Can bi-cubic surfaces be class A?

Kęstutis Karčiauskas Jörg Peters

Vilnius University University of Florida

NSF CCF-0728797

Context

- 37 years since Catmull & Clark suggested a simple way to smooth quad meshes by an infinite sequence of nested rings of bi-3 splines.

  CC surfaces are an important modeling tool (often the last stage of the geometry processing pipeline) but

  We know that CC surfaces have shape deficiencies.
- 25 years of geometric continuity: joining (a finite number of) surface pieces by change of variables

  1990s focused on algebraic challenges to keep polynomial degree low – shape secondary
- $<$ 10 years of focus on shape

  G^1 construction of high quality (class A) of degree bi-5
- NEW SGP 2015: bi-3 high quality and sacrifice smoothness (a bit).

  $\rightarrow$ alternative to CC: better shape, few bi-3 pieces.

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Outline

1. What are class A surfaces?
2. Smoothness and low degree – class A?
3. Algorithm
4. Choices and Examples
   - Challenging Input
   - Valence $n = 3$ and $n > 7$
5. Discussion

Class A in the blogosphere

Correct Answer
by Frederic Hodshon on Nov 19, 2014 10:21 AM

here's another good one:

A Class surfacing and its importance: A class surfaces are those aesthetic/ free form surfaces, which are visible to us (interior/exterior), having an optimal aesthetic shape and high surface quality.
Mathematically class A surface are those surfaces which are curvature continuous while providing the simplest mathematical representation needed for the desired shape/form and does not have any undesirable waviness.

- aesthetic visible surface
- no undesirable waviness

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Class A in the blogosphere

Zoran Andjelic about 3 years ago

The distance between each point of the edges of two neighboring patches must comply with the following limits:

- For class A: no more than 0.01 mm.
- For class B: no more than 0.02 mm.
- For class C: no more than 0.05 mm.

Tangents continuity

The angle between the tangents to the surface on the edges of two neighboring patches must comply with the following limits:

- For class A: no more than 6' (0.1◦).

- gap at join $<$ 0.01mm (*SGP 2015 will be watertight)
- angle between normals $<$ 0.1 degrees

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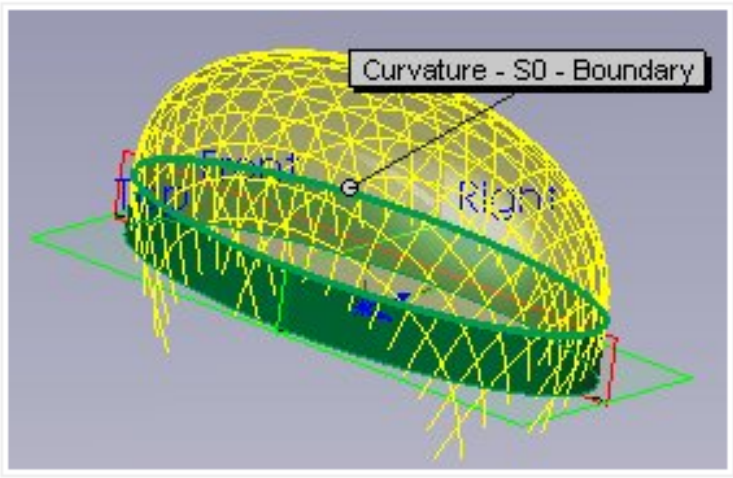
Surface Filling

where do gaps come from?

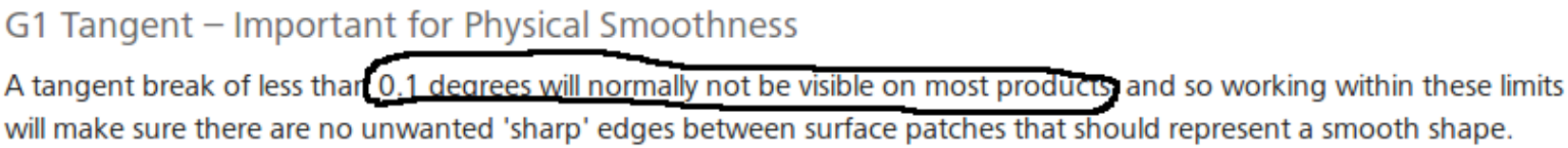
guldo 12:09 pm on September 10, 2012 Permalink | Log in to leave a Comment

What is the correct way to adjust the height or boldness of the dome created when using the Filled Surface tool on an extruded surface? How are the parameters set?
Also, what is the “Constraint Curves” section all about?



Class A in the blogosphere

Autodesk user examples


G1 Tangent – Important for Physical Smoothness

A tangent break of less than 0.1 degrees will normally not be visible on most products, and so working within these limits will make sure there are no unwanted 'sharp' edges between surface patches that should represent a smooth shape.

- no unwanted sharp edges
- angle between normals < 0.1 degrees not visible

Class A in the blogosphere

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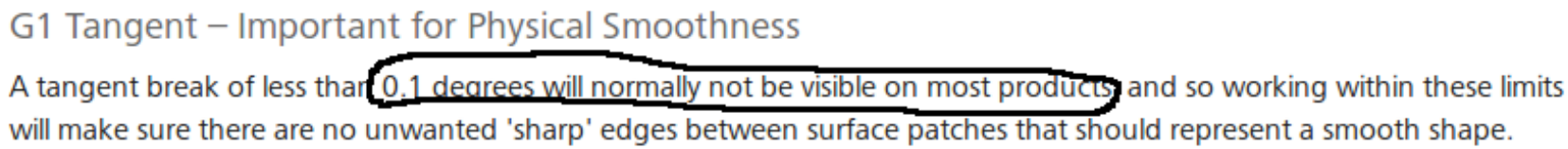
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- no unwanted sharp edges
- angle between normals $<$ 0.1 degrees not visible

Class A in wikipedia

Class A surfaces

From Wikipedia, the free encyclopedia

Class A surfaces is a term used in automotive design to describe a set of freeform surfaces of high efficiency and quality. Although, strictly, it is nothing more than saying the surfaces have curvature and tangency alignment - to ideal aesthetical reflection quality, many people interpret class A surfaces to have G2 (or even G3) curvature continuity to one another (see freeform surface modelling).

- automotive design
- surfaces of high quality (= curvature and tangency *alignment*)
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Reflection lines – highlight lines

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reflection lines

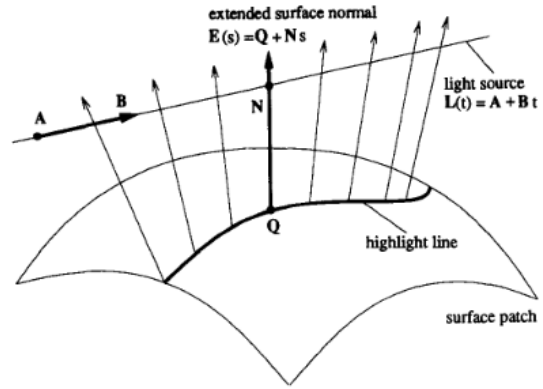


Figure 1 Definition of highlight line

highlight lines [BC94]

Reflection lines → highlight lines

Beier,Chen 94: place zebra stripes above surface;
surface color := color reached by extending point's normal

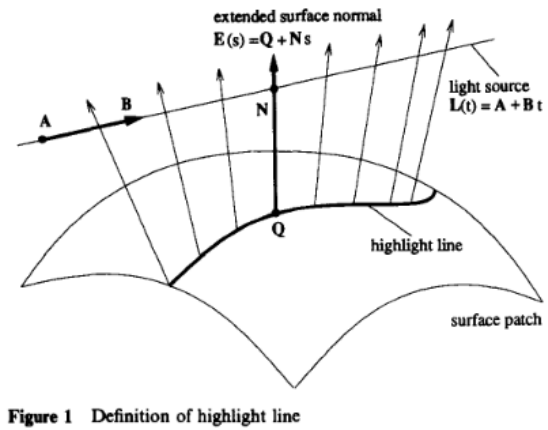
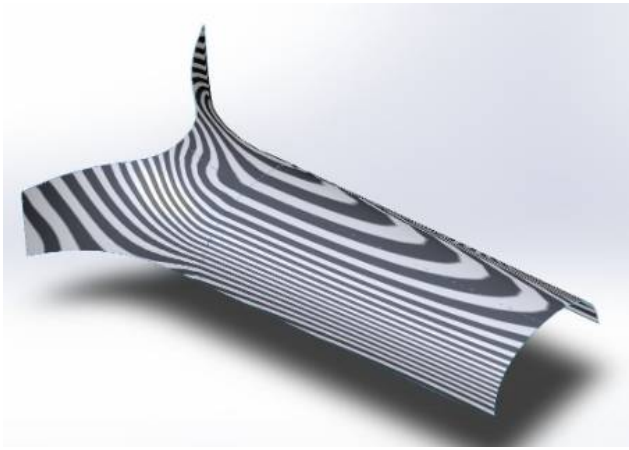


Figure 1 Definition of highlight line

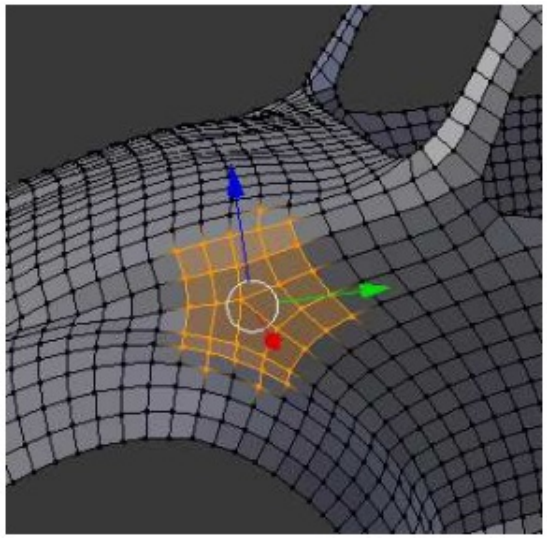
highlight lines



highlight lines

Setup

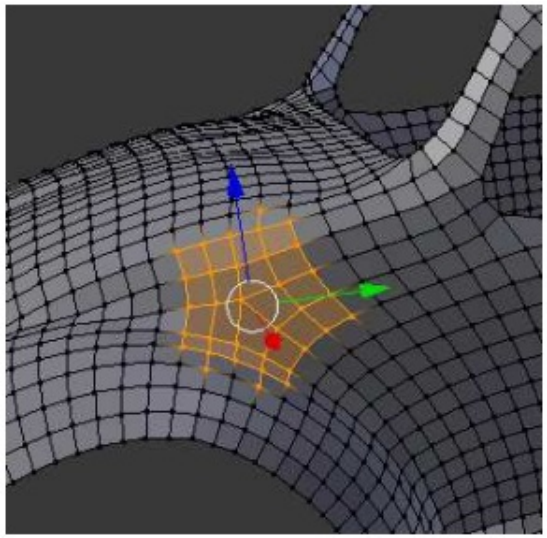
rectangular tensor grid $\rightarrow$ univariate



$n = 5$ net

Setup

rectangular tensor grid $\rightarrow$ univariate $\rightarrow$ focus on irregularities



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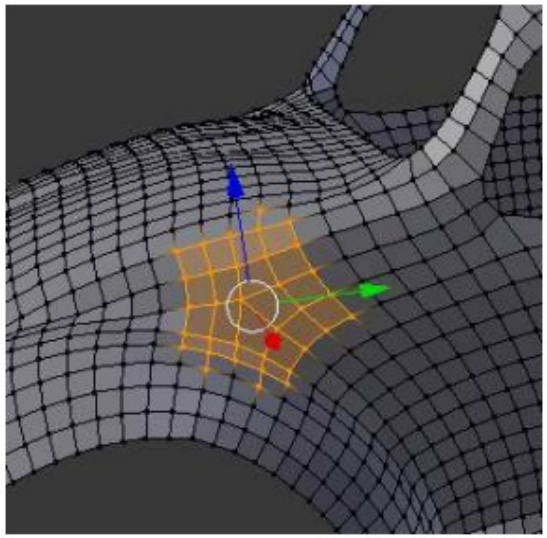

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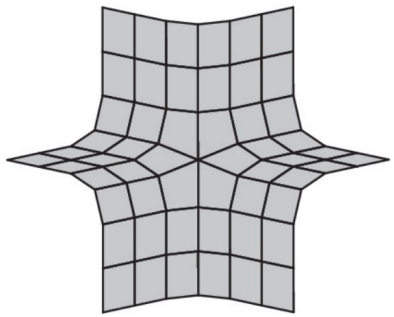

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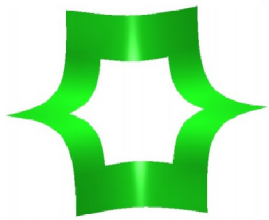

bi-3 ring

Setup

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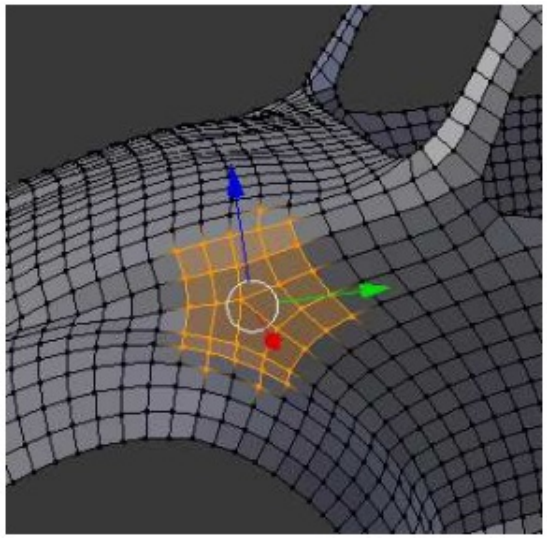

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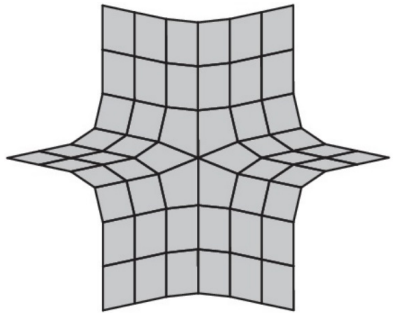

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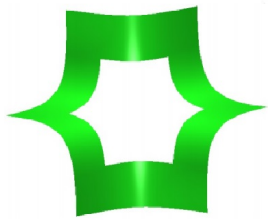
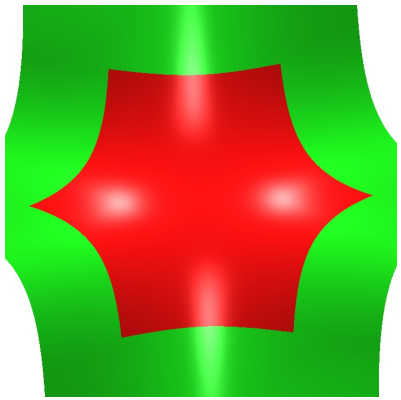
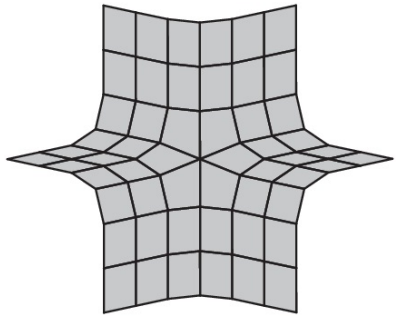


bi-3 ring + cap



Can *smooth* bi-cubic surfaces be class A?



