Semi-structured splines for design & analysis

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INdAM  2020, Rome, Italy

Support:  INdAM, DARPA
Overview

- Irregularities
- Classification of splines on meshes with irregularities
  - Semi-structured G-splines
  - Subdivision Surfaces
  - → hybrid for IGA
Assumes you are familiar with

- Bernstein-Bezier
- B-spline
- (generalized) Subdivision Surfaces
- $G^k$ continuity of surfaces
- $\mathbb{R}^d$
Surface Quality vs $C^k$

uniform, parallel = good (unless feature)

(a) reflection lines
Surface Quality vs $C^k$

Splines on Meshes with Irregularities  Jorg Peters

(a) reflection lines

(b) G-spline surface + highlight lines on the surface

Zebra $C^1$ Uniform, parallel $\rightarrow$ good
Surface Quality vs $C^k$ in automobile styling

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(a) reflection lines

Uniform, parallel $\rightarrow$ good
Irregularities

merge parameter directions

transition between coarse and fine meshes
Efficient Pixel-accurate Rendering of Animated Curved Surfaces [YBP12]
Thin shell analysis

heat dissipation on thin shell
14K bi-3 pieces, 27.5K spline-dof, 3.6 secs
Classification of Splines for Irregular Layout

\[ G^k: \]
- geom smooth
- singular:
  - polar
  - subdivision
  - singular jet
- rational:
  - transfinite
  - Gregory
  - barycentric
  - orbifold
Overview

➢ Motivation
➢ Classification of splines on meshes with irregularities
  ○ Semi-structured G-splines
  ○ Subdivision Surfaces
Irregularities

merge parameter directions

transition between coarse and fine meshes

polar
Irregularities $\rightarrow$ semi-structured G-splines
Semi-structured G splines

- trimmed $C^{-1}$
- conforming $C^0$
- $C^{1-\epsilon}$
- $C^k$
- singular
- rational

SMOOTH
### Semi-structured G-splines

Splines on Meshes with Irregularities  
Jorg Peters

Constructions with good shape = smooth highlight lines, curvature distribution

<table>
<thead>
<tr>
<th>smooth</th>
<th>regular</th>
<th>irregular</th>
<th>valence</th>
<th>split</th>
<th>reference</th>
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<td>any</td>
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<td>bi-2</td>
<td>bi-3</td>
<td>3,5</td>
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<td>MVS</td>
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</tbody>
</table>

irregularity surrounded by quads

← as many as spline families

each is subtly different to achieve special properties
Good highlight lines? Large industrial data sets

Thanks to Martin Marinov
Good highlight lines?  Shape obstacle course

(a) $n = 5$
convex

(b) $n = 5$
House corner

(c) $n = 6$
chrysler

(d) $n = 7$
3 beams

(e) $n = 8$
origami n=10

(f) $n = 9$
mitsubishi

(g) $n = 3$
convex

(h) $n = 6$
peak
Basic Functions: Polynomial pieces joined with matching derivatives after change of variables (geometric continuity)

- Polynomial pieces joined with matching derivatives after change of variables (geometric continuity)

- Tabulate as Bezier

- Boundary G-spline

- valence \( n=3 \)

- near \( n=3 \)

- near \( n=5 \)

- near \( n=5 \) and \( n=3 \)
General Theorem for gIGA

Matched $G^k$-constructions always yield $C^k$-continuous (isogeometric) finite elements \[ [GP15] \]

- in any number of variables,
- for any smoothness $k$,
- any manifold

(including, of course, planar ones)

IGA=isogeometric analysis (texture mapping)
G-splines \textbf{gIGA} \textit{(generalized IGA)} on manifolds [NKP13,15]

Splines on Meshes with Irregularities \hspace{1cm} Jorg Peters

Finite Element Obstacle course: \hspace{1cm} meshing-less analysis

use spline for geometry \hspace{1cm} and \hspace{1cm} displacement function

(a) Octant of a spherical shell \hspace{1cm} (b) Scordelis-Lo roof thin shell
G-splines  Layered tri-variate manifolds

