Generalizing bicubic splines for modelling and IGA with irregular layout

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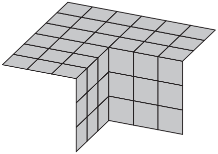
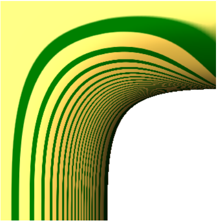
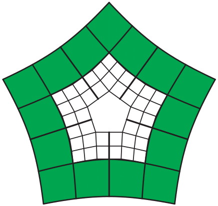
Overview

- Irregularities in a C^2 bi-3 spline surface



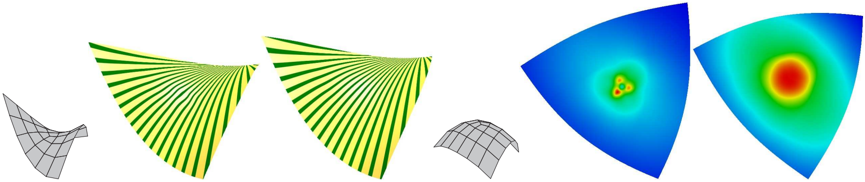




highlight lines

Overview

- Irregularities in a C^2 bi-3 spline surface
- G^1 completion with good distribution of highlight lines, curvature



CC-net G^2 bi-7 G^1 bi-4 G^2 bi-7 G^1 bi-4

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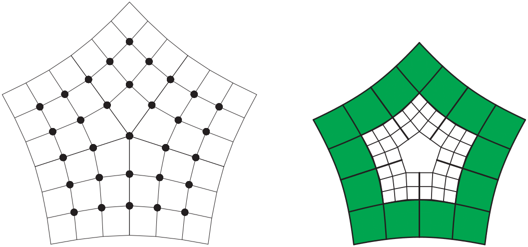
- Degrees of freedom = vertices of the irregular quad mesh



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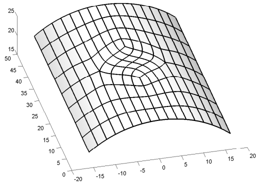
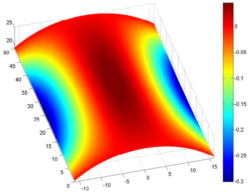
- generalized isogeometric analysis (gIGA) elements





Outline

1. Setup
2. G^1 bi-4 cap construction for $n = 5, 6, 7$
3. Obstacle Course
4. Generalized Isogeometric Analysis (gIGA) elements

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Surface cap

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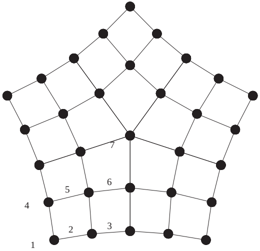

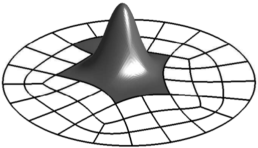

- Polynomial tensor-product pieces **f** of bi-degree 3 in BB-form

$$\mathbf{f}(u,v) := \sum_{i=0}^{d}\sum_{j=0}^{d} \mathbf{f}_{ij} B_i^d(u) B_j^d(v)\,, \quad (u,v) \in \square := [0..1]^2,$$

$B_k^d(t)$ is the kth Bernstein-Bézier polynomial of degree d.

Surface cap

- Smoothly joined to form B-spline-like functions.
- Polynomial tensor-product pieces $\mathbf{f}$ of bi-degree 3 in BB-form
- Geometric Continuity – G^1 constraints for abutting patches $\tilde{\mathbf{f}}$ and $\mathbf{f}$

$$\partial_v \tilde{\mathbf{f}}(u,0) - a(u)\, \partial_v \mathbf{f}(0,u) - b(u)\, \partial_u \mathbf{f}(0,u) = 0$$

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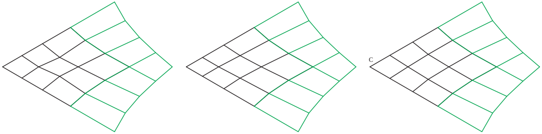
- Minimizing distortion:
  parameterization – minimize functionals
  Characteristic parameterizations σ when $n = 6$



$\mathcal{F}_2$ $\mathcal{F}_{2,3}$ default $\mathcal{F}^3_{2,3}$ map σ_4

$$\mathcal{F}_k f := \int_0^1 \int_0^1 \sum_{i+j=k, i,j \geq 0} \frac{k!}{i!j!} (\partial_s^i f(s,t) \partial_t^j f(s,t))^2 ds dt$$

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- G^k constructions always C^k [Peters 2014, Groisser+P 2015]

