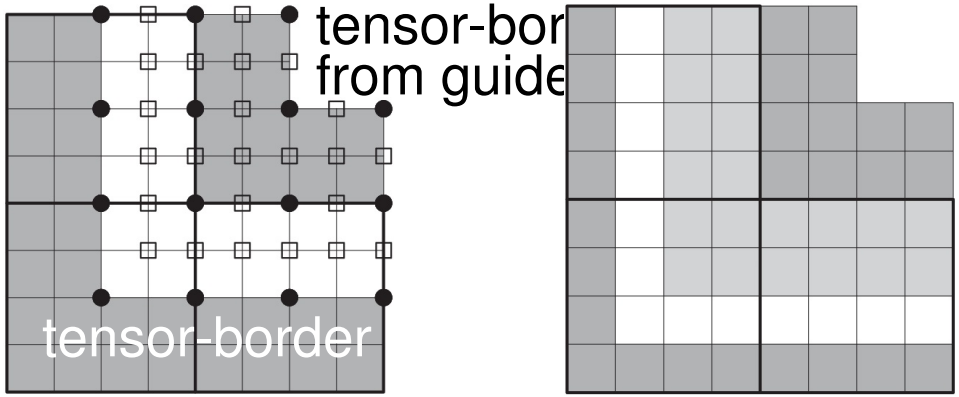
Contracting bi-4 rings



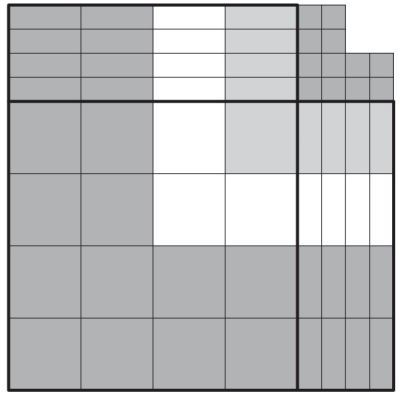
transition: from bi-3 from bi-2

Contracting bi-4 rings

bi-4 sector



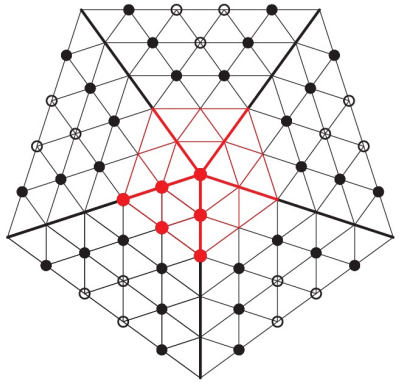
rapid contraction



transition: from bi-3 from bi-2 main body

Good shape from a Guide Surface

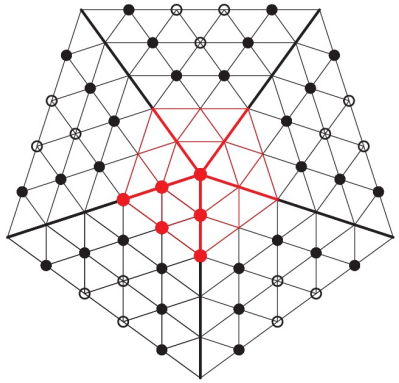
C^1 guide $\mathbf{g}$ (degree 5) computed by *linear operator* from CC-net or DS-net



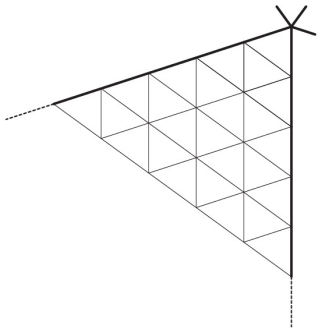
$\mathbf{g}$

Good shape from a Guide Surface

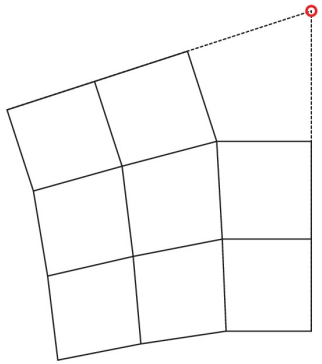
C^1 guide **g** (degree 5) computed by *linear operator* from CC-net or DS-net



g



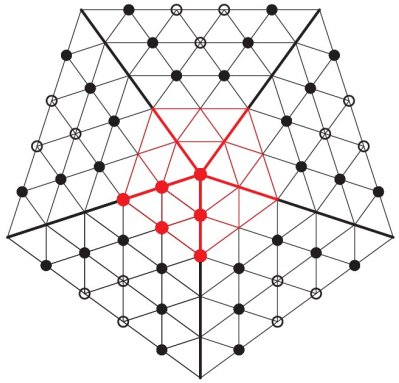
sector of **g**



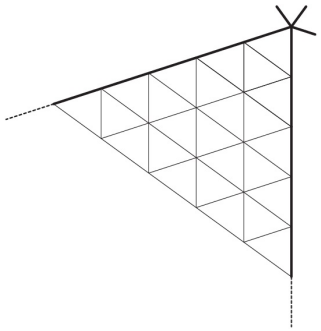
χ_σ

Good shape from a Guide Surface

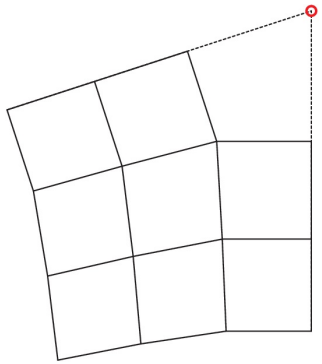
C^1 guide **g** (degree 5) computed by *linear operator* from CC-net or DS-net