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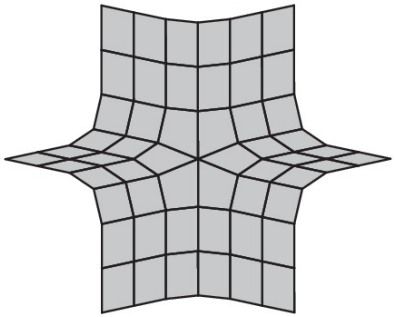

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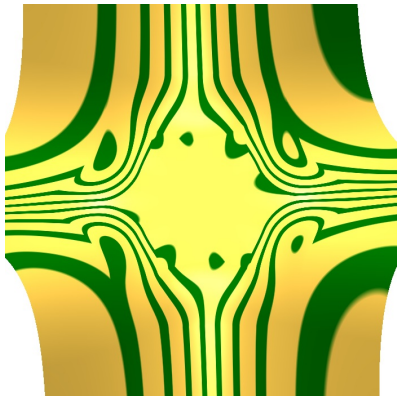

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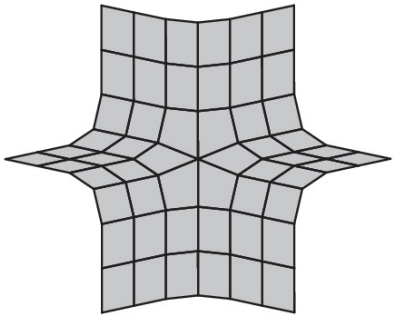

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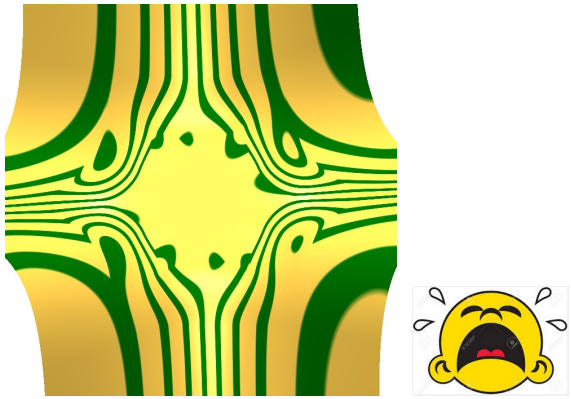

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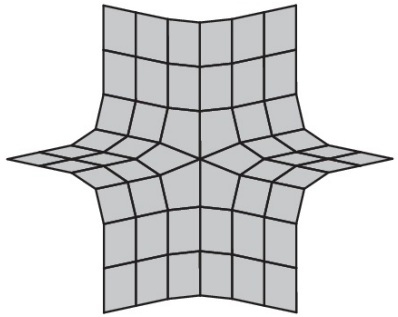

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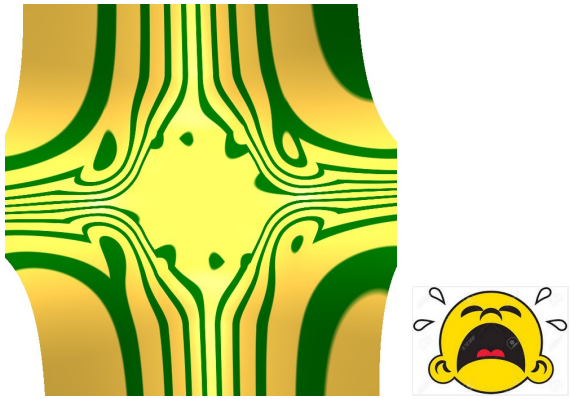

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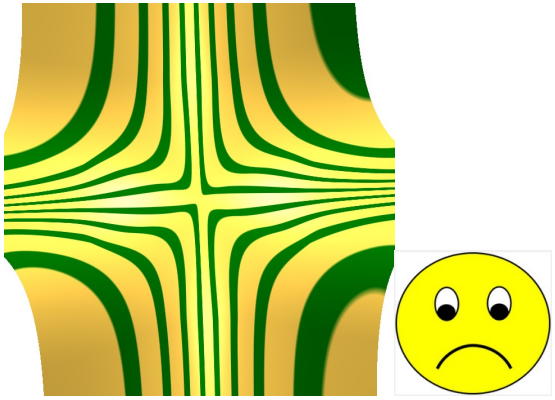

Gregory patches [LSNC09]

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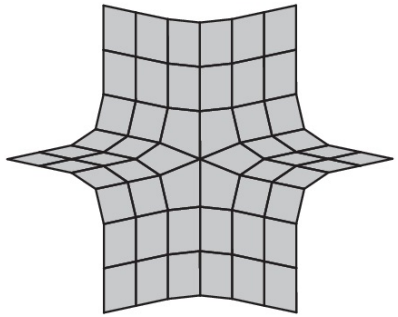

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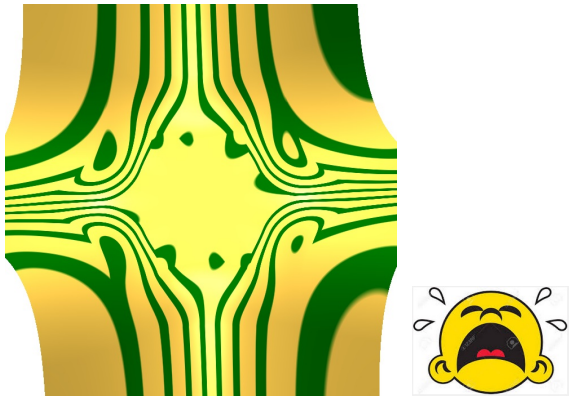

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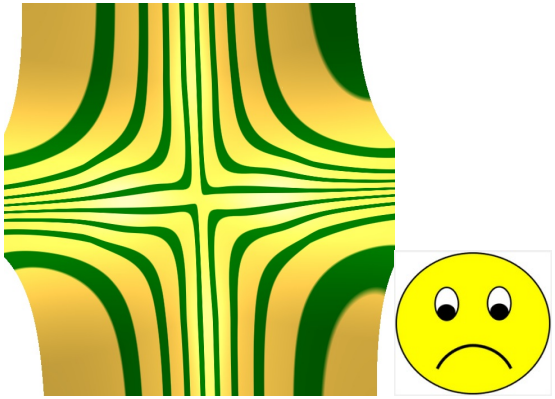

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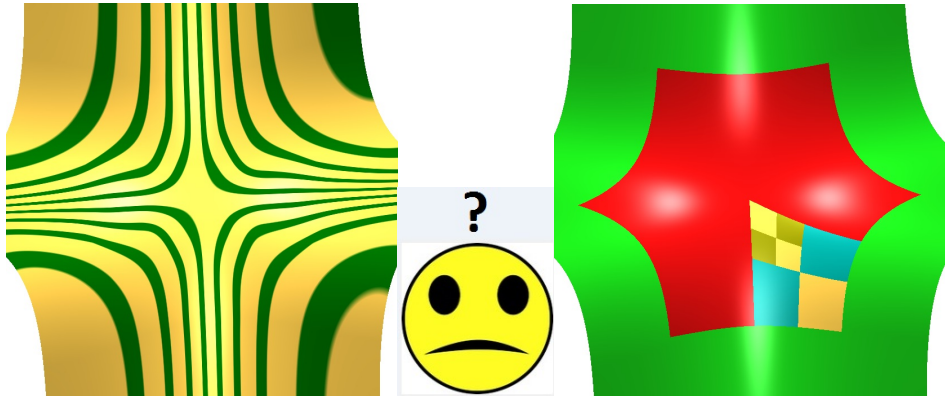
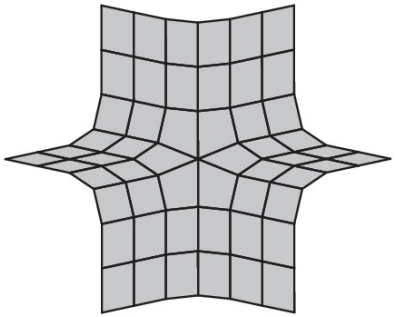


7 bi-3 pieces per sector

Can *smooth* bi-cubic surfaces be class A? NO



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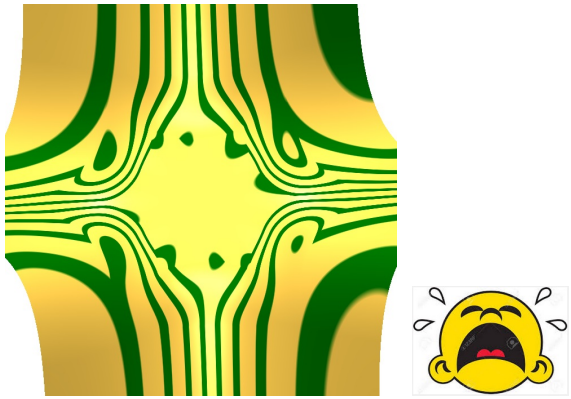

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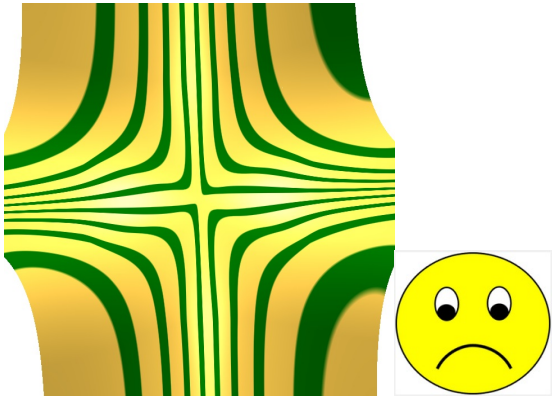

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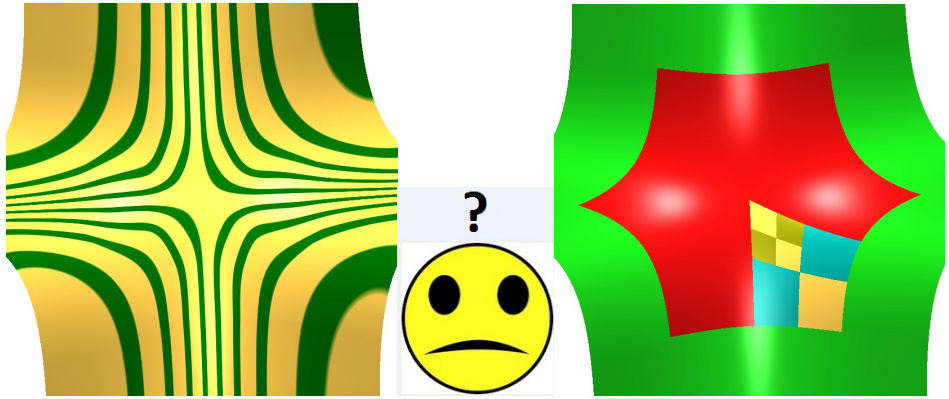
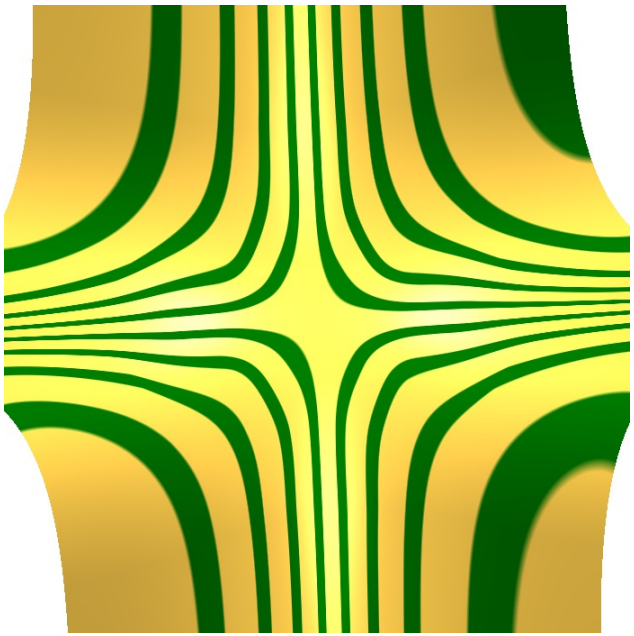


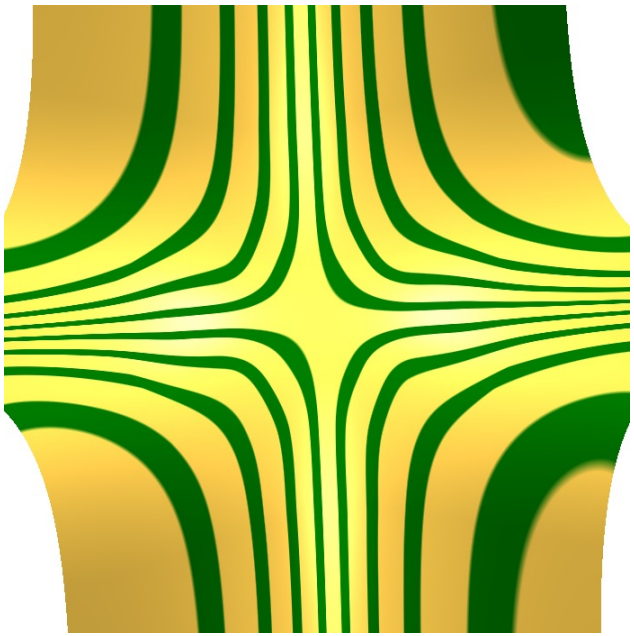
7 bi-3 pieces per sector

Can C^0 bi-cubic surfaces be class A?



best bi-3 G^1 surface

Can C^0 bi-cubic surfaces be class A? YES!



