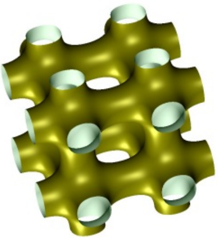
Curvature continuous bi-4 constructions for scaffold- and sphere-like surfaces

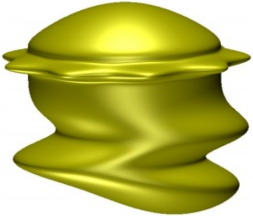
Kęstutis Karčiauskas

Vilnius University

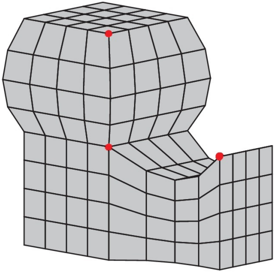
Jörg Peters

University of Florida





Completion (filling the holes) of B-spline surfaces



B-spline mesh

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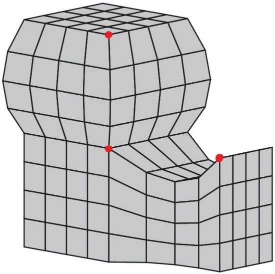

B-spline mesh

regular bi-3 surface

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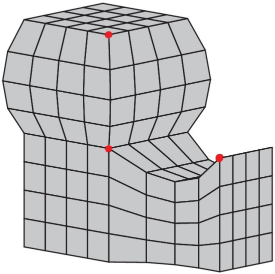

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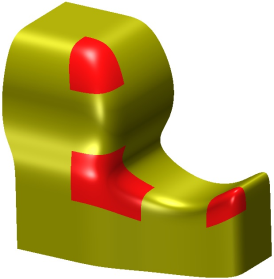
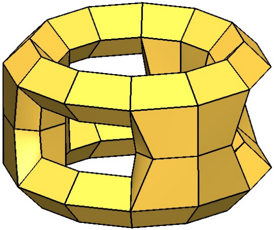


completion with multi-sided caps

Surfaces from Minimal Single Valence meshes

Minimal Single Valence (MSV) mesh := quad mesh where
(Minimal) each quad containes exactly one extraordinary node (eon);
(Single Valence) all eons have the same valence n.



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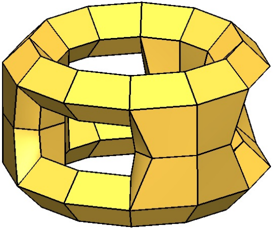

MSV mesh

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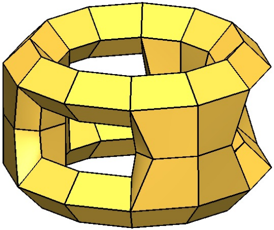
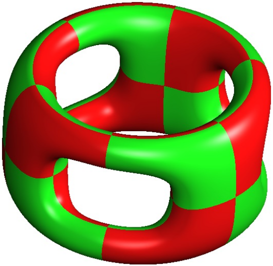


MSV mesh

regular bi-3 surface

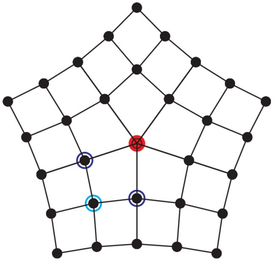


completion with multi-sided caps

Outline

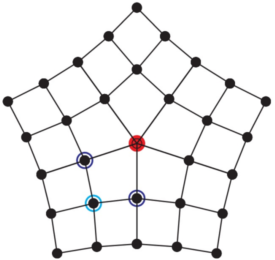
1. MSV meshes and surfaces
2. Examples
3. Sphere-like surfaces
4. Discussion