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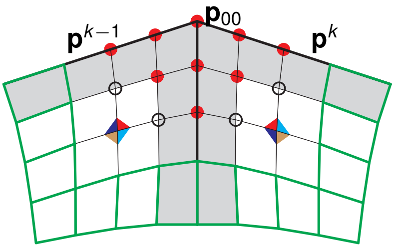
G^1 continuity of the cap



$$\grave{\mathbf{p}}_{10} := \left(1 - \frac{1}{c}\right)\grave{\mathbf{p}}_{00} + \frac{\grave{\mathbf{p}}_{01} + \acute{\mathbf{p}}_{01}}{2c};$$

$$\grave{\mathbf{p}}_{20} := \frac{(3c-4)\grave{\mathbf{p}}_{10} + 2(\grave{\mathbf{p}}_{11} + \acute{\mathbf{p}}_{11})}{3c};$$

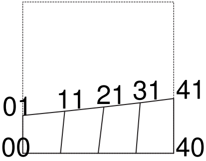
$$\acute{\mathbf{p}}_{21} := (2-c)\grave{\mathbf{p}}_{20} + c\grave{\mathbf{p}}_{30} - \grave{\mathbf{p}}_{21};$$

$$\grave{\mathbf{p}}_{30} := \frac{2(\grave{\mathbf{p}}_{31} + \acute{\mathbf{p}}_{31}) - c\grave{\mathbf{p}}_{40}}{4-c};$$

$$\grave{\mathbf{p}}_{40} := \frac{\grave{\mathbf{p}}_{41} + \acute{\mathbf{p}}_{41}}{2}.$$

G^1 join with the tensor-border **b**

- Reparameterization across the boundary
  $\mathbf{b} \circ \beta$ of degree 4:



$$\mathbf{p}_{11} := \frac{(1-7c)\mathbf{b}_{00} + 3(\mathbf{b}_{10} + \mathbf{b}_{01}) + 9(1-c)\mathbf{b}_{11}}{16(1-c)},$$

$$\mathbf{p}_{21} := \frac{(1-6c)\mathbf{b}_{10} + (1+c)\mathbf{b}_{20} + 3\mathbf{b}_{11} + 3(1-c)\mathbf{b}_{21}}{8(1-c)},$$

$$\mathbf{p}_{31} := \frac{(3-15c)\mathbf{b}_{20} + (1+2c)\mathbf{b}_{30} + 9\mathbf{b}_{21} + 3(1-c)\mathbf{b}_{31}}{16(1-c)},$$

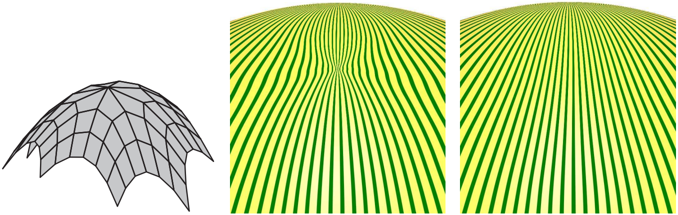
$$\mathbf{p}_{41} := \frac{(1-4c)\mathbf{b}_{30} + 3\mathbf{b}_{31}}{4(1-c)},$$

G^1 join with the tensor-border **b**

- Reparameterization across the boundary $\mathbf{b} \circ \beta$ of degree 4:
- Interleaved G^1 constraints (non-linear) explicitly solved

Construction via a bi-5 guide

- central quadratic from guide surface [KP15] (bi-5):



(a) $n = 8$ (b) Catmull-Clark (c) KP15

Construction via a bi-5 guide

- central quadratic from guide surface [KP15] (bi-5):
- Why not use functionals?

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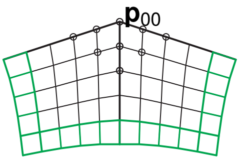
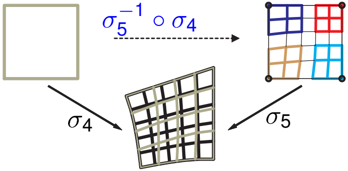


bi-5 guide **g**

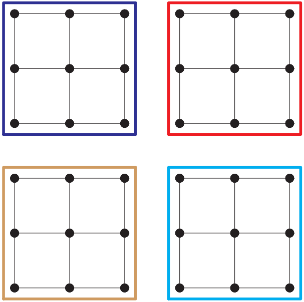


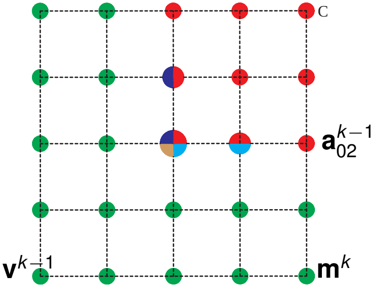
sampling map

Construction via a bi-5 guide

- central quadratic from guide surface [KP15] (bi-5):
- Why not use functionals?