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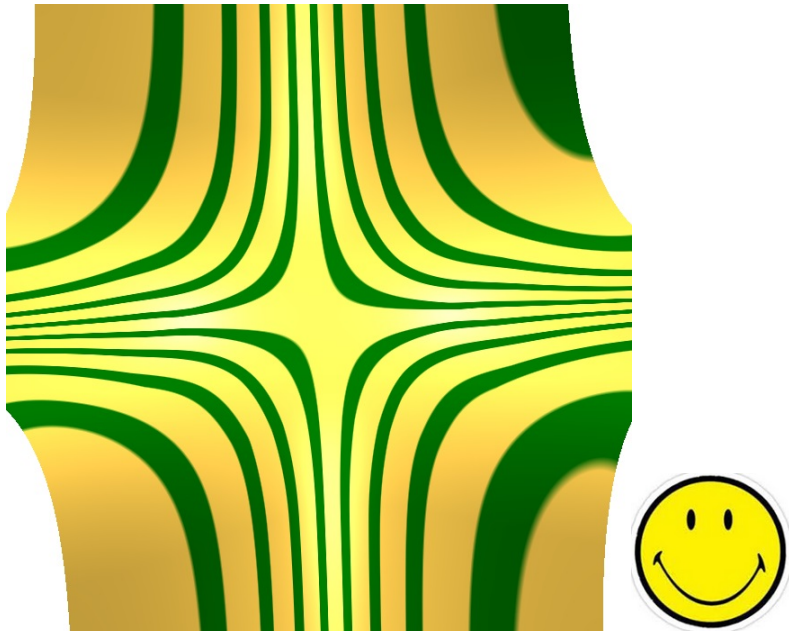
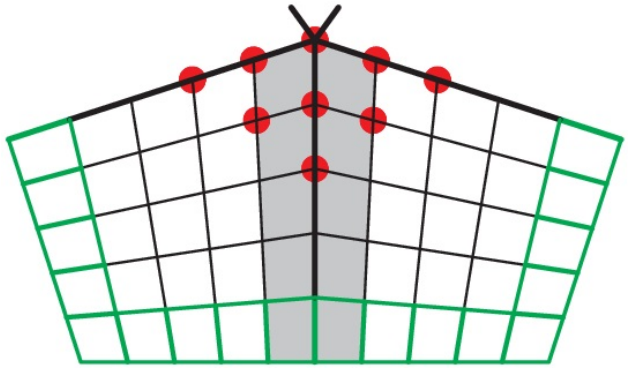


C^0 surface

Smooth (G^1) Bi-5 reference cap

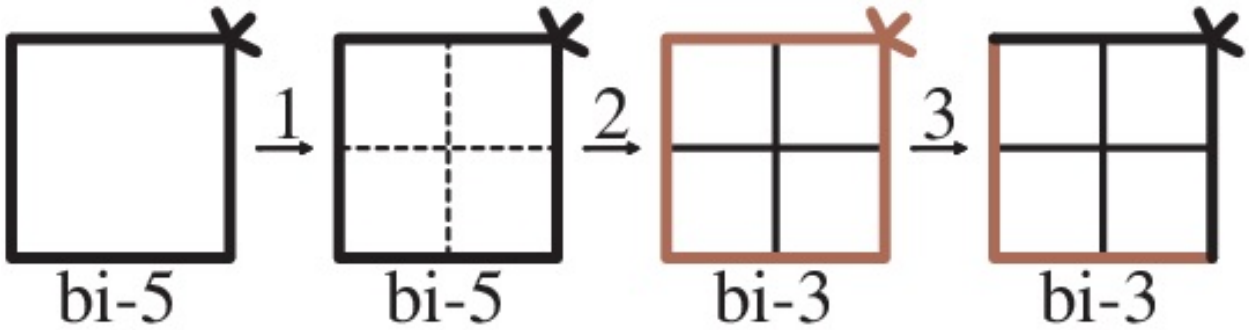
Dag14 [KP]: G^1 surface construction with good curvature distribution:



Gray $\Rightarrow$ G^1 connected bi-5 sectors.
Red $\Rightarrow$ well-defined curvature at extraordinary point.
Green $\Rightarrow$ G^1 (not C^1 as in most constructions) connection to input data.

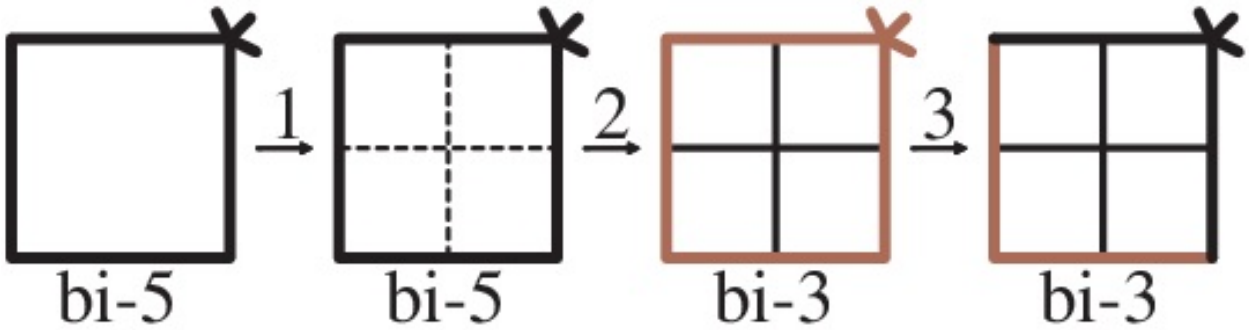
Pipeline

C^0 $\mathbf{G^1}$ C^1



Pipeline

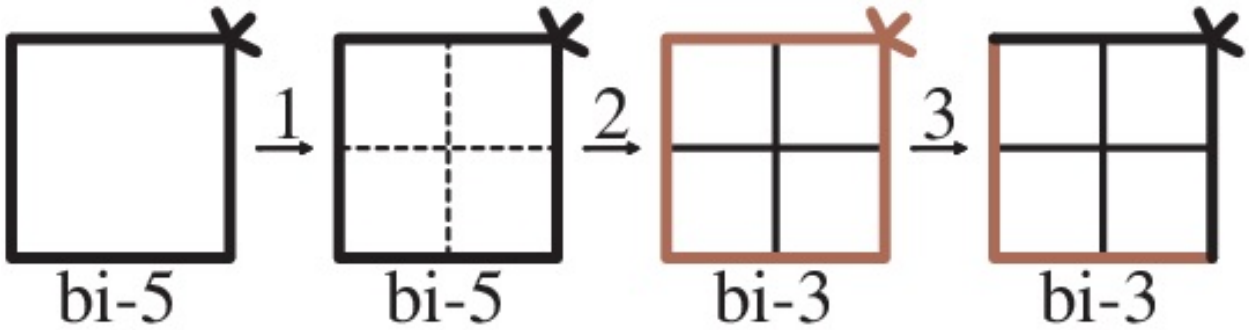
C^0 $\mathbf{G^1}$ C^1



Step 1: deCasteljau split $\Rightarrow$ four bi-5 pieces.

Pipeline

C^0 $\mathbf{G^1}$ C^1



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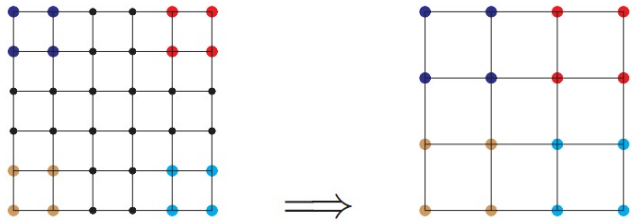
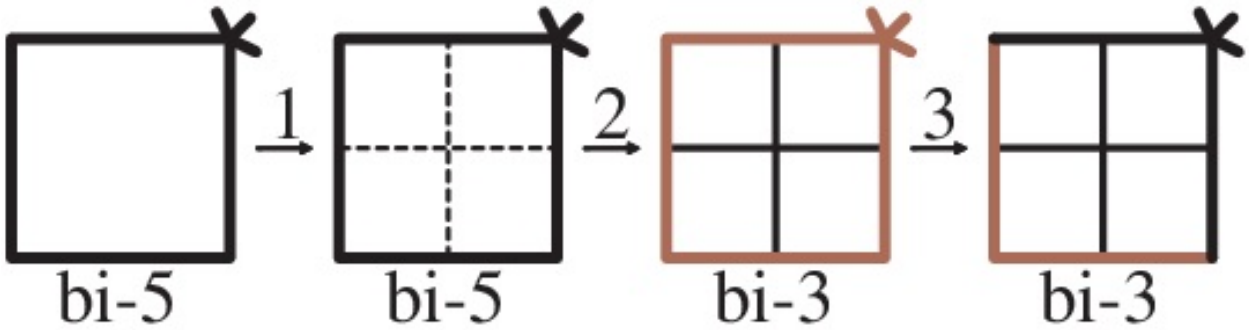


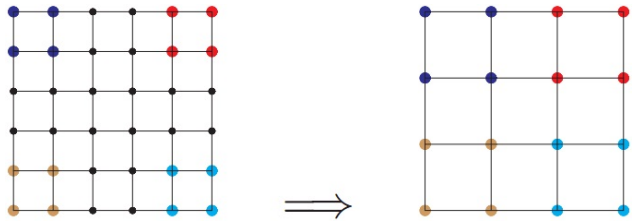
Step 2: bi-5 pieces are transformed to bi-3 patches $\Rightarrow$ C^0 junctions

Pipeline

C^0 $\mathbf{G^1}$ C^1

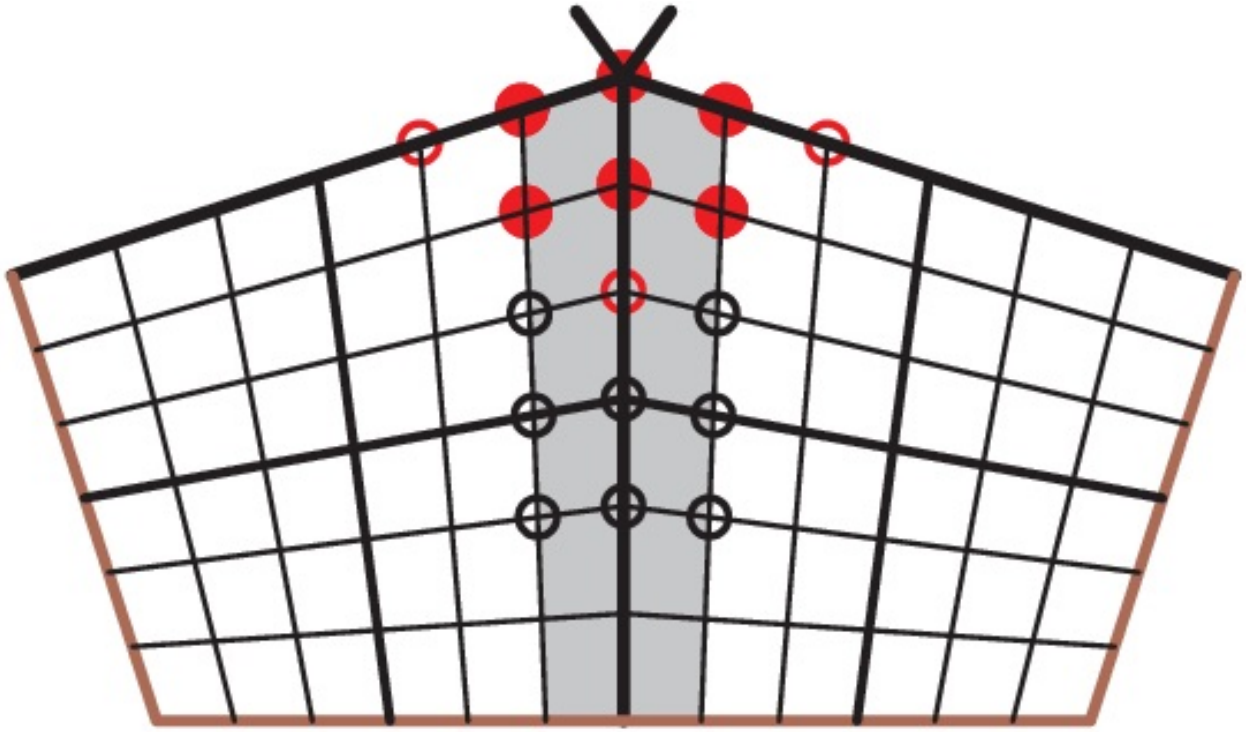


Step 1: deCasteljau split $\Rightarrow$ four bi-5 pieces.