Cattmull-Clark (CC) nets

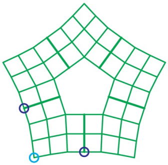


CC net

Cattmull-Clark (CC) nets

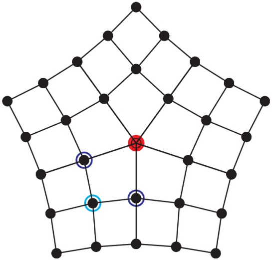


CC net



second-order
Hermite data

Cattmull-Clark (CC) nets



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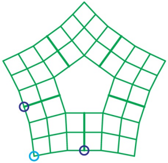

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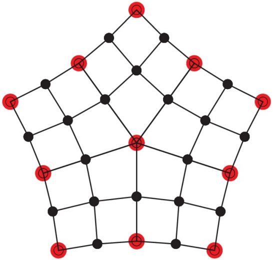
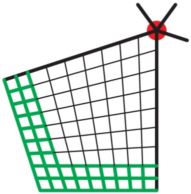


MSV net

Polynomial G^2 constructions of good quality



bi-9: X. Ye 1997;
P. Kiciak 2013

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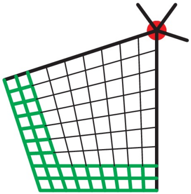

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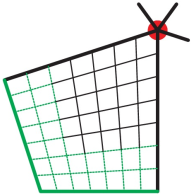

bi-7: C. Loop &
S. Schaefer 2008

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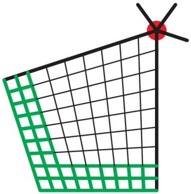

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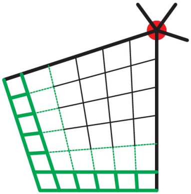
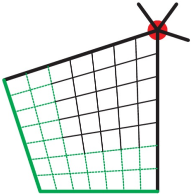


bi-6: KP 2016



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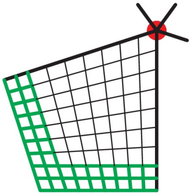

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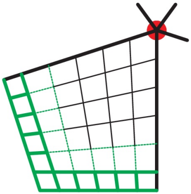

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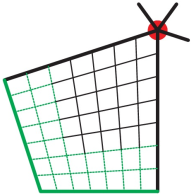

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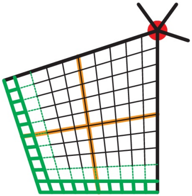
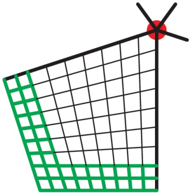


bi-5: KP 2013

Polynomial G^2 constructions of good quality



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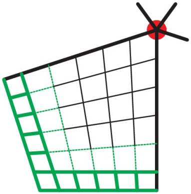

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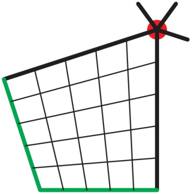
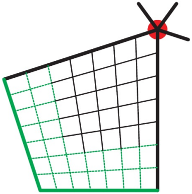


bi-5: SPM16



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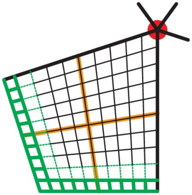

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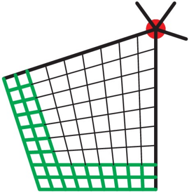

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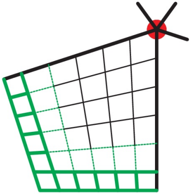

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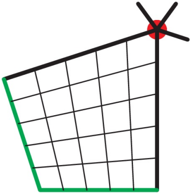

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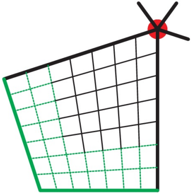

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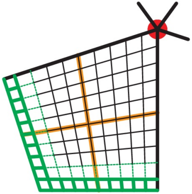

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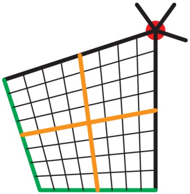
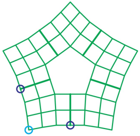