g

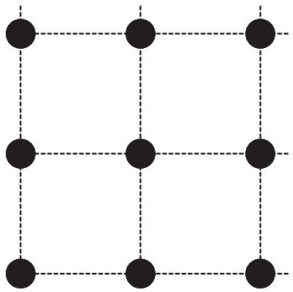


sector of **g**



χ_σ

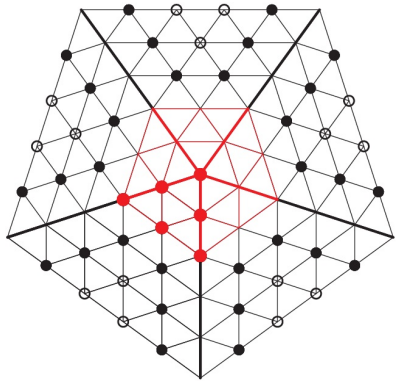
$$\begin{pmatrix} \partial_v^2 F & \partial_u \partial_v^2 F & \partial_u^2 \partial_v^2 F \\ \partial_v F & \partial_u \partial_v F & \partial_u^2 \partial_v F \\ F & \partial_u F & \partial_u^2 F \end{pmatrix} \rightarrow$$



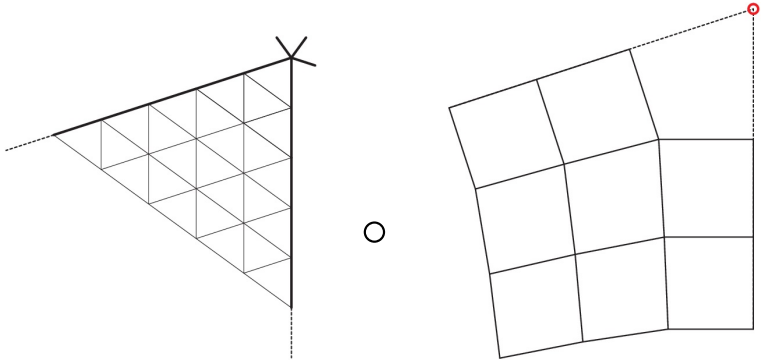
Hermite data (derivatives) $\rightarrow$ to BB-form

Good shape from a Guide Surface

C^1 guide **g** (degree 5) computed by *linear operator* from CC-net or DS-net

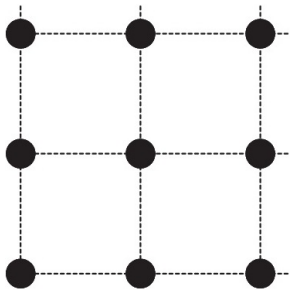


g

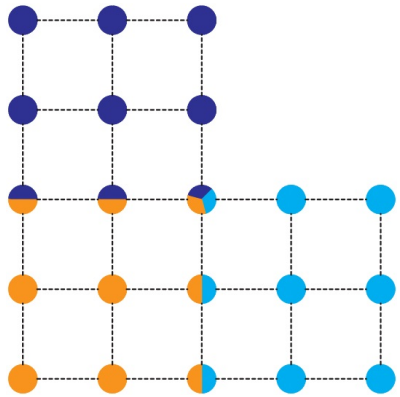


sector of **g** $\quad \chi_\sigma$

$$\begin{pmatrix} \partial_v^2 F & \partial_u \partial_v^2 F & \partial_u^2 \partial_v^2 F \\ \partial_v F & \partial_u \partial_v F & \partial_u^2 \partial_v F \\ F & \partial_u F & \partial_u^2 F \end{pmatrix} \rightarrow$$