Step 2: bi-5 pieces are transformed to bi-3 patches $\Rightarrow$ C^0 junctions
Step 3: repair internal cap smoothness

Step 3



G^1 between sectors restored by setting circle points.
Red $\Rightarrow$ well-defined curvature at extraordinary point.
Brown: deviation of normals $< 0.1°$.

Claim: $<$ 0.1 degree mismatch

not strictly true, but

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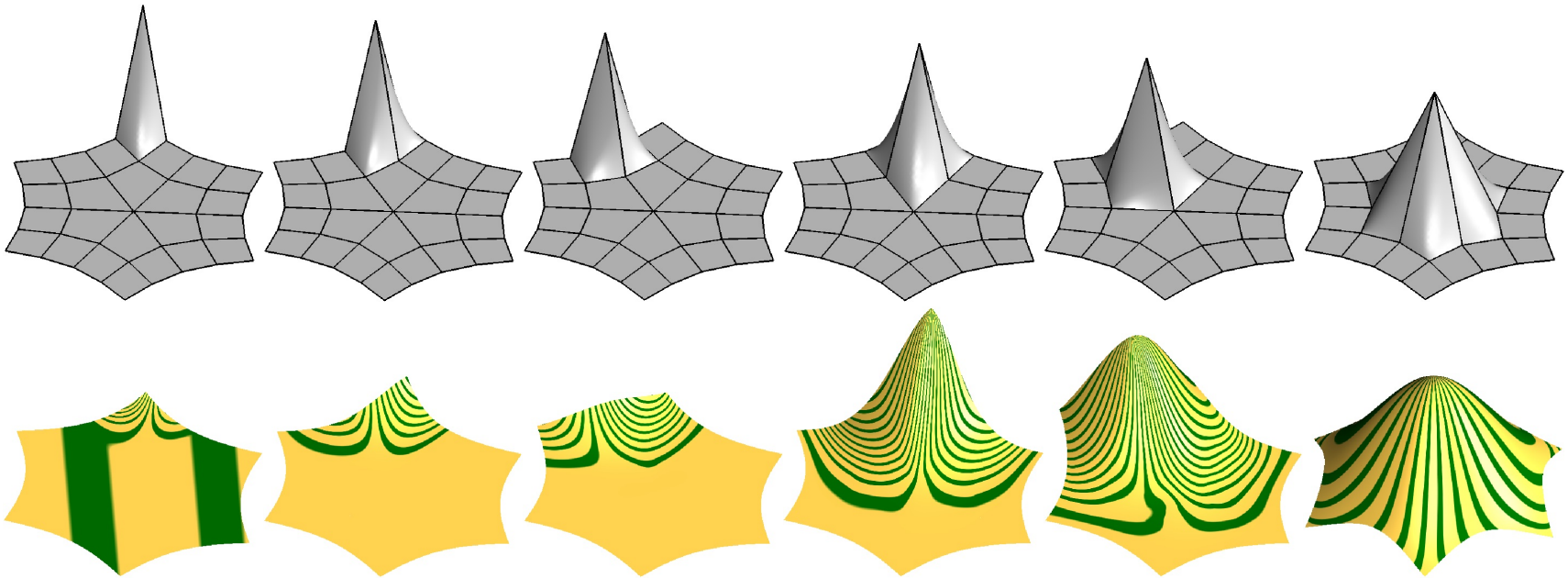
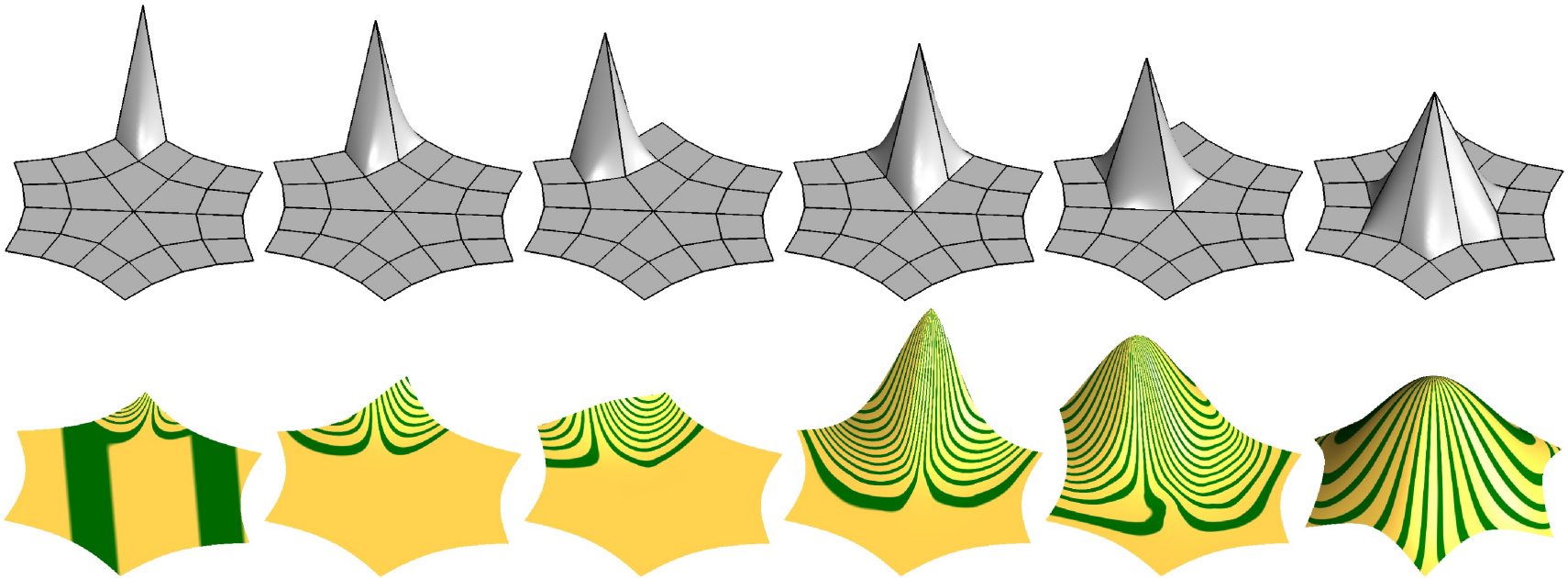
not strictly true, but



Basic functions satisfy $<$ 0.1 degrees (for $n < 60$)

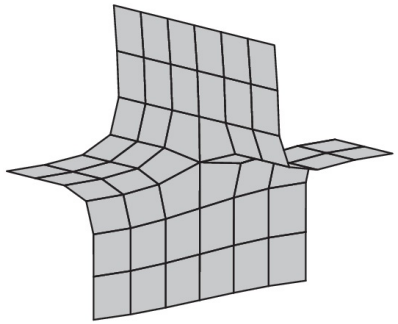
Claim: < 0.1 degree mismatch

not strictly true, but



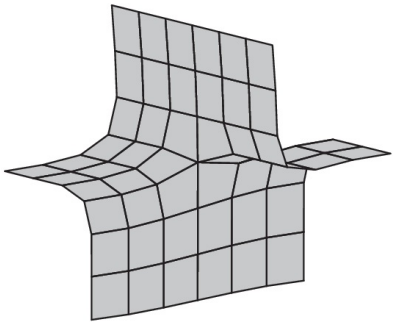
Basic functions satisfy < 0.1 degrees (for $n < 60$)
Obstacle course of challenging nets satisfy < 0.1 degrees

Joining two crossing beams

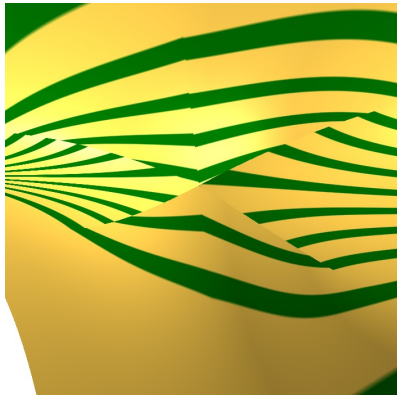


$n = 6$ net

Joining two crossing beams

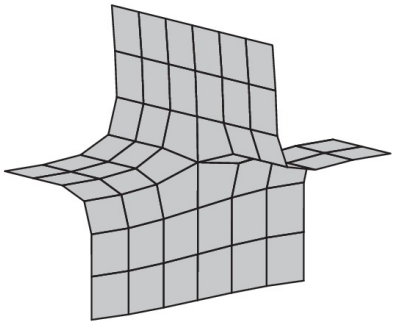


$n = 6$ net

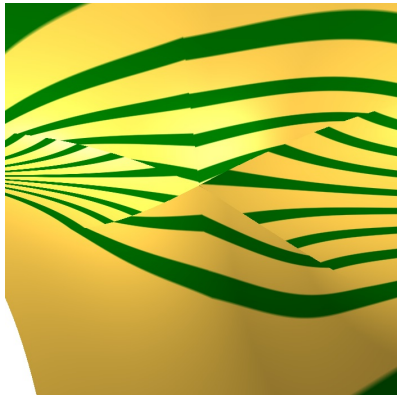


ACC [LS08]

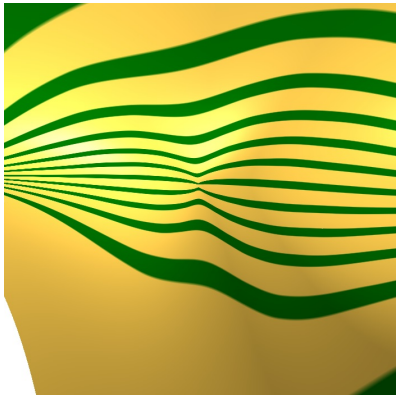
Joining two crossing beams



$n = 6$ net

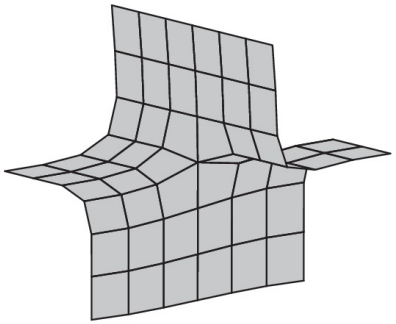


ACC [LS08]

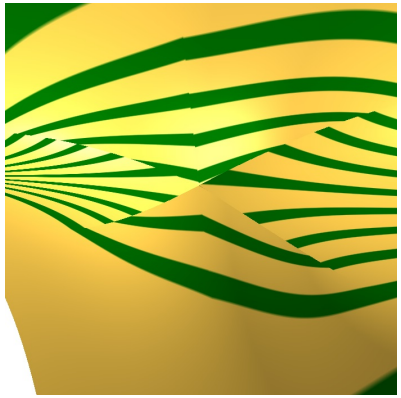


Catmull-Clark

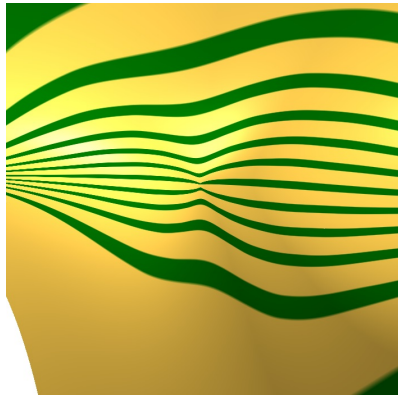
Joining two crossing beams



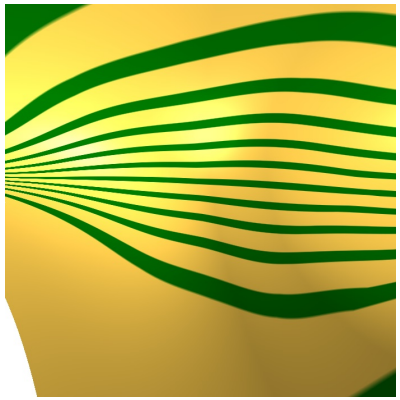
$n = 6$ net



ACC [LS08]