Stacking (=tensoring) surfaces

[NKP15]

(a) A cross-section  (b) Turbine blade (solid)  (c) Computed solution
d.o.f. for analysis under refinement

Along G-edges:

- Irregularly distributed
- Asymmetrically distributed
Semi-structured G-splines

Pro:
Semi-structured control nets
Piecewise polynomial
Good shape

Con:
Refinement not uniform
Overview

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Subdivision Surfaces

Splines on Meshes with Irregularities

Jorg Peters

Sven Eberwein

Pixar

SMOOTH

trimmed $C^{-1}$

conforming $C^0$

$C^{1-\epsilon}$

$G^k$: geom smooth

singular: polar

subdivision singular jet

c rational: transfinite

Gregory barycentric orbifold

Pixar
Catmull-Clark Subdivision [CC78] is not class A

[KPR04] n>4:
convex input $\rightarrow$ saddle

[KPP17] T-junction
convex input $\rightarrow$ flat

[KP14] n>5
saddle $\rightarrow$ pinched highlight lines
New idea: Guided Subdivision

Separate shape finding from mathematical constraints of final output surface

Splines on Meshes with Irregularities  Jorg Peters

(a) control net (central valence $n = 7$)  (b) $G^1$ guide surface  (c) $C^2$ subdivision rings

(e) Catmull-Clark: highlight line distribution and zoom

(f) Guided Subdivision: highlight line distribution and zoom
Separate shape-finding from mathematical constraints of final output surface

Evolution: Subdivision $\rightarrow$ Guided Subdivision

CC-net, $n = 5$

Eg: G-spline

bicubic ring

guide

Splines on Meshes with Irregularities  Jorg Peters
Separate shape-finding from mathematical constraints of final output surface.
Separate shape-finding from mathematical constraints of final output surface.

Subdivision $\rightarrow$ Guided Subdivision

CC-net, $n = 5$
B-spline like

bicubic ring + guide

guides rings

highlight lines
Guided Subdivision

Shape obstacle course

Splines on Meshes with Irregularities  Jorg Peters

(a) \( n = 5 \)
(b) \( n = 5 \)
(c) \( n = 6 \)
(d) \( n = 7 \)

(a) 4 bi-5 guided rings
(b) highlight lines
(c) Gauss curvature
(d) 4 bi-6 guided rings
(e) 6 guided rings
(f) highlight lines

(g) zoom of (d): highlights and mean curvature
Guided Subdivision

Shape obstacle course

Splines on Meshes with Irregularities

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(d) \( n = 7 \)

3 beams

2 beams

(a) 4 guided rings of degree bi-6

(b) zoom; mean curvature

(c) 4 guided rings of degree bi-5

(d) zoom; Gauss curvature
Guided Subdivision

(f) $n = 9$

(e) $n = 8$

(b) 5 guided rings, $n = 9$, bi-5

(a) 4 guided rings, $n = 8$, bi-5

(c) 3 guided rings, $n = 3$, bi-5, Gauss curvature
Guided Subdivision

Stationary subdivision with larger, denser stencil than CC
Guided Subdivision: refinable d.o.f. for analysis

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(a) d.o.f. of bi-5 $C^2$ ring

(b) $f_1$

(c) $f_2$

(d) $f_3$
Guided Subdivision

Pro:
Semi-structured control nets
Uniform refinability: all transitions $C^k$
Good shape (guided)

Con:
Not finite
Can choose contraction speed!
Evolution

subdivision $\Rightarrow$ refinable $C^2$ surfaces;
guided subdivision $\Rightarrow$ good highlight line distribution;
accelerated guided subdivision $\Rightarrow$ essentially finite!
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Recommendation for glGA:
Smooth polynomial spaces with semi-structured layout for design & analysis

accelerated guided subdivision + G-cap
smaller than max refinement
Semi-structured splines for design & analysis

Smooth polynomial spaces with semi-structured layout for design & analysis

Gaussian smile

https://www.cise.ufl.edu/research/SurfLab/pubs.shtml