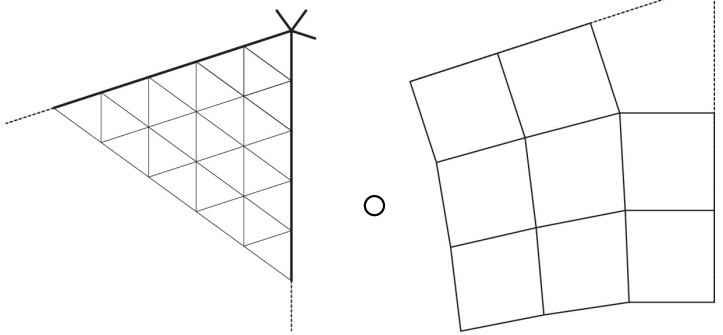


Hermite data (derivatives) $\rightarrow$ to BB-form

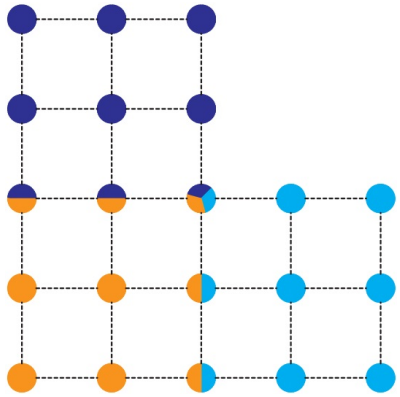


L-shaped bi-4 tensor-border

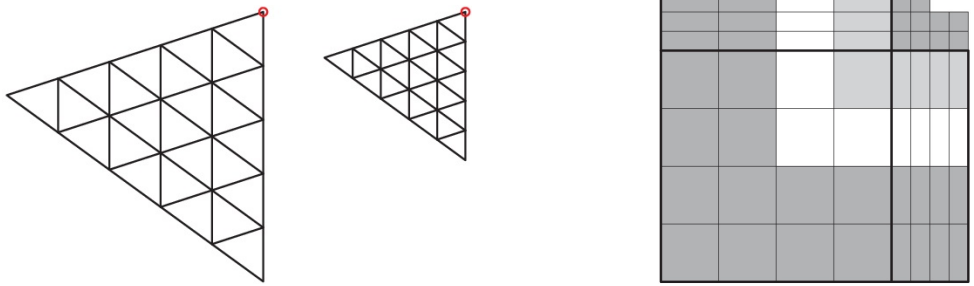
Good shape from a Guide Surface



$\mathbf{g}$ $\circ$ χ_σ

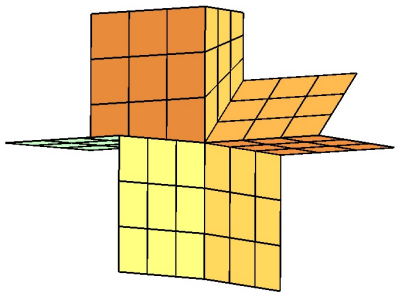


$H^{samp}(\mathbf{g} \circ \chi_\sigma)$

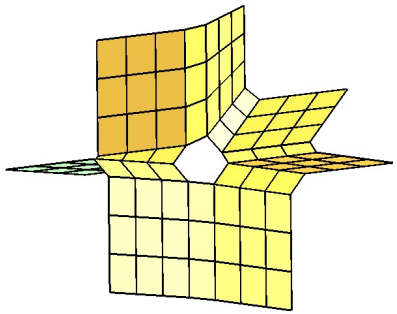


recursion via domain scaling

Surfaces from quad meshes ($n = 7$)

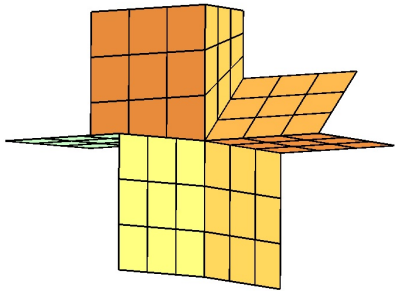


extented CC-net

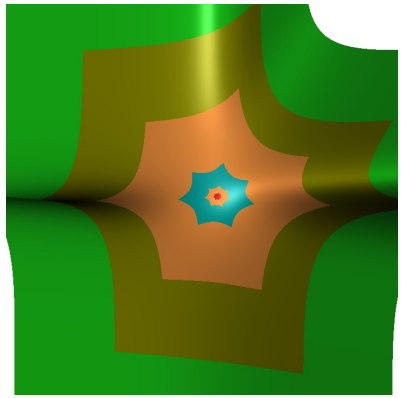


extended DS-net

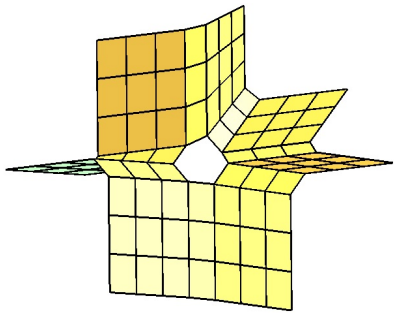
Surfaces from quad meshes ($n = 7$)



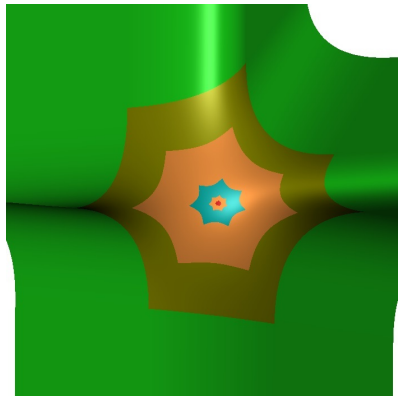
extented CC-net



layout

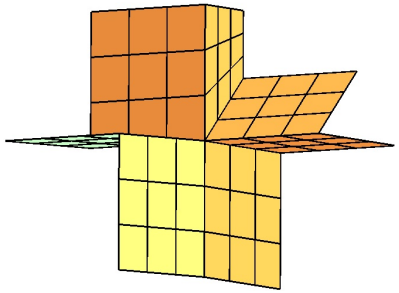


extended DS-net

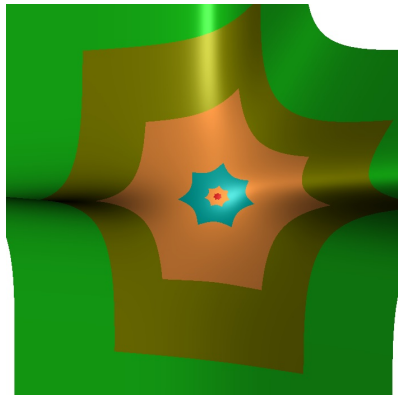


layout

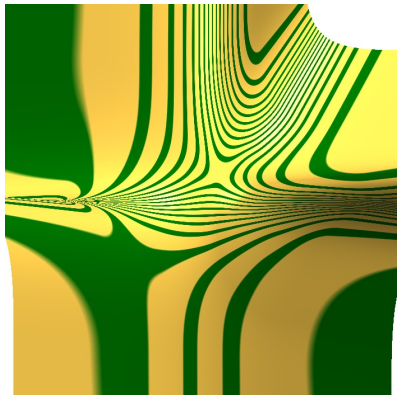
Surfaces from quad meshes ($n = 7$)