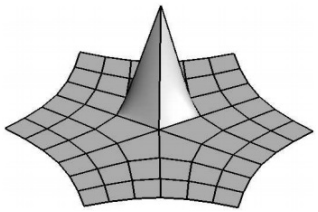


Catmull-Clark



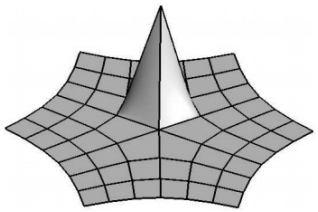
SGP2015

Benefits of one Catmull-Clark-subdivision step

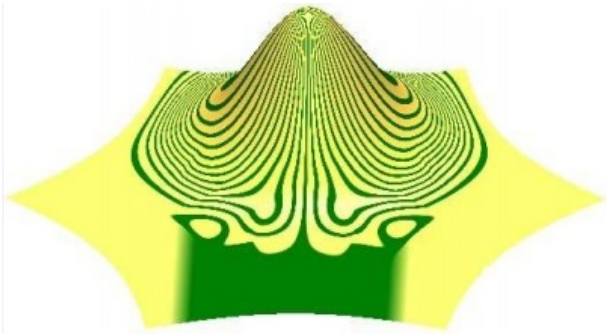


$n = 6$ net

Benefits of one Catmull-Clark-subdivision step

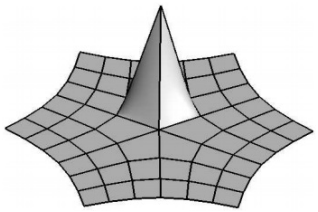


$n = 6$ net



no CC

Benefits of one Catmull-Clark-subdivision step



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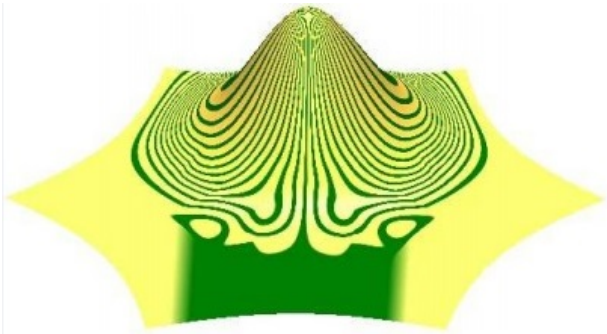
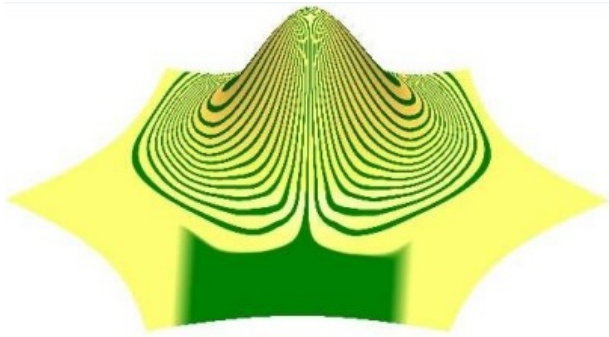
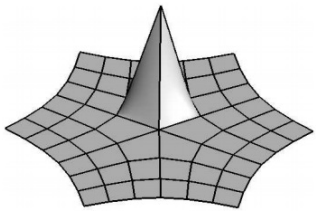


no CC

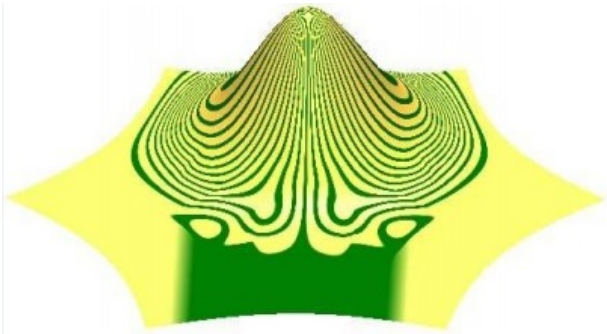


one CC-step

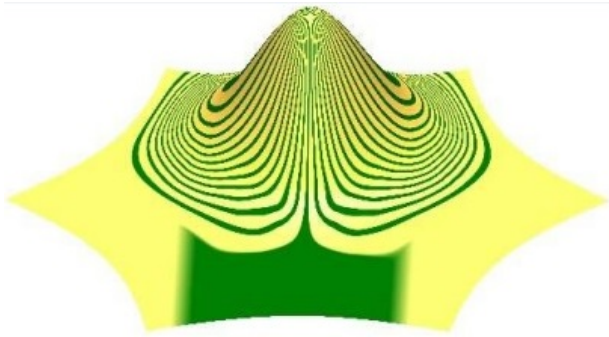
Benefits of one Catmull-Clark-subdivision step



$n = 6$ net



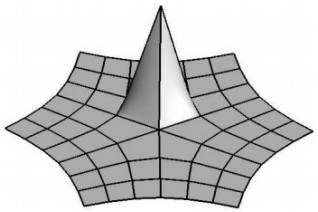
no CC



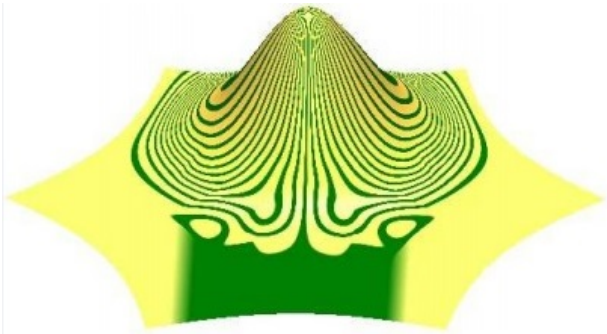
one CC-step

one CC step 

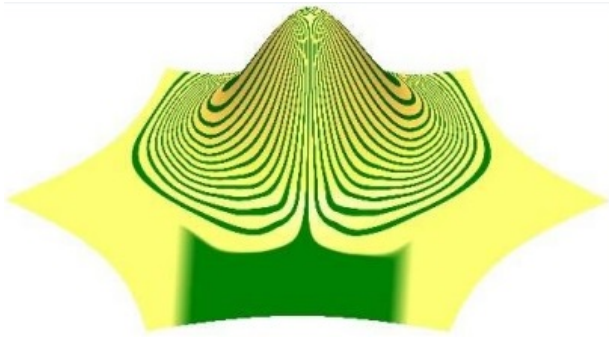
Benefits of one Catmull-Clark-subdivision step



$n = 6$ net



no CC

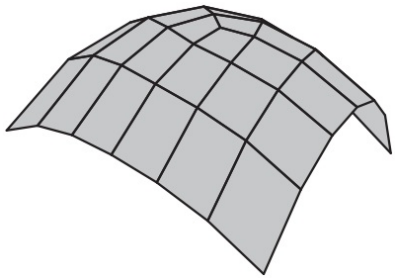


one CC-step

one CC step 

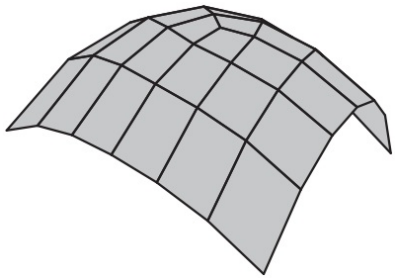
multiple CC steps 

Valence $n = 3$ case

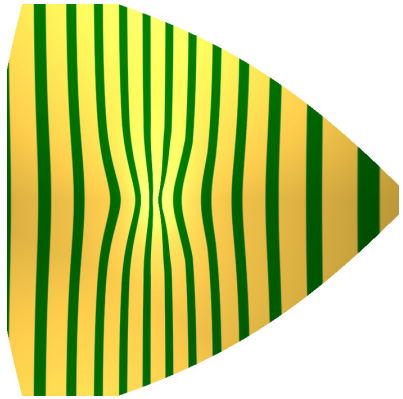


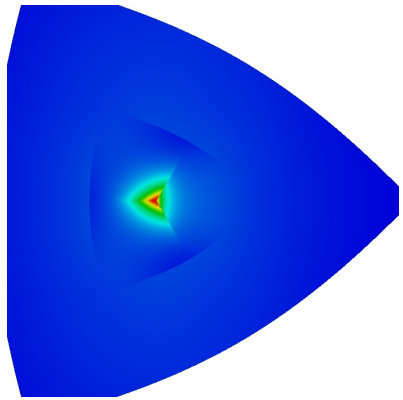
$n = 3$ net

Valence $n = 3$ case



$n = 3$ net

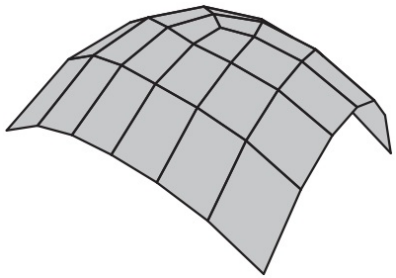




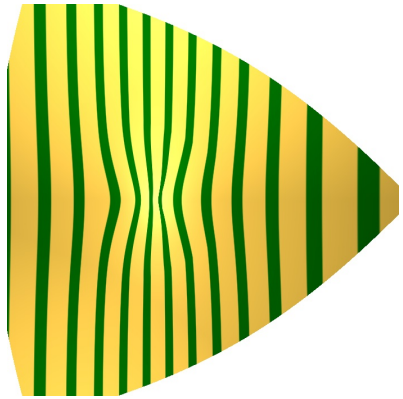
Gaussian curvature:

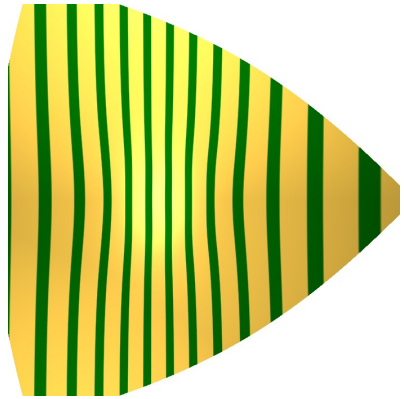
[GZ94]