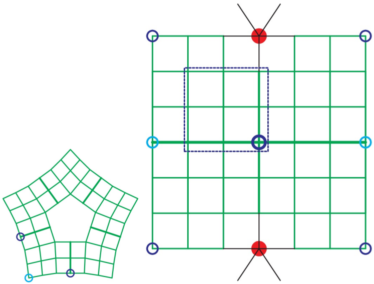
bi-4: SPM16

Second-order smoothness constraints



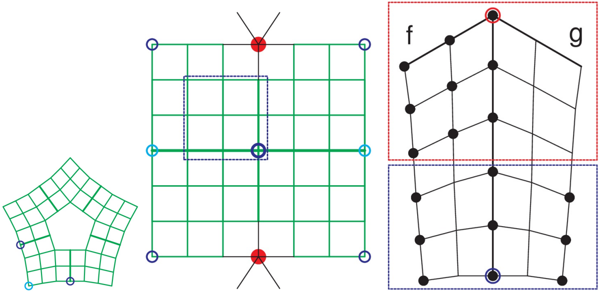
around eon

Second-order smoothness constraints



around eon between eons

Second-order smoothness constraints

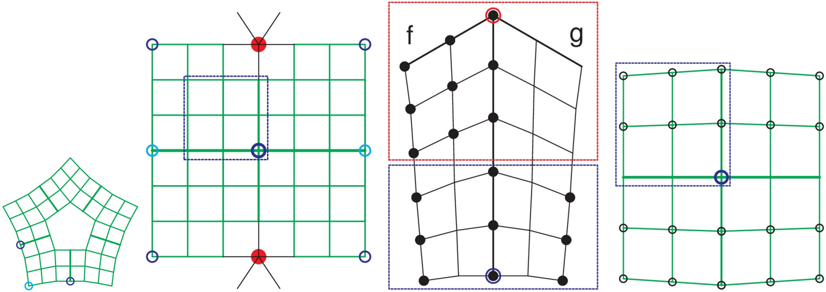


around eon   between eons G^2 constraints
u-direction $\circ \to \circ$

$$g_v = -f_v + 2c(1-u)f_u, \quad c := \cos\frac{2\pi}{n};$$

$$g_{vv} = f_{vv} - 4c(1-u)uf_{uv} + 4c^2(1-u)^2 f_{uu} - 4c^2(1-u)uf_u\ .$$

Second-order smoothness constraints



around eon | between eons | G^2 constraints u-direction $\circ \to \circ$ | modification

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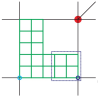
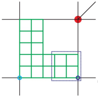
Key steps of construction



1. Degree raise; boundaries will stay fixed.

Key steps of construction



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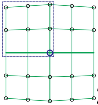
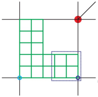


2. Symmetrically modify the black circled Bézier coefficients (10 free parameters).

Key steps of construction



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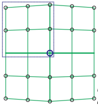

2. Symmetrically modify the black circled Bézier coefficients (10 free parameters).

3. Hard symbolic computation $\Rightarrow$ linear condition on free parameters $\Rightarrow$ system of G^2 constraints becomes solvable.

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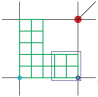

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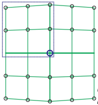

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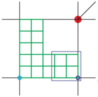
4. Reduce 9 of remaining free parameters to 3.

Key steps of construction



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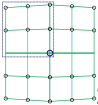

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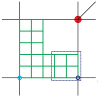
5. Fix remaining 3 parameters to optimize shape.

Key steps of construction



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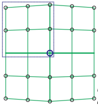

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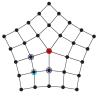
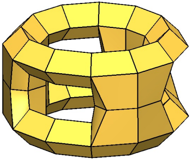
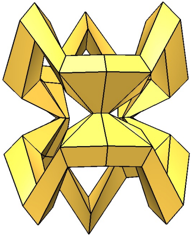
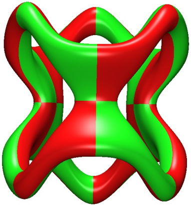


Implementation. Pre-calculated generating functions.