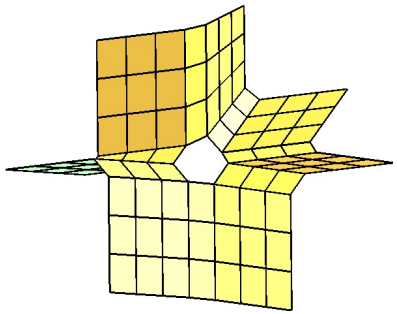
extented CC-net



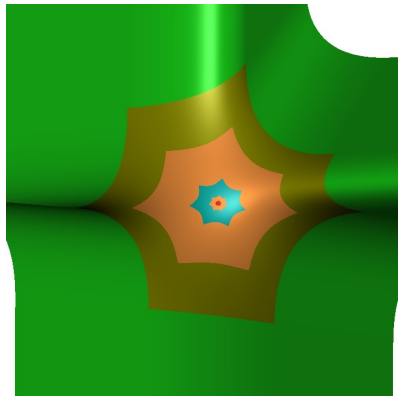
layout



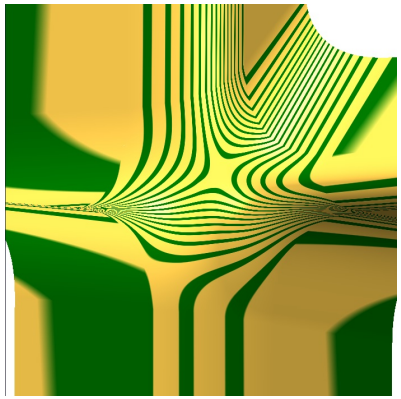
bi-3 + unified



extended DS-net

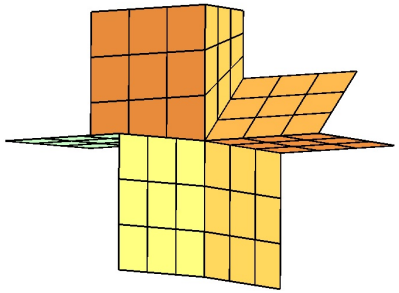


layout

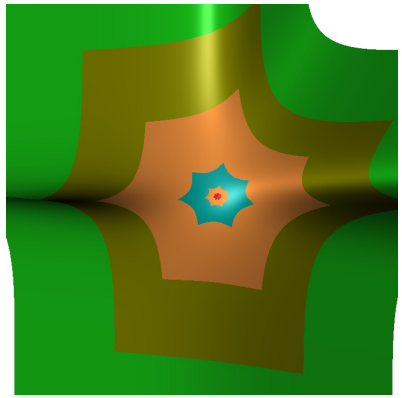


bi-2 + unified

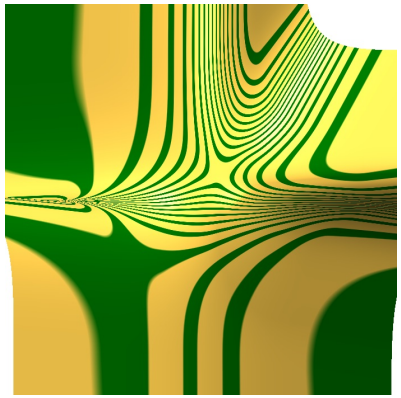
Surfaces from quad meshes ($n = 7$)



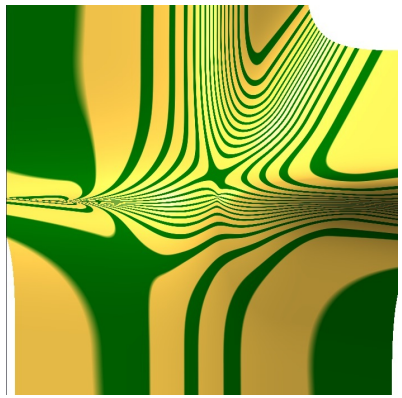
extented CC-net



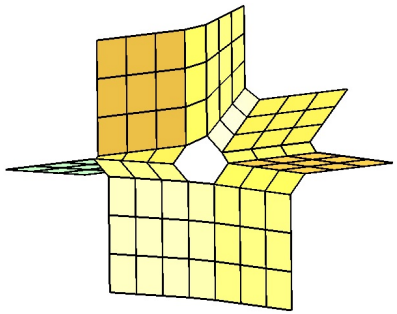
layout



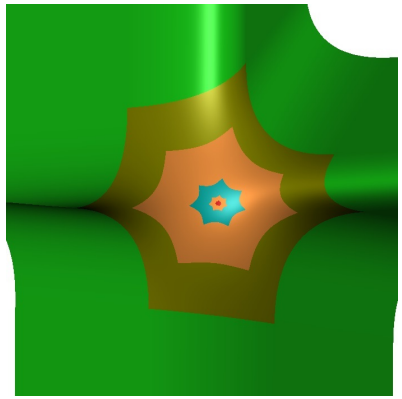
bi-3 + unified



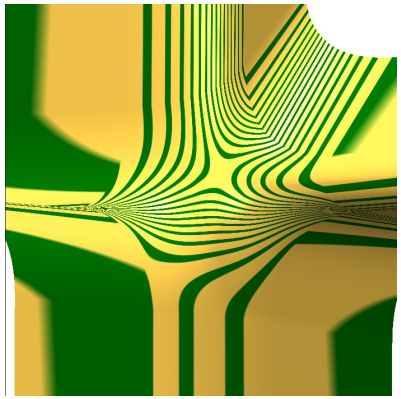
Catmull-Clark



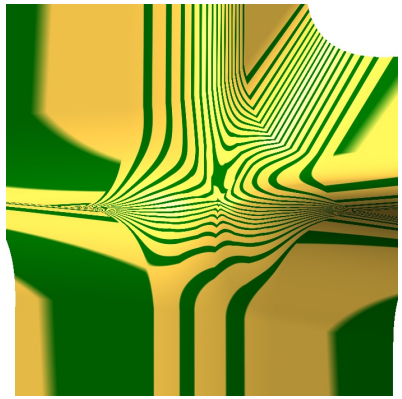
extended DS-net



layout



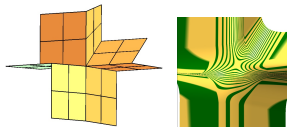
bi-2 + unified



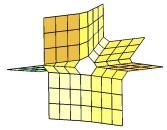
Doo-Sabin

Summary

- One algorithm for capping both CC-nets (bi-3) and DS-net (bi-2).
- Parameterically C^1 or C^2 joined rings (except for G^1 tiny cap).
- Bi-4 rings are refinable, converge rapidly
- Can interpret CC-net as bi-2 + cap control net (see paper)