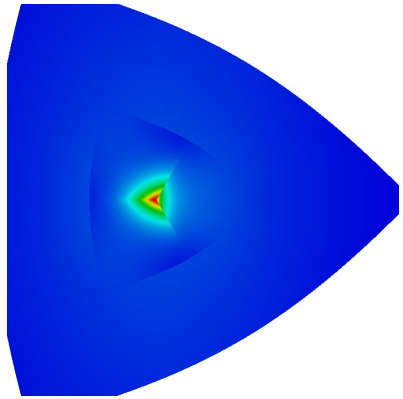
Valence $n = 3$ case

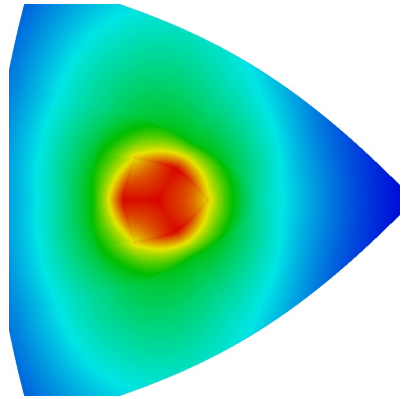


$n = 3$ net



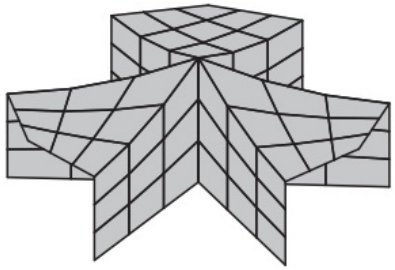




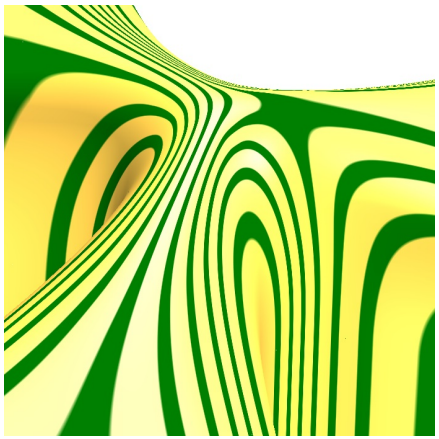


Gaussian curvature: [GZ94] SGP2015

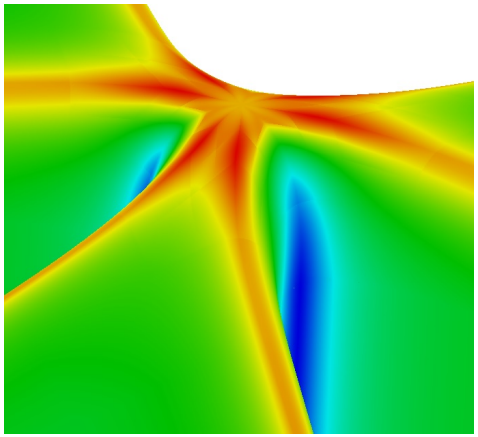
Valences $n > 7$



$n = 9$ net

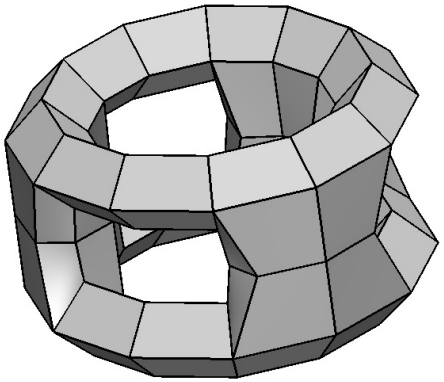


highlight shading



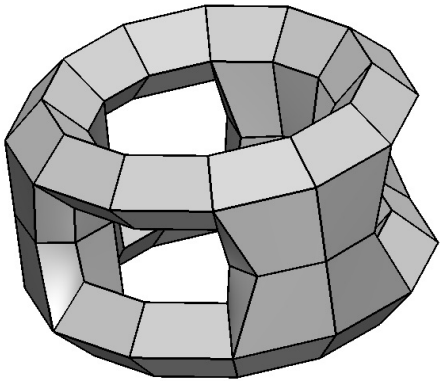
mean curvature

'Platonic' surfaces from 5-sided patches

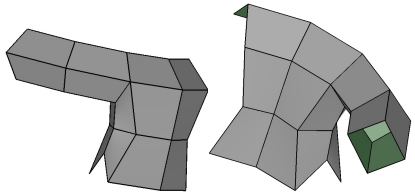


mesh

'Platonic' surfaces from 5-sided patches

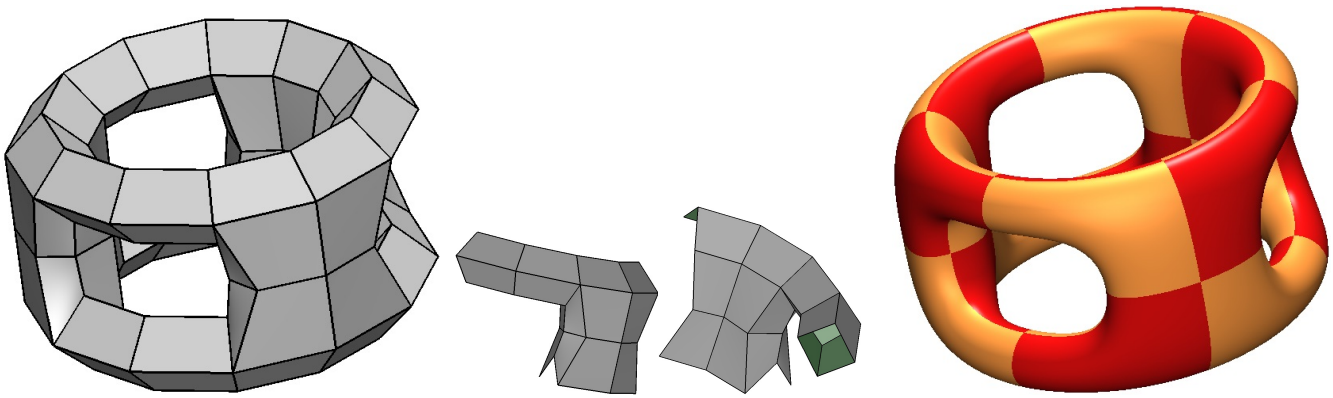


mesh



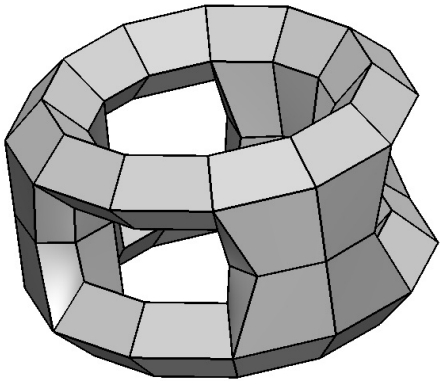
$n = 5$ nets

'Platonic' surfaces from 5-sided patches

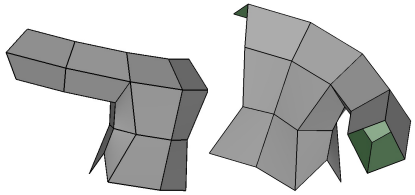


mesh $n = 5$ nets layout

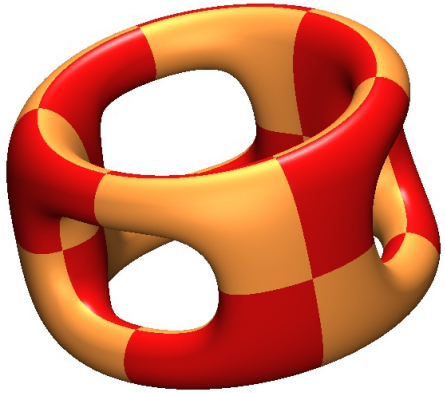
'Platonic' surfaces from 5-sided patches



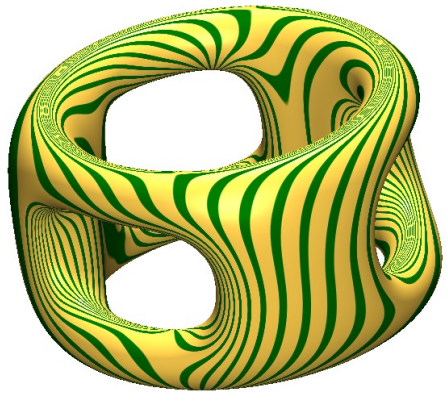
mesh



$n = 5$ nets

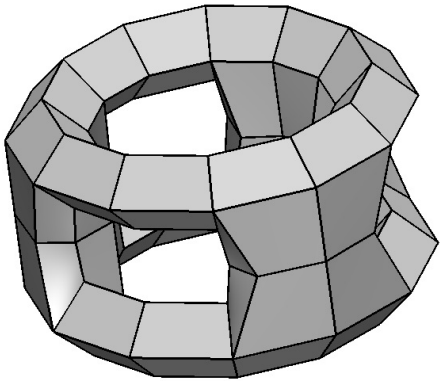


layout

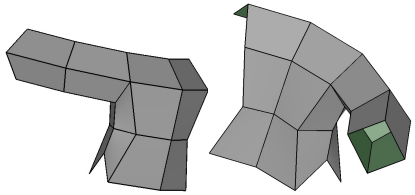


highlight lines

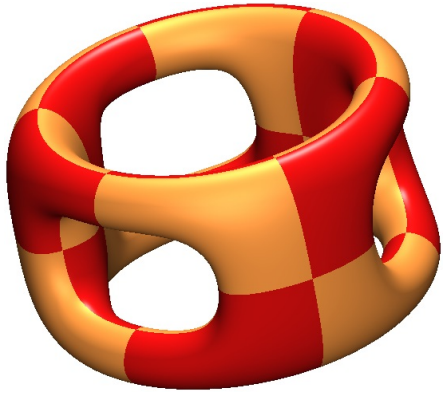
'Platonic' surfaces from 5-sided patches



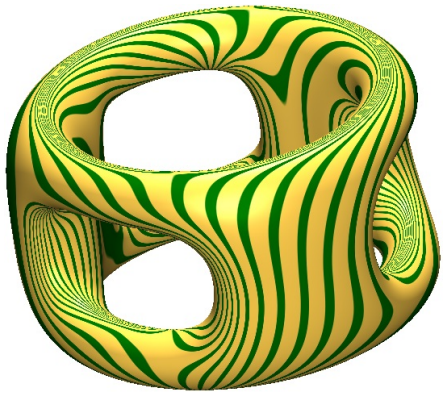
mesh



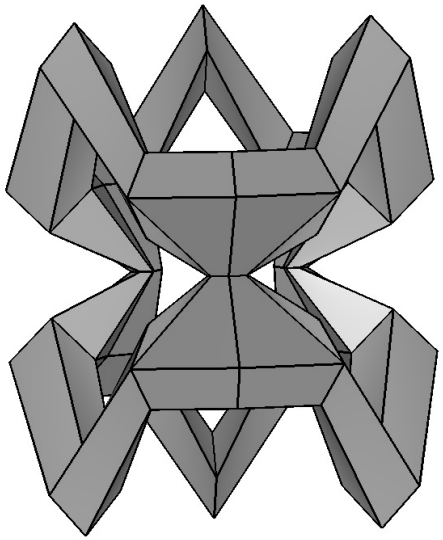
$n = 5$ nets



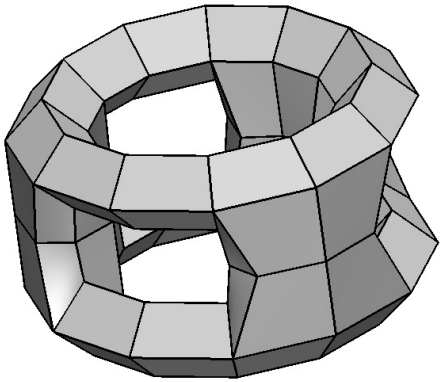
layout



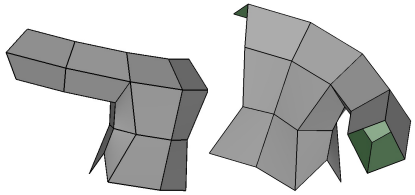
highlight lines



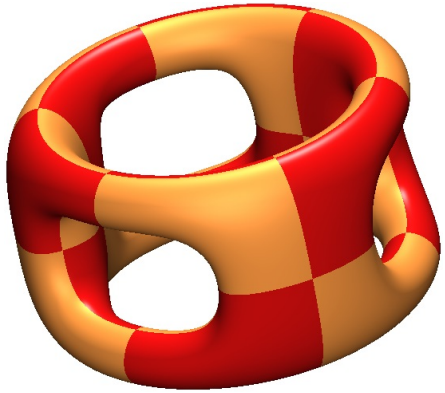
'Platonic' surfaces from 5-sided patches



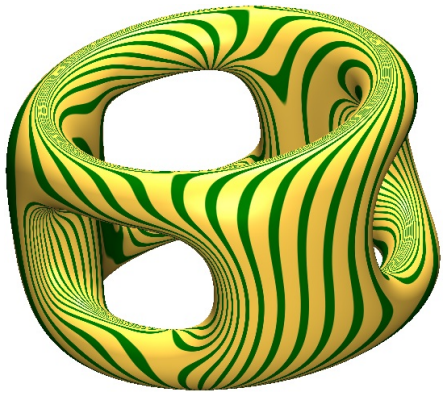
mesh



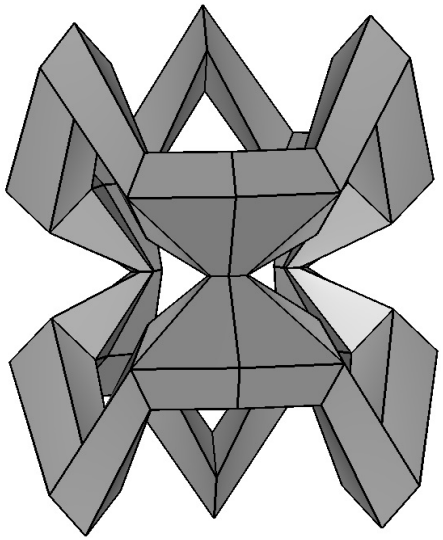
$n = 5$ nets

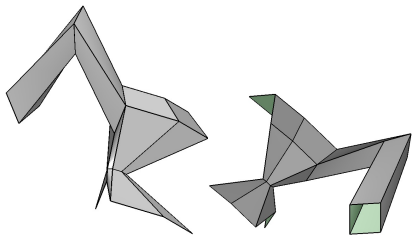


layout

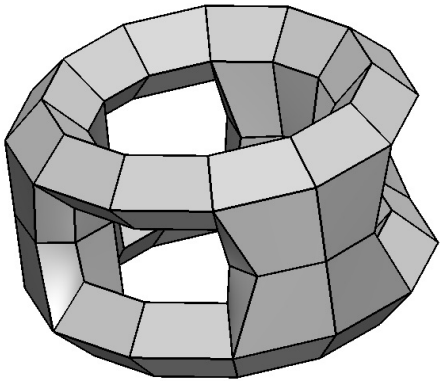


highlight lines

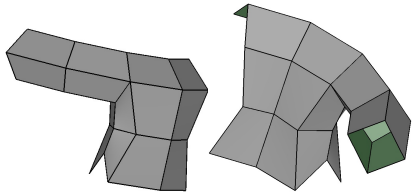




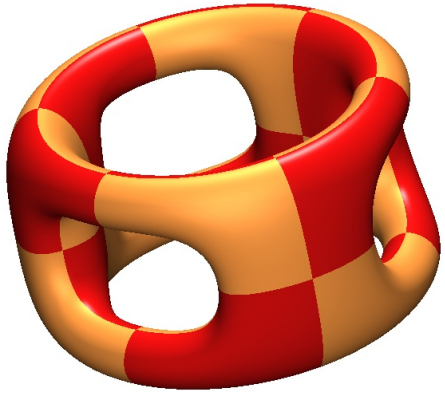
'Platonic' surfaces from 5-sided patches



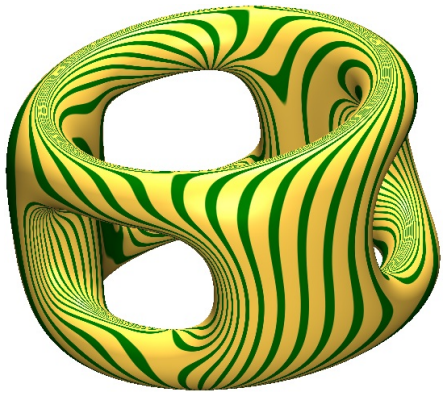
mesh



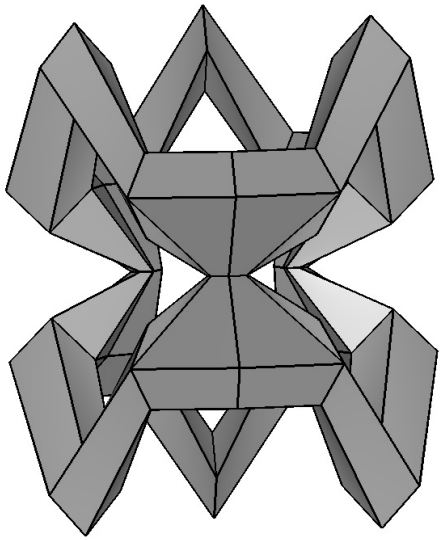
$n = 5$ nets

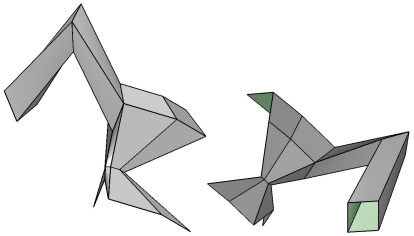


layout



highlight lines





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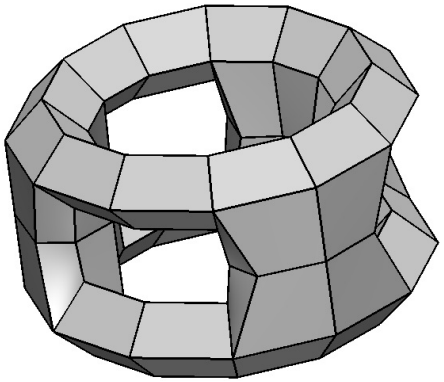
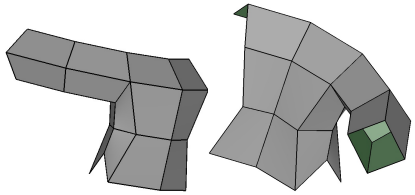


mesh



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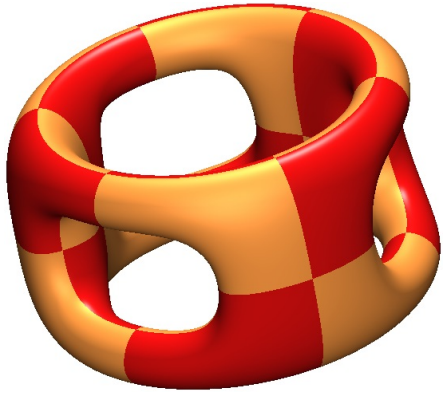
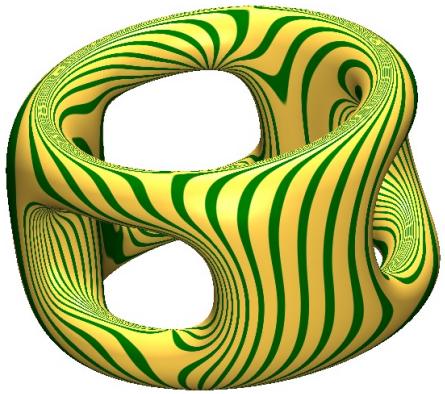


layout



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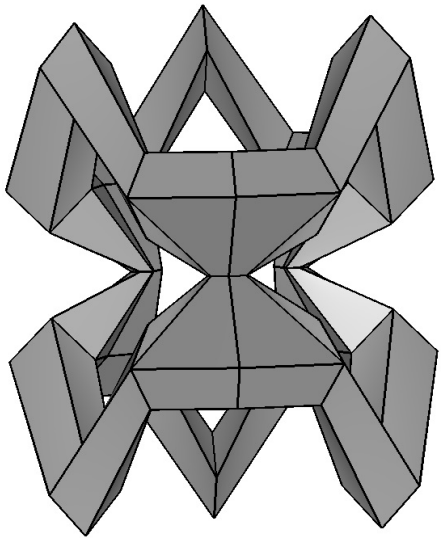