Direct design with MSV meshes ($n = 5$)

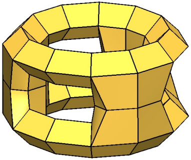
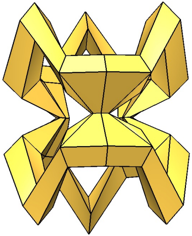
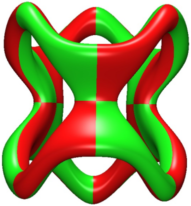






genus 4: mesh perturbed mesh surface

Direct design with MSV meshes ($n = 5$)

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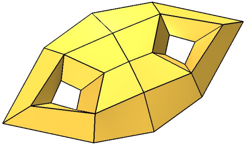
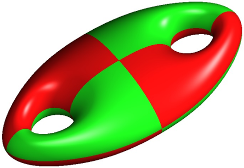
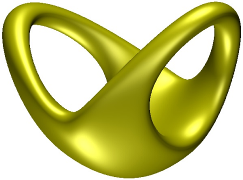
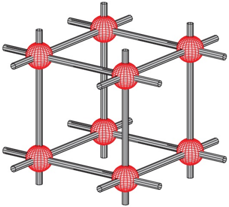




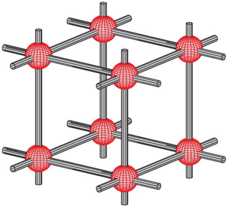
genus 2: mesh surface perturbation

Scaffolds as thick wireframes

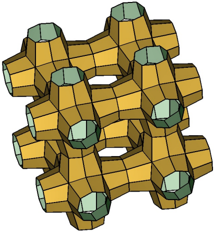


balls & sticks

Scaffolds as thick wireframes

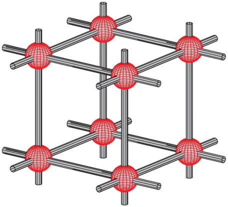


balls & sticks

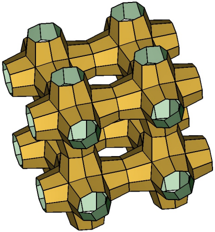


MSV mesh ($n = 6$)

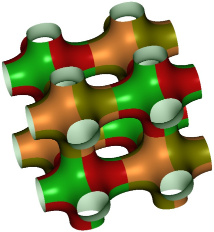
Scaffolds as thick wireframes

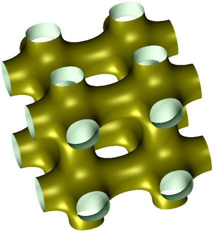


balls & sticks

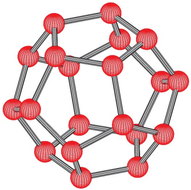


MSV mesh ($n = 6$)



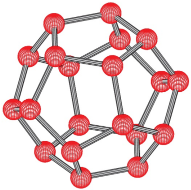


Dodecahedron-like scaffolds

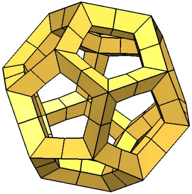


wireframe

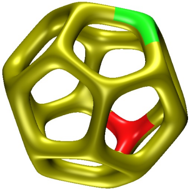
Dodecahedron-like scaffolds



wireframe

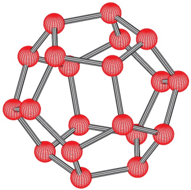


MSV mesh ($n = 6$)

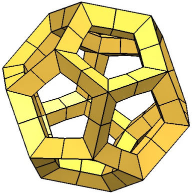


surface

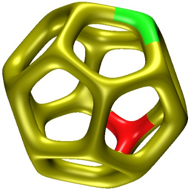
Dodecahedron-like scaffolds



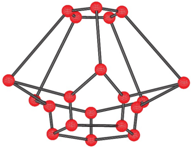
wireframe



MSV mesh ($n = 6$)