- Can also interpret DS-net as bi-3 + cap control net
  (not in paper: requires more machinery to obtain good shape)



- **Questions?**

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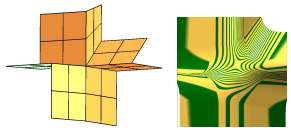

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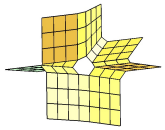

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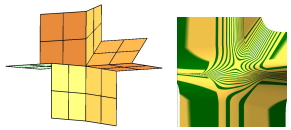

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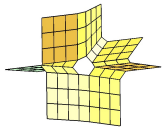

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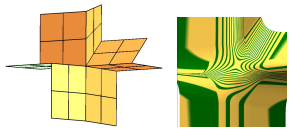

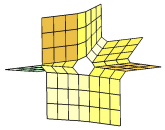

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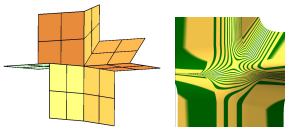

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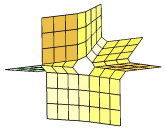

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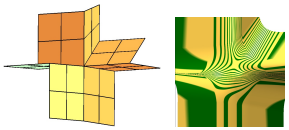

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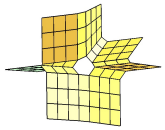
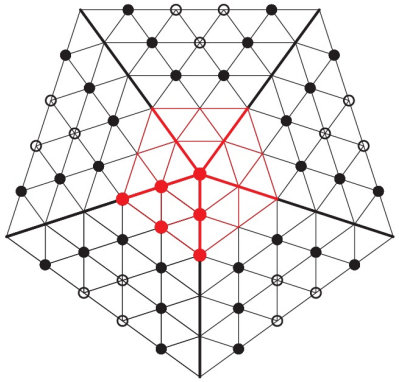


- **Questions?**

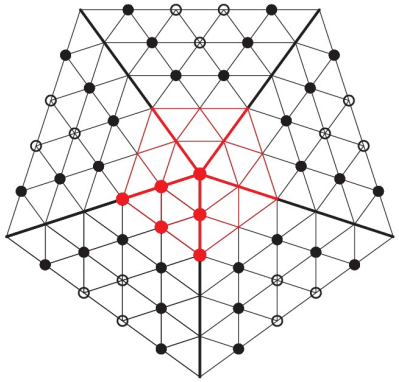
Blank

Good shape via a guide surface: computed by linear operator from CC-net or DS-net



C^1 guide $\mathbf{g}$ (degree 5)

Good shape via a guide surface: computed by linear operator from CC-net or DS-net



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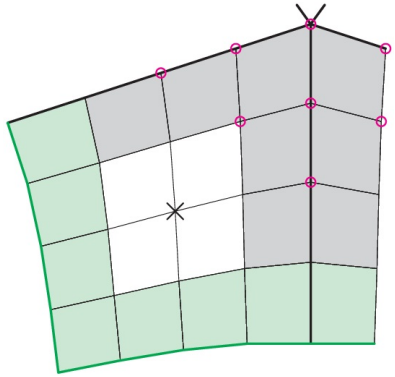

auxiliary G^1 bi-4 guide **h**

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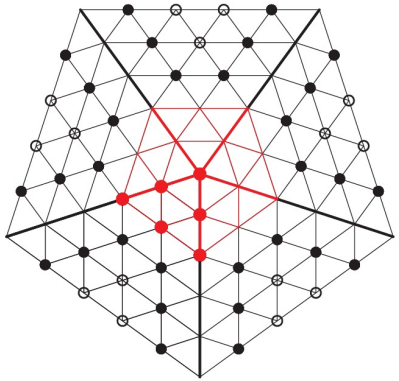

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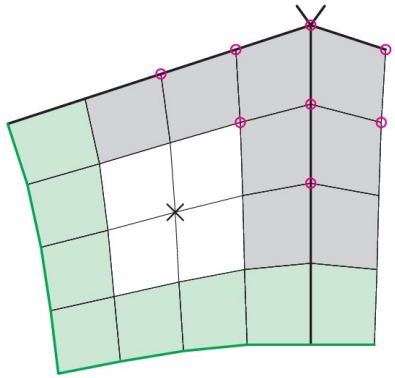

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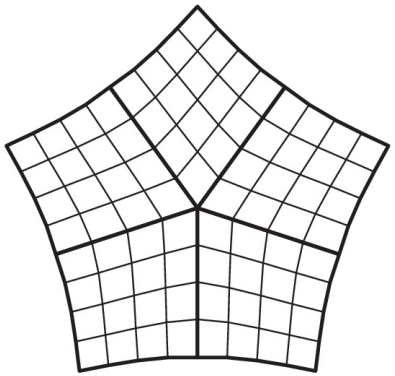
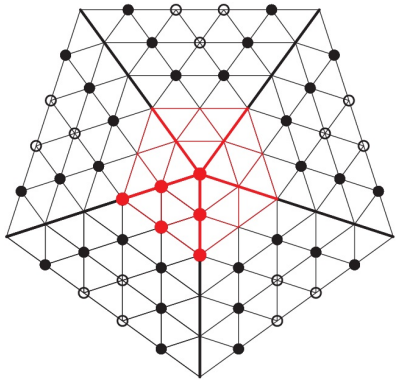
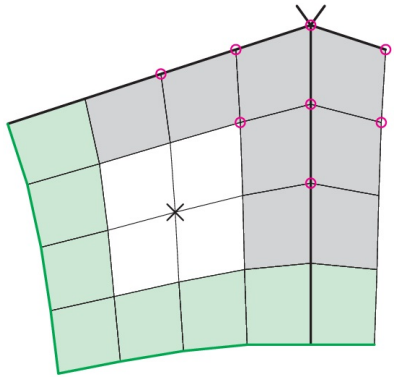


parameterization $\mathbf{h}^{char}$

Good shape via a guide surface: computed by linear operator from CC-net or DS-net