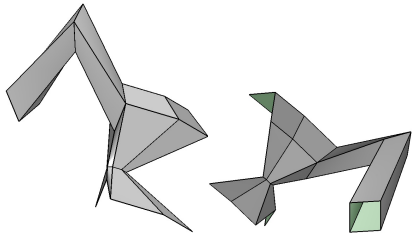

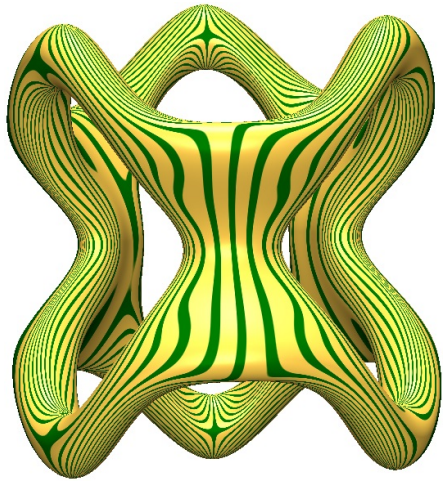

When C^0 beats G^1

- Mismatch of normals: $< 0.1^\circ$ is acceptable in industry.
- One local CC step improves shape for difficult input.

  Quad meshes from reverse engineering typically do not require a CC step (neither for mismatch nor for shape)
- C^0 continuity along cap boundary.

  C^0 continuity between sectors does not work! The deviation of normals becomes large.
- The guiding bi-5 cap is essential for deriving A class surface.
- Method is a finite alternative to CC subdivision.

Thank You!

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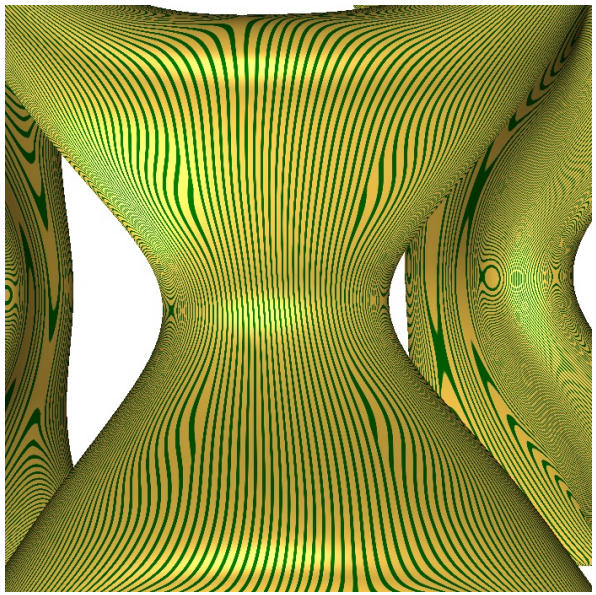
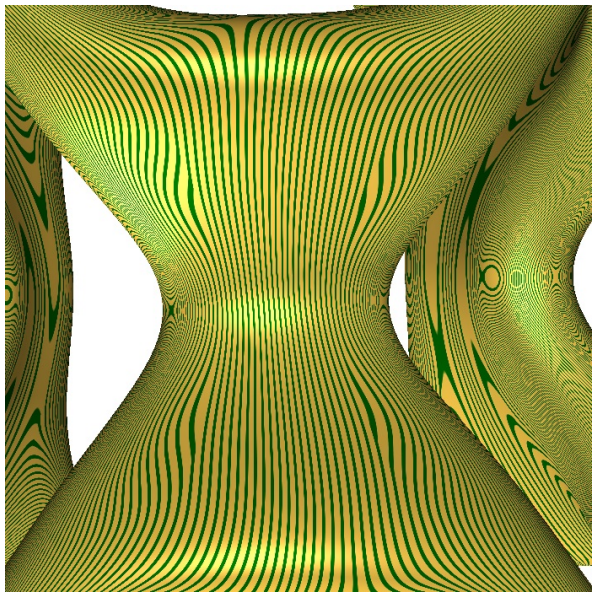
Magnified view

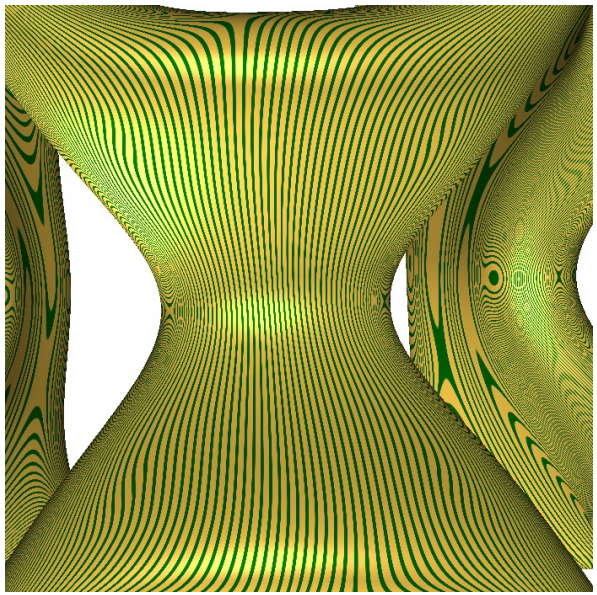


SGP 2015, no CC

Magnified view

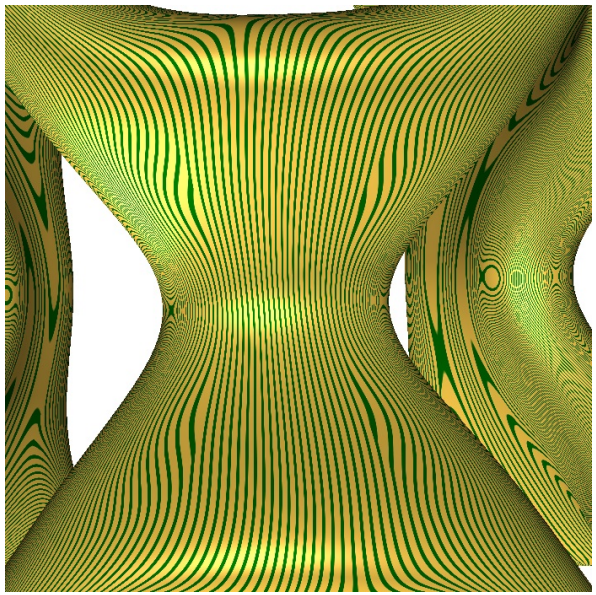


SGP 2015, no CC



one CC step + SGP 2015

Magnified view



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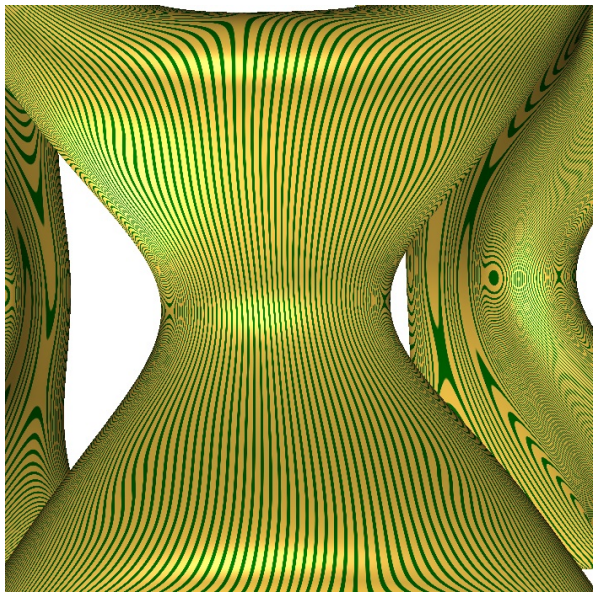
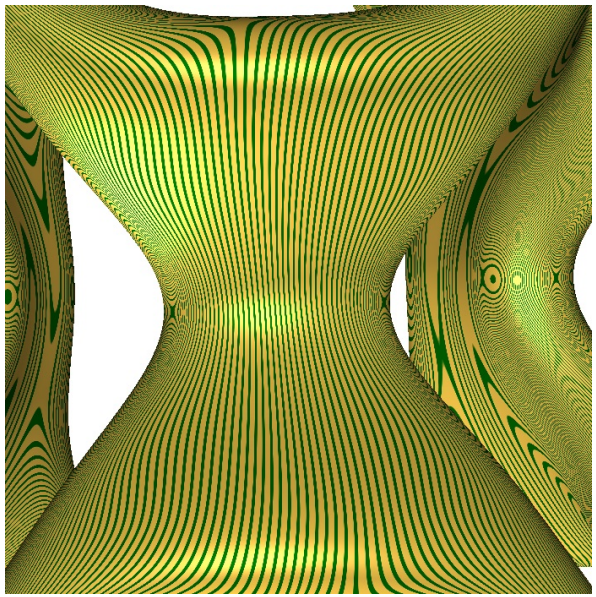
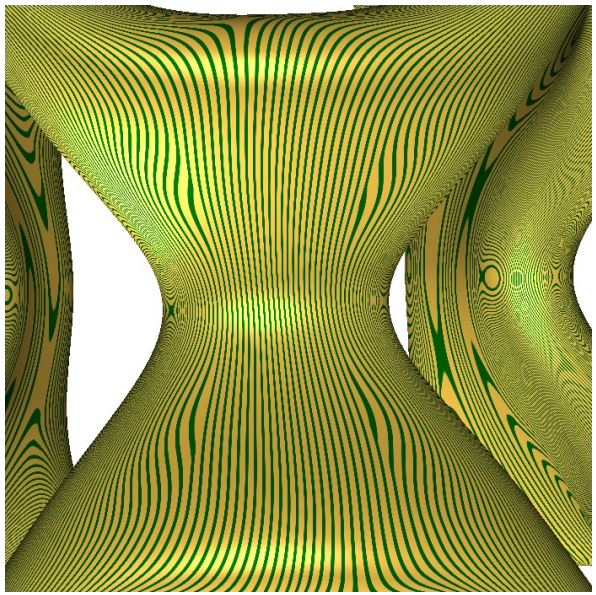


one CC step + SGP 2015



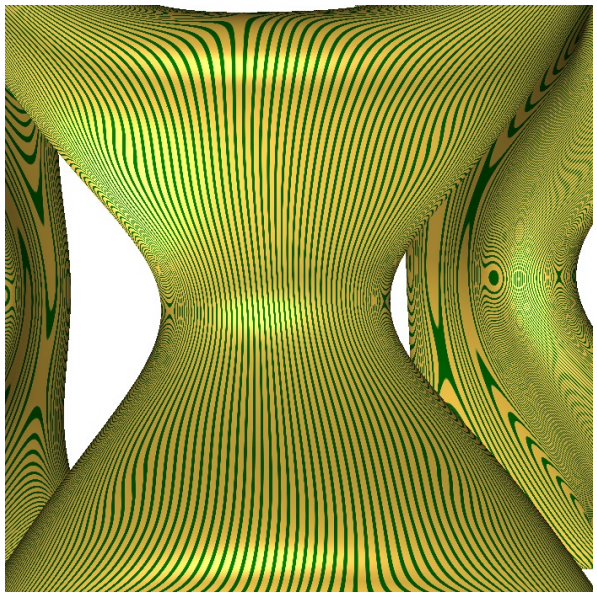
bi-5 guide, no CC

Magnified view



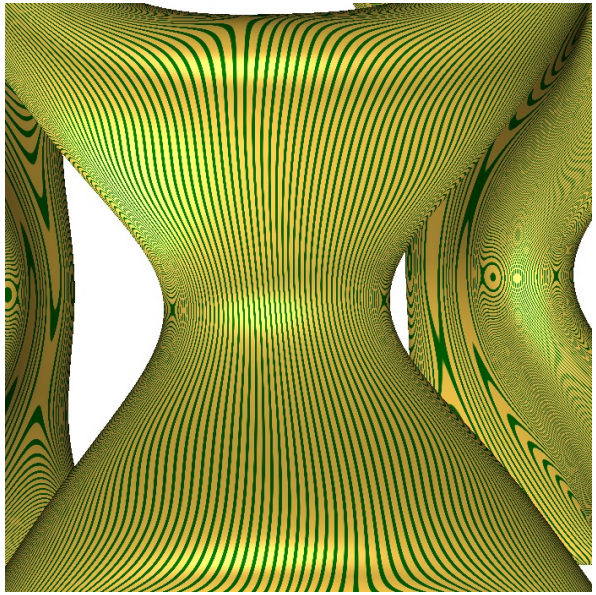
SGP 2015, no CC

A-



one CC step + SGP 2015

A

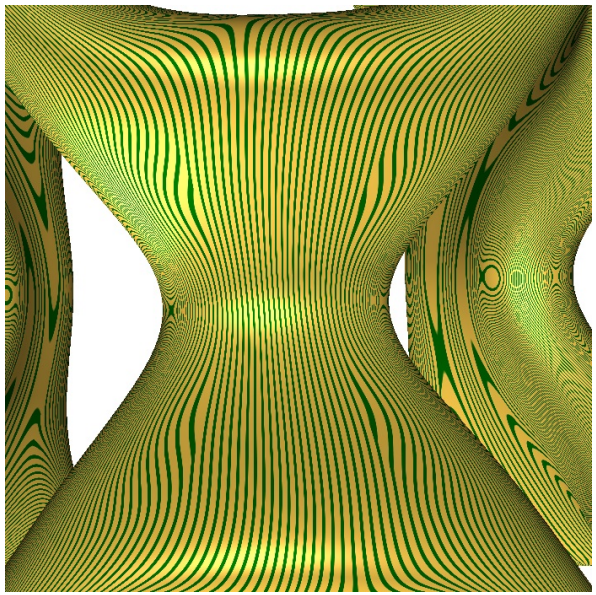


bi-5 guide, no CC

AA

(Standard & Poor's rating ;-)