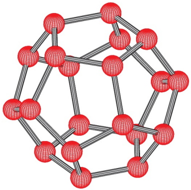


surface

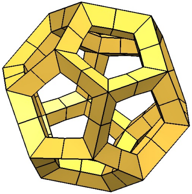


perturbation

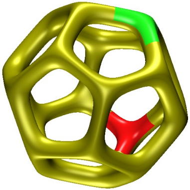
Dodecahedron-like scaffolds



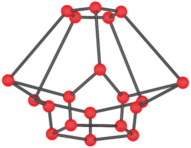
wireframe



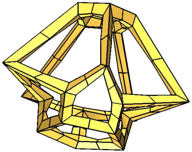
MSV mesh ($n = 6$)

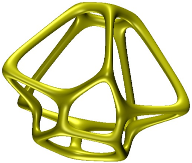


surface

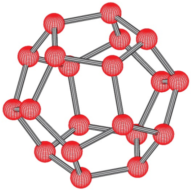


perturbation

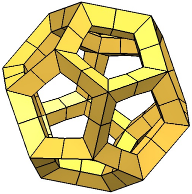




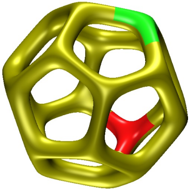
Dodecahedron-like scaffolds



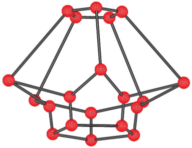
wireframe



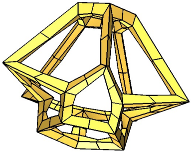
MSV mesh ($n = 6$)

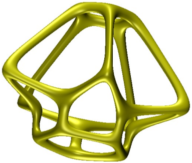


surface



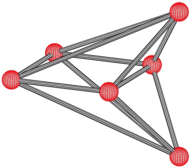
perturbation





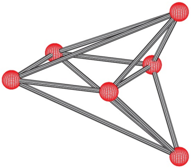
$$genus = \frac{V(n-4)}{8} + 1\,, \qquad \text{V} - \text{number of eons}$$

Perturbed octahedron, icosahedron

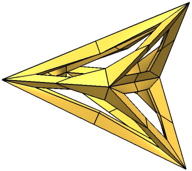


perturbed octahedron

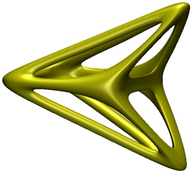
Perturbed octahedron, icosahedron



perturbed octahedron

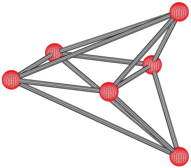


MSV mesh ($n = 8$)

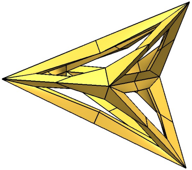


surface

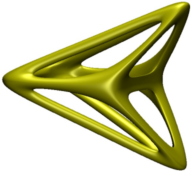
Perturbed octahedron, icosahedron



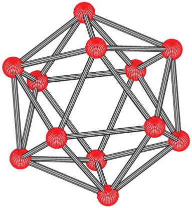
perturbed octahedron



MSV mesh ($n = 8$)

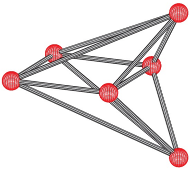


surface

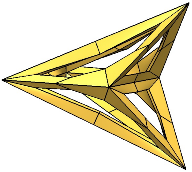


icosahedron

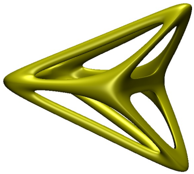
Perturbed octahedron, icosahedron



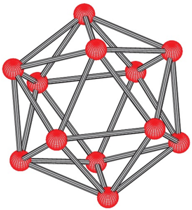
perturbed octahedron



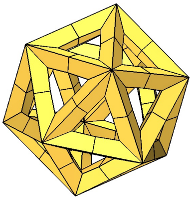
MSV mesh ($n = 8$)



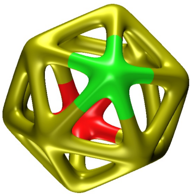
surface



icosahedron

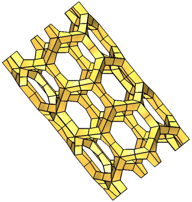


MSV mesh ($n = 10$)