C^1 guide $\mathbf{g}$ (degree 5)



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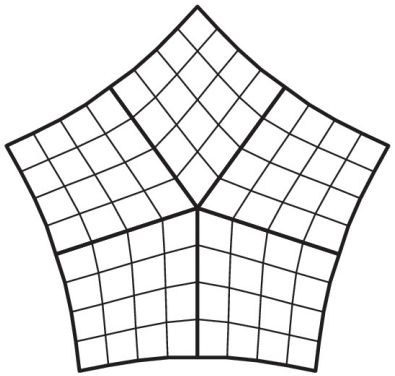
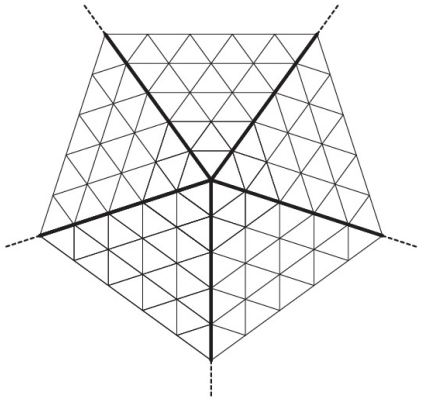


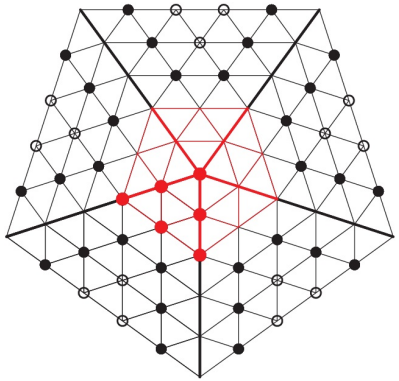
parameterization $\mathbf{h}^{char}$

$\rightarrow$

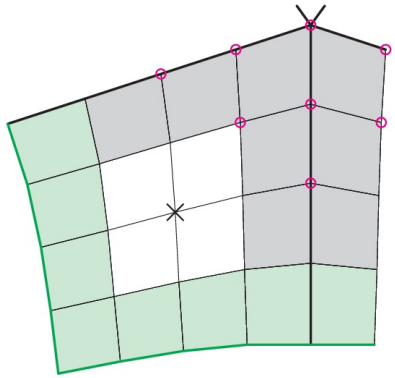


$\mathbf{g}$, $9n + 5$ dofs

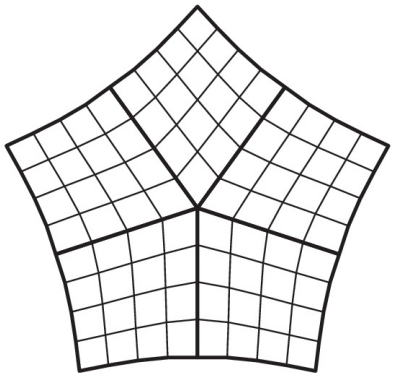
Good shape via a guide surface: computed by linear operator from CC-net or DS-net



C^1 guide $\mathbf{g}$ (degree 5)



auxiliary G^1 bi-4 guide $\mathbf{h}$



parameterization $\mathbf{h}^{char}$

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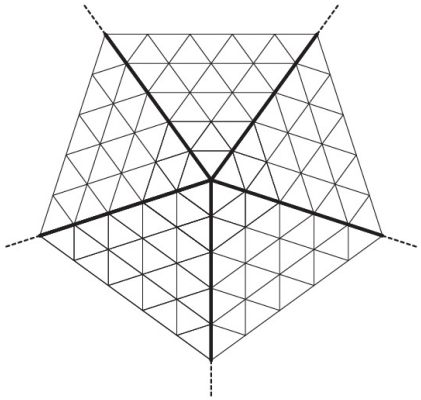

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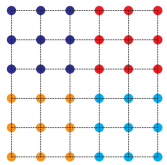
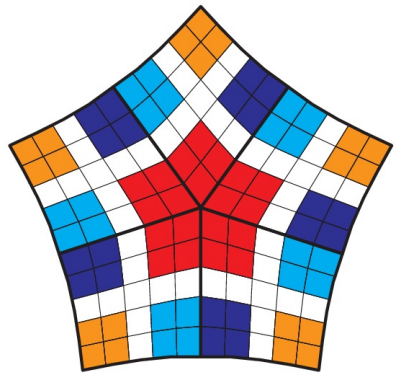
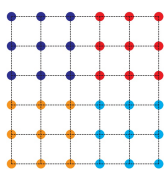
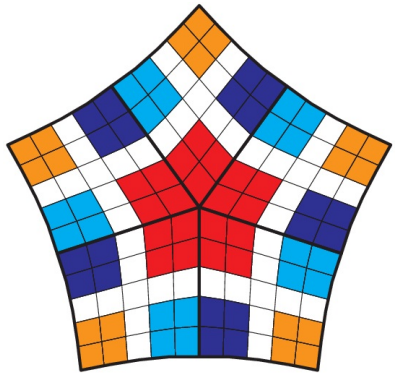




$H^{samp}(\mathbf{g} \circ \mathbf{h}^{char})$

Good shape via a guide surface: computed by *linear operator* from CC-net or DS-net