- jets $\mathbf{J}^4_{\mathbf{s},2}(\mathbf{g} \circ \sigma_5^{-1} \circ \sigma_4)$ sampled from the guide $\mathbf{g}$





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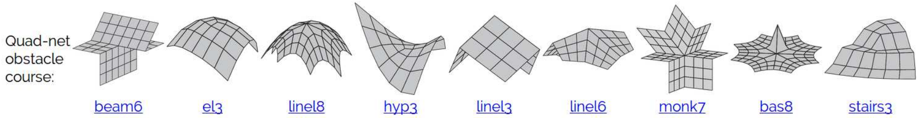
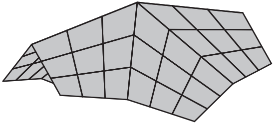
Challenge input

http://www.cise.ufl.edu/research/SurfLab/shape_gallery.shtml

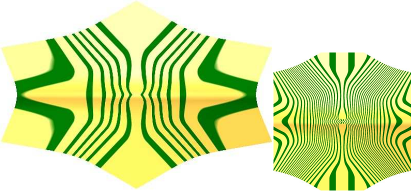


Convex input net

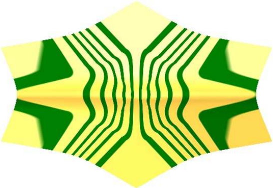
‘deceptively simple’



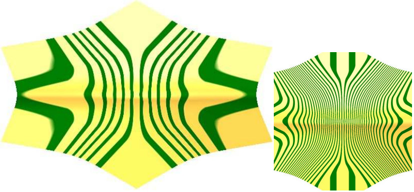
(a) $n = 6$



(b) Catmull-Clark– magnified

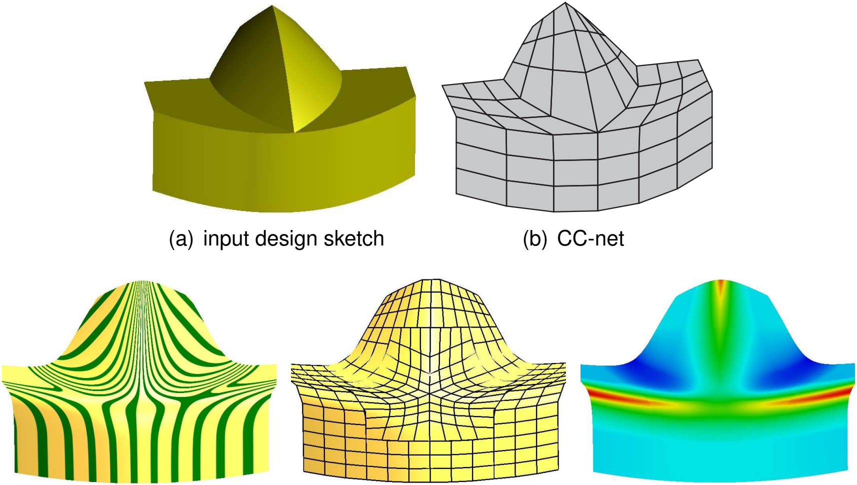


(c) G^2 bi-7



(d) this paper – magnified

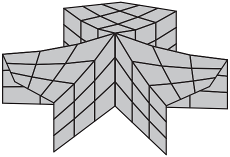
Surface from design sketch



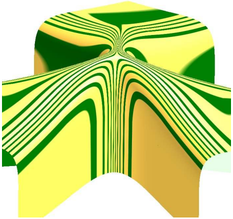
(a) input design sketch (b) CC-net

(c) this paper (d) BB-net (e) mean curvature

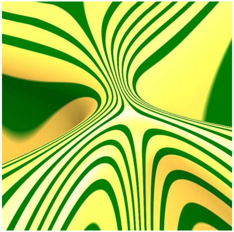
High valence



(a) CC-net $n = 9$



(b) G^1 2×2 cap



(c) cap magnified

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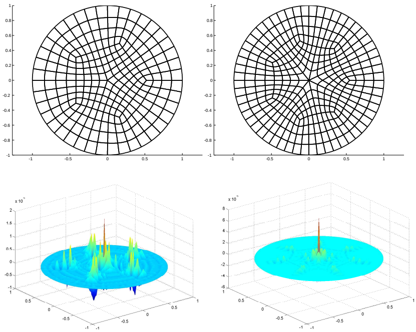
Poisson's equation on the disk

Matched G^k-constructions always yield C^k-continuous isogeometric elements [P14, GP 15]

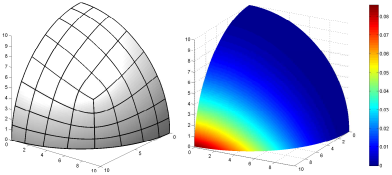


(a) $n = 5$ (b) $n = 7$

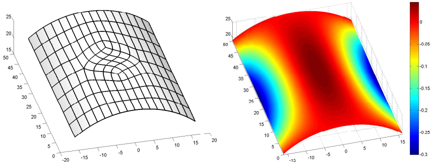
L^2 convergence: higher than $O(h^3)$ L^∞ convergence: higher than $O(h^2)$.

Koiter's shell model, 4th order PDE

Kirchhoff-Love assumptions that lines normal to the 'middle surface' in the original configuration.



(a) Octant of a spherical shell



(b) Scordelis-Lo roof thin shell

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Thank You & Questions?