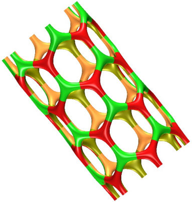


surface

Scaffolds from cylinder

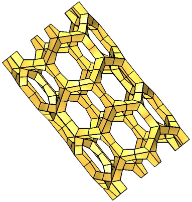


MSV mesh ($n = 6$)

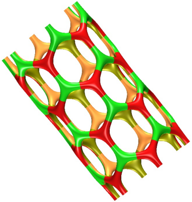


surface

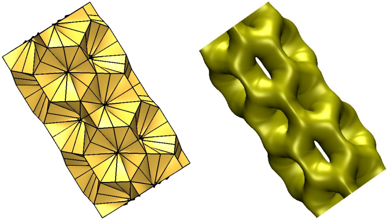
Scaffolds from cylinder



MSV mesh ($n = 6$)

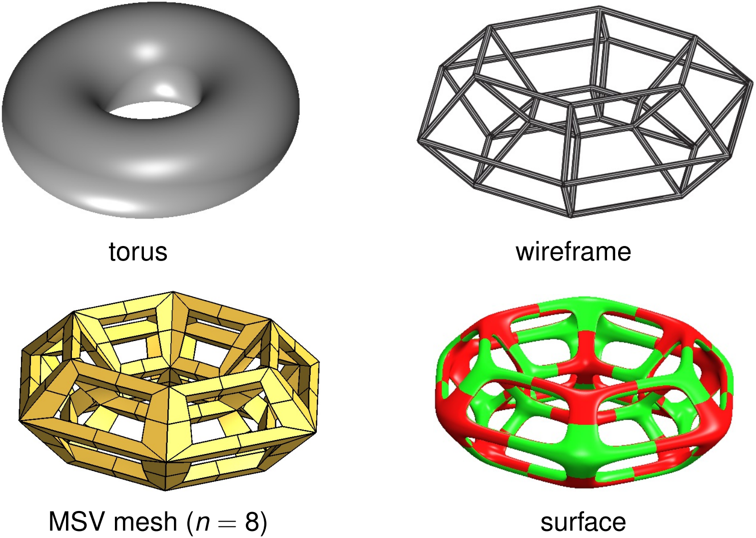


surface



thickening

Scaffolds from torus

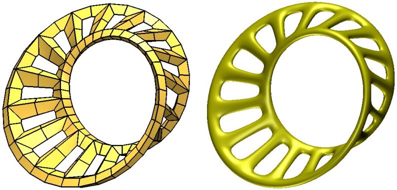


Strips (demo for topology course)

inspiration

Strips (demo for topology course)

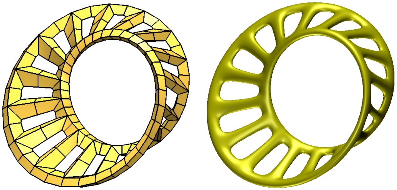
inspiration



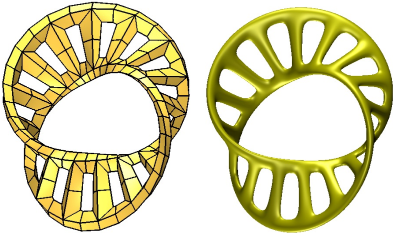
twisted strip

Strips (demo for topology course)

inspiration



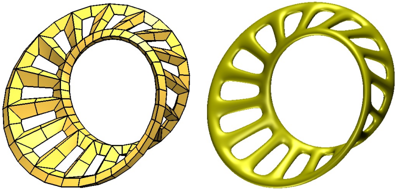
twisted strip



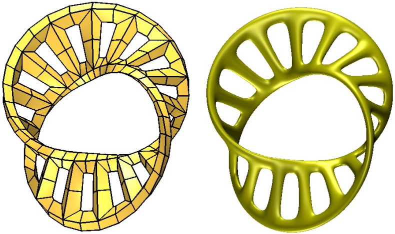
double twist

Strips (demo for topology course)