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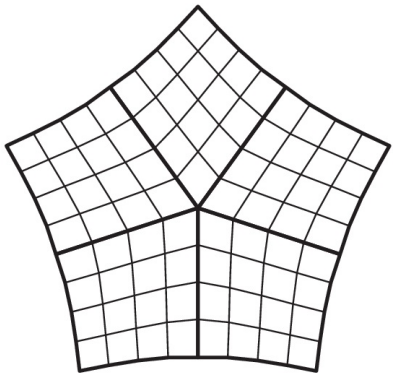
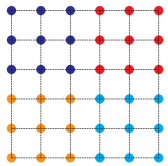
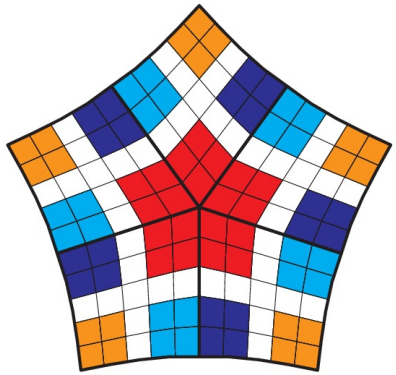


$\mathbf{h}^{char} : ([0..1]^2)^n \to \mathbb{R}^2$





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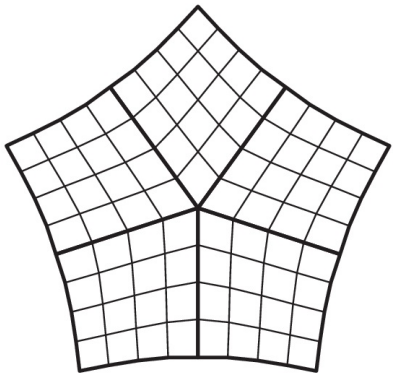

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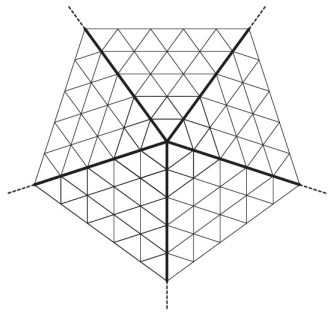
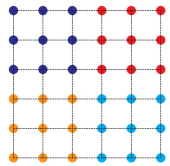
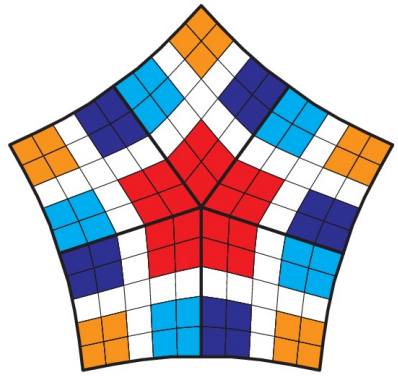
$\circ$



$\mathbf{g}$, $9n + 5$ dofs

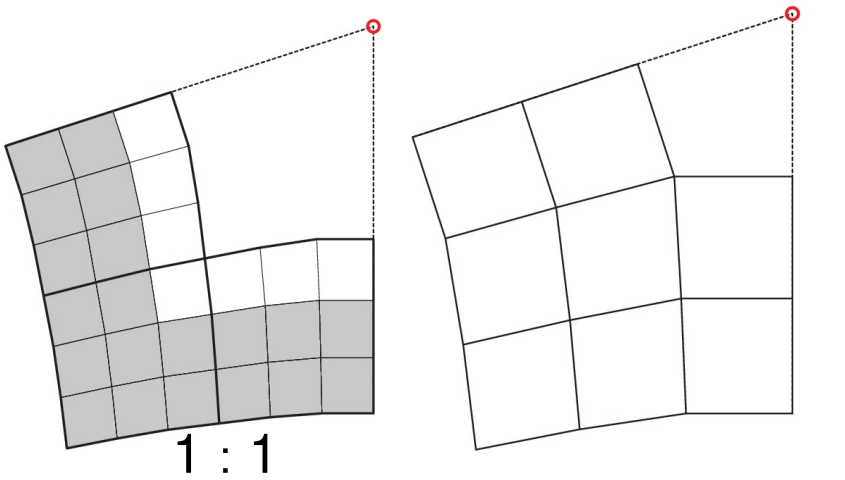




$H^{samp}(\mathbf{g} \circ \mathbf{h}^{char})$

Parameterizations: characteristic maps and tensor-borders

Catmull-Clark subdivision ($\sigma := \frac{1}{2}$)

