Curvature continuous bi-4 constructions for scaffold- and sphere-like surfaces

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Completion (filling the holes) of B-spline surfaces

B-spline mesh
Completion (filling the holes) of B-spline surfaces

B-spline mesh

regular bi-3 surface
Completion (filling the holes) of B-spline surfaces

- B-spline mesh
- Regular bi-3 surface
- Completion with multi-sided caps
Surfaces from Minimal Single Valence meshes

Minimal Single Valence (MSV) mesh := quad mesh where (Minimal) each quad contains exactly one extraordinary node (eon); (Single Valence) all eons have the same valence $n$. 

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MSV mesh

regular bi-3 surface

completion with multi-sided caps
Outline

1. MSV meshes and surfaces
2. Examples
3. Sphere-like surfaces
4. Discussion
1 MSV meshes and surfaces

2 Examples

3 Sphere-like surfaces

4 Discussion
Cattmull-Clark (CC) nets
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CC net

second-order Hermite data
Cattmull-Clark (CC) nets

CC net

second-order Hermite data

MSV net
Polynomial $G^2$ constructions of good quality

bi-9: X. Ye 1997; P. Kiciak 2013
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bi-4: SPM16
Second-order smoothness constraints

around eon
Second-order smoothness constraints

\[
g_{uv} = f_{uv} - 4c(1-u)f_{uu} + 4c^2(1-u)^2f_{uu} - 4c^2(1-u)uf_{uv}
\]

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Second-order smoothness constraints

around eon  between eons  $G^2$ constraints
$u$-direction $\circ \rightarrow \circ$

\[ g_v = -f_v + 2c(1 - u)f_u, \quad c := \cos \frac{2\pi}{n}; \]
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Second-order smoothness constraints

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\( G^2 \) constraints
\( u \)-direction \( \circ \rightleftharpoons \circ \)

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Key steps of construction

1. Degree raise; boundaries will stay fixed.
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3. Hard symbolic computation $\Rightarrow$ linear condition on free parameters $\Rightarrow$ system of $G^2$ constraints becomes solvable.
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4. Reduce 9 of remaining free parameters to 3.
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Implementation. Pre-calculated generating functions.
1. MSV meshes and surfaces

2. Examples

3. Sphere-like surfaces

4. Discussion
Direct design with MSV meshes ($n = 5$)

- genus 4: mesh
- perturbed mesh
- surface
Direct design with MSV meshes ($n = 5$)

genus 4: mesh

perturbed mesh

surface

genus 2: mesh

surface

perturbation
Scaffolds as thick wireframes

balls & sticks
Scaffolds as thick wireframes

balls & sticks

MSV mesh ($n = 6$)
Scaffolds as thick wireframes

balls & sticks

MSV mesh \( (n = 6) \)
Dodecahedron-like scaffolds

wireframe
Dodecahedron-like scaffolds

- Wireframe
- MSV mesh \((n = 6)\)
- Surface

Examples

Curvature continuous bi-4
Dodecahedron-like scaffolds

- wireframe
- MSV mesh \((n = 6)\)
- surface

\[ \text{genus} = V - \left( n - 4 \right) - 1 + 2g \]

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Curvature continuous bi-4
Dodecahedron-like scaffolds

wireframe

MSV mesh ($n = 6$)

surface

perturbation
Dodecahedron-like scaffolds

wireframe

MSV mesh ($n = 6$)

surface

perturbation

\[ \text{genus} = \frac{V(n - 4)}{8} + 1, \quad V - \text{number of eons} \]
Perturbed octahedron, icosahedron

perturbed octahedron
Perturbed octahedron, icosahedron

perturbed octahedron

MSV mesh \((n = 8)\)

surface
Perturbed octahedron, icosahedron

perturbed octahedron

MSV mesh \((n = 8)\)

surface

icosahedron
Perturbed octahedron, icosahedron

perturbed octahedron  MSV mesh \((n = 8)\)  surface

icosahedron  MSV mesh \((n = 10)\)  surface
Examples

Scaffolds from cylinder

MSV mesh \((n = 6)\)  surface
Scaffolds from cylinder

MSV mesh ($n = 6$)  surface  thickening
Scaffolds from torus

- Torus
- Wireframe
- MSV mesh ($n = 8$)
- Surface
Strips (demo for topology course)

inspiration
Examples

Strips (demo for topology course)

inspiration

twisted strip
Strips (demo for topology course)

- Inspiration
- Twisted strip
- Double twist
Strips (demo for topology course)

- Inspiration
- Twisted strip
- Double twist
- Triple twist
1. MSV meshes and surfaces

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Sphere-like surfaces

Polar structure and examples

MSV mesh $n = 3$
Polar structure and examples

MSV mesh $n = 3$  polar mesh
Polar structure and examples

MSV mesh \( n = 3 \)  
polar mesh  
structure
Sphere-like surfaces

Polar structure and examples

MSV mesh $n = 3$

polar mesh

structure

top view
Polar structure and examples

MSV mesh $n = 3$
	polar mesh

structure
top view

minimal mesh

surface
Polar structure and examples

MSV mesh $n = 3$

polar mesh

structure

top view

minimal mesh

surface

mesh

surface
1. MSV meshes and surfaces

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Why not subdivision?

'octahedron'  SPM 2016  Catmull-Clark
Relaxed scaffolds

Motivation of MSV surfaces – locally simple scaffold-like surfaces.
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Relaxing $M=\text{Minimal}$ we get more flexible surfaces. For them general constructions are applied $\Rightarrow$ the degree moves up.
Relaxed scaffolds

Motivation of MSV surfaces – locally simple scaffold-like surfaces.

Relaxing $M=\text{Minimal}$ we get more flexible surfaces. For them general constructions are applied $\Rightarrow$ the degree moves up. **Isogeometric analysis** MSV surfaces have refinable bi-5 $G^2$ functions, while for relaxed scaffolds a corresponding degree is bi-6.
Is difficult to imagine how new world record – bi-4 $G^2$ free-form surfaces of good quality – could be achieved without a help of true friend Maple.
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Thank You!
Impact of overtight MSV structure

'energetic' meshes
Impact of overtight MSV structure

‘energetic’ meshes

‘ignorant’ surfaces