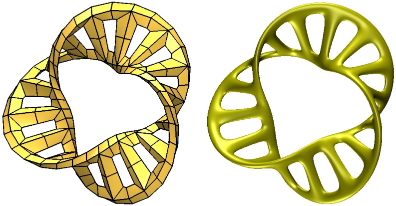
inspiration



twisted strip

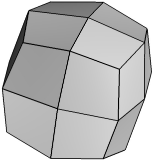


double twist



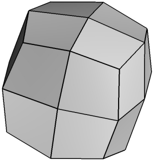
triple twist

Polar structure and examples

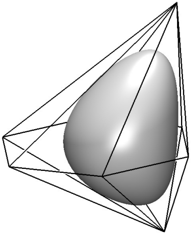


MSV mesh $n = 3$

Polar structure and examples

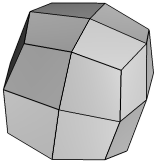


MSV mesh $n = 3$

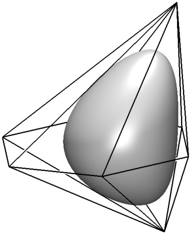


polar mesh

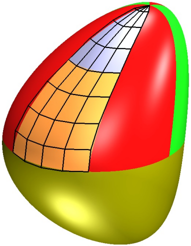
Polar structure and examples



MSV mesh $n = 3$

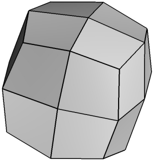


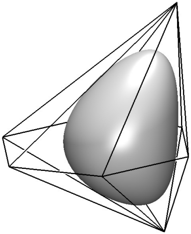
polar mesh

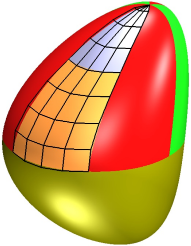


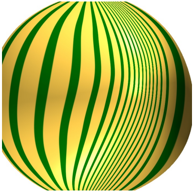
structure

Polar structure and examples

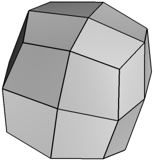

MSV mesh $n = 3$

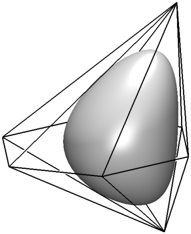

polar mesh

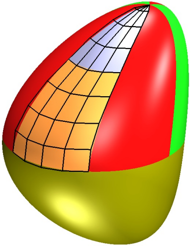

structure

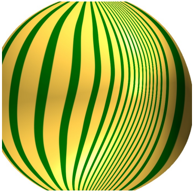

top view

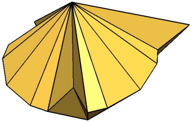
Polar structure and examples

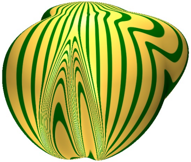

MSV mesh $n = 3$


polar mesh

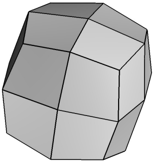

structure


top view

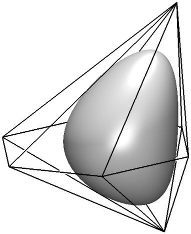

minimal mesh


surface

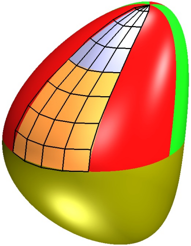
Polar structure and examples



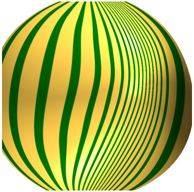
MSV mesh $n = 3$



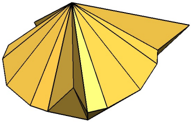
polar mesh



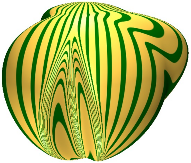
structure



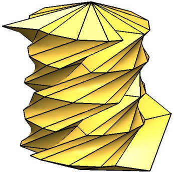
top view



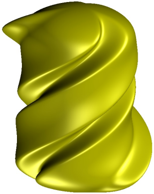
minimal mesh



surface

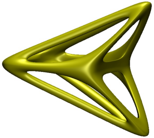