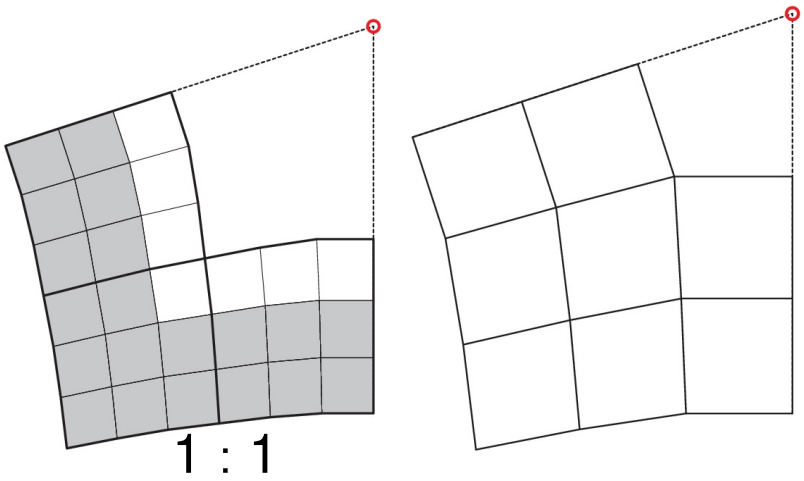


1 : 1

char-map tensor-border χ_σ

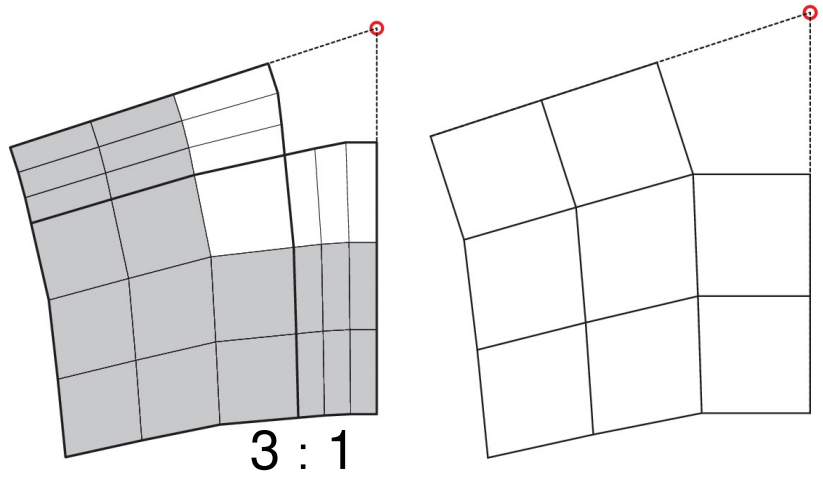
Parameterizations: characteristic maps and tensor-borders

Catmull-Clark subdivision ($\sigma := \frac{1}{2}$)



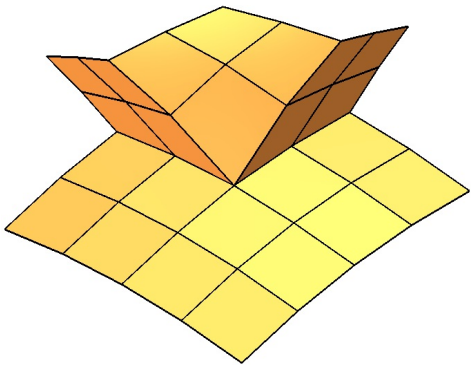
char-map tensor-border χ_σ

adjustable speed subdivision ($\sigma := \frac{3}{4}$)



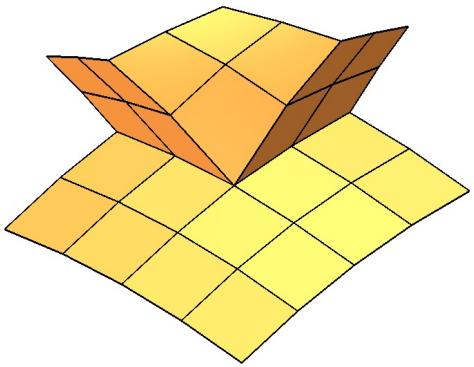
char-map tensor-border χ_σ

Quad-net $\Rightarrow$ DS-net

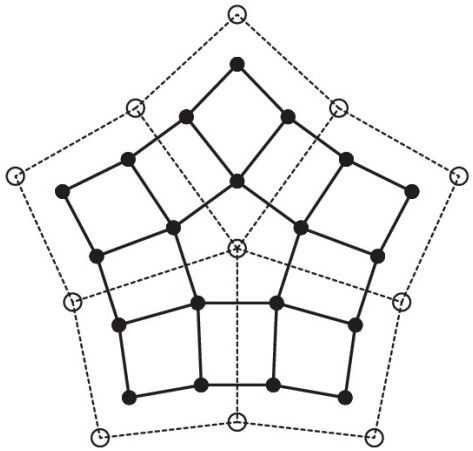


quad-net

Quad-net $\Rightarrow$ DS-net

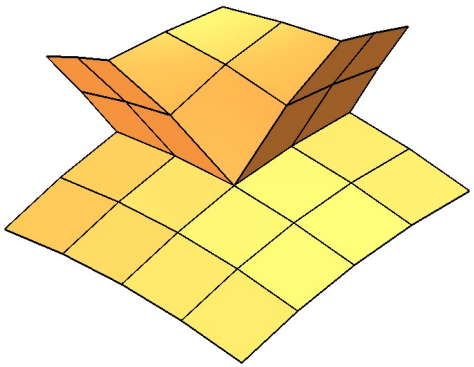


quad-net

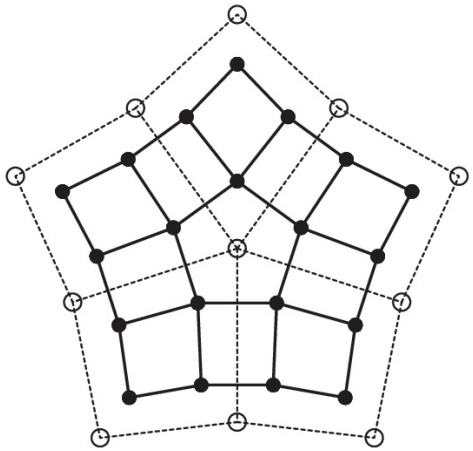


regular refinement

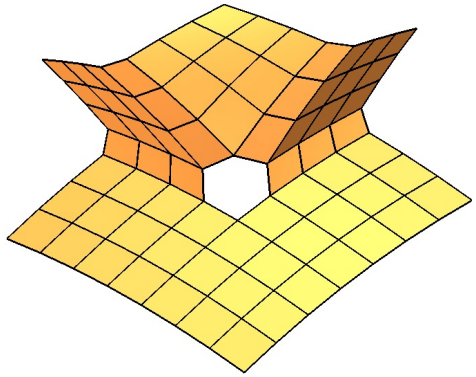
Quad-net $\Rightarrow$ DS-net



quad-net



regular refinement



DS-net