mesh

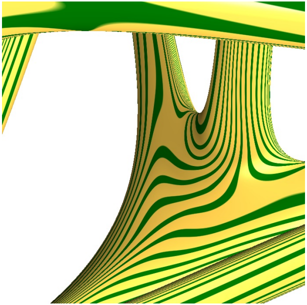


surface

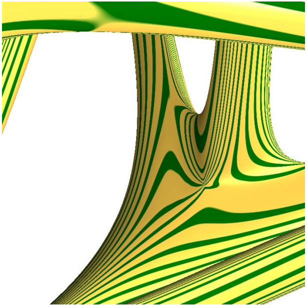
Why not subdivision?



'octahedron'



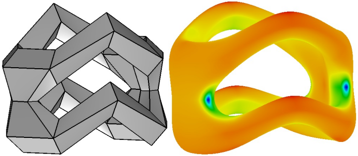
SPM 2016



Catmull-Clark

Relaxed scaffolds

Motivation of MSV surfaces – locally simple scaffold-like surfaces.



Relaxed scaffolds

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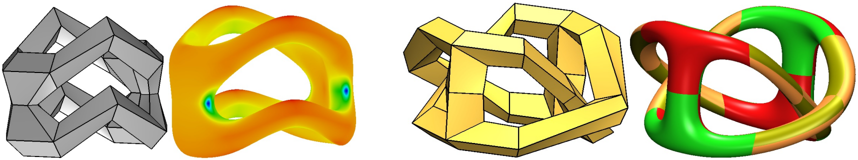
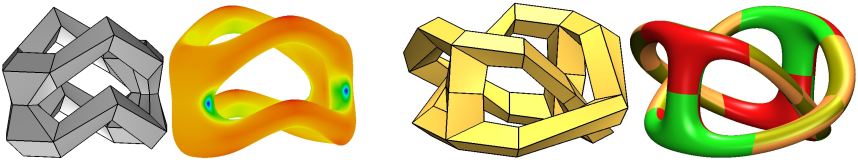


Relaxing M=Minimal we get more flexible surfaces.
For them general constructions are applied $\Rightarrow$ the degree moves up.

Relaxed scaffolds

Motivation of MSV surfaces – locally simple scaffold-like surfaces.



Relaxing M=Minimal we get more flexible surfaces.
For them general constructions are applied $\Rightarrow$ the degree moves up.
Isogeometric analysis MSV surfaces have refinable bi-5 G^2 functions, while for relaxed scaffolds a corresponding degree is bi-6.

Acknowledgments

Is difficult to imagine how new world record
– bi-4 G^2 free-form surfaces of good quality –
could be achieved without a help of true friend Maple.

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SAGA
ShApes Geometry Algebra

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SAGA
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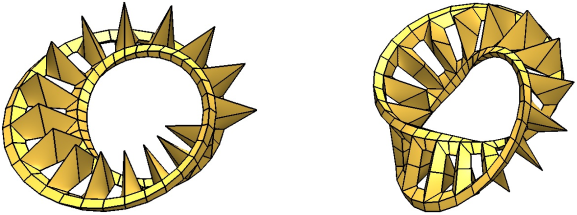
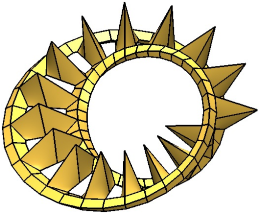
Thank You!

Impact of overtight MSV structure



'energetic' meshes

Impact of overtight MSV structure



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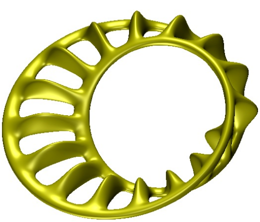

'ignorant' surfaces