Magnified view



SGP 2015, no CC

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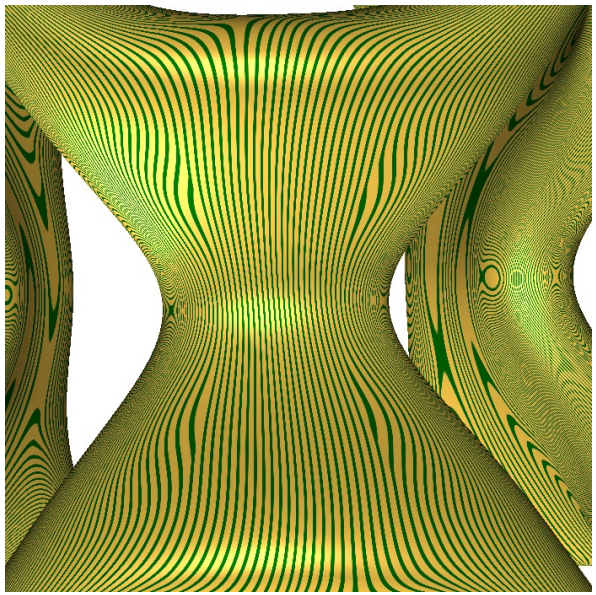

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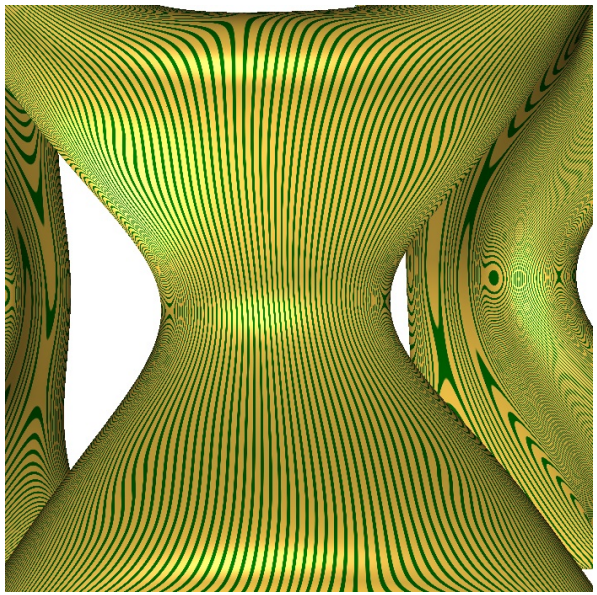

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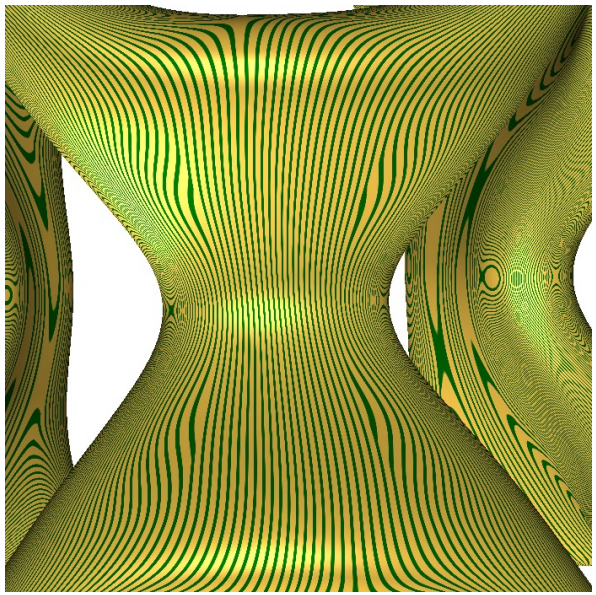

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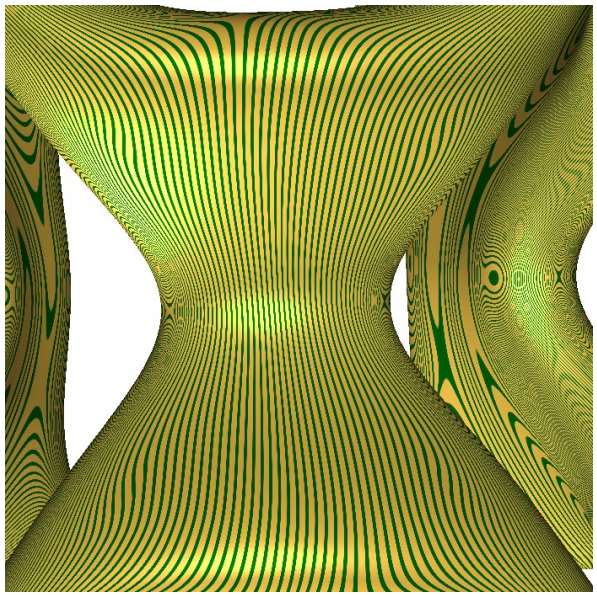
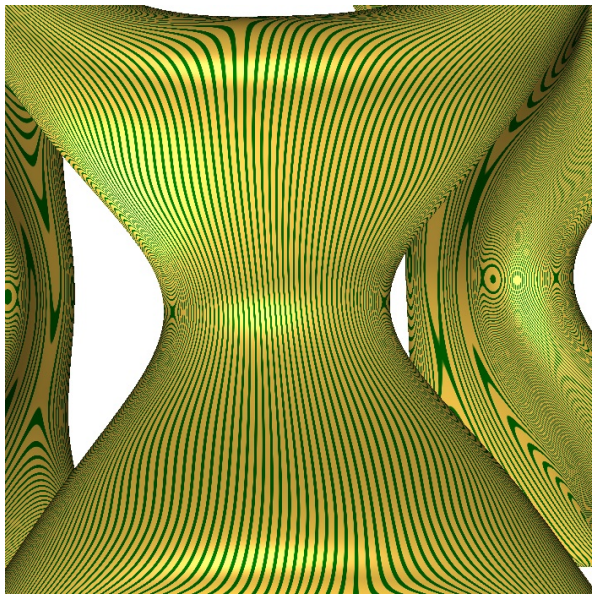


one CC step + SGP 2015



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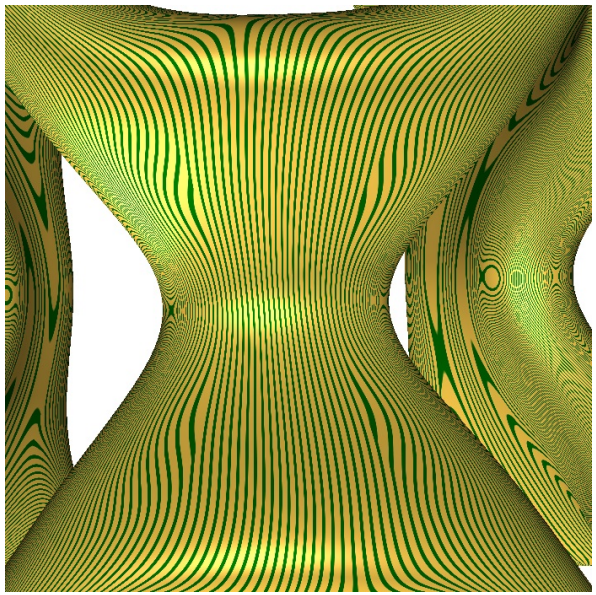
Magnified view



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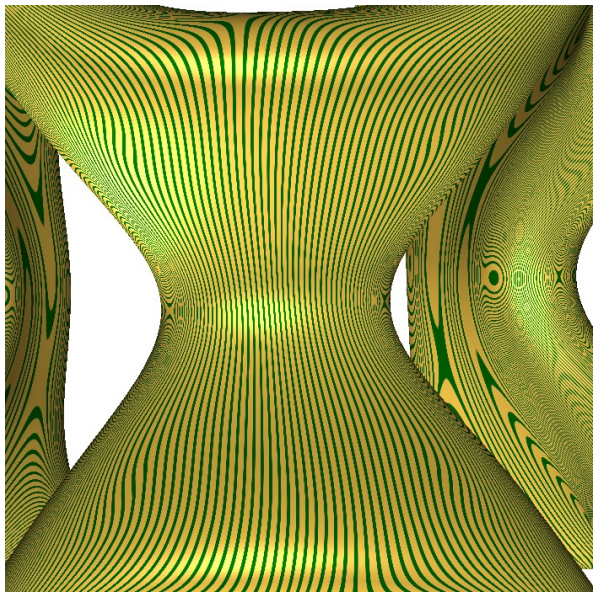

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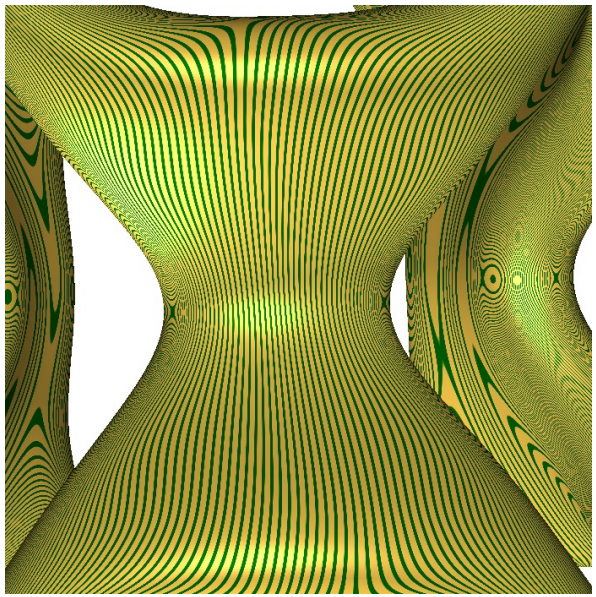
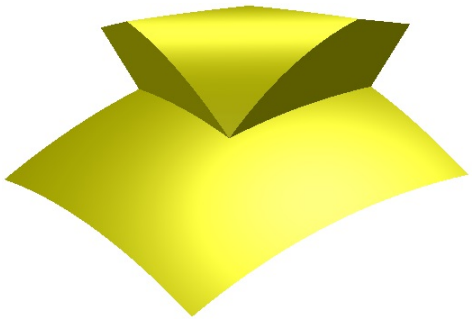


bi-5 guide, no CC

AA

(Standard & Poor's rating ;-)

Design with sketches



sketch

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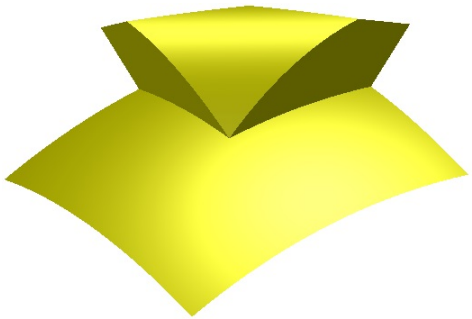

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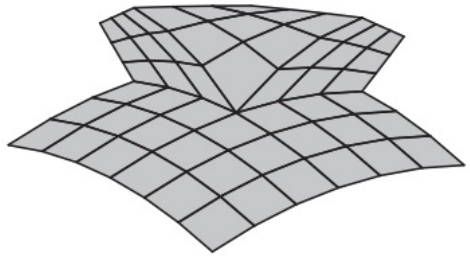
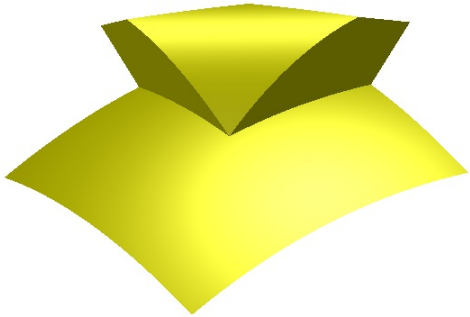
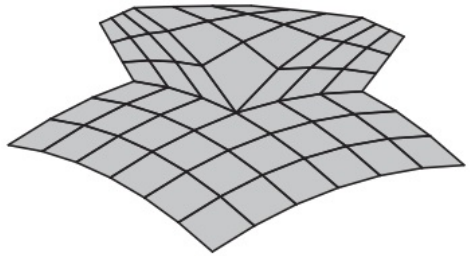


net

Design with sketches



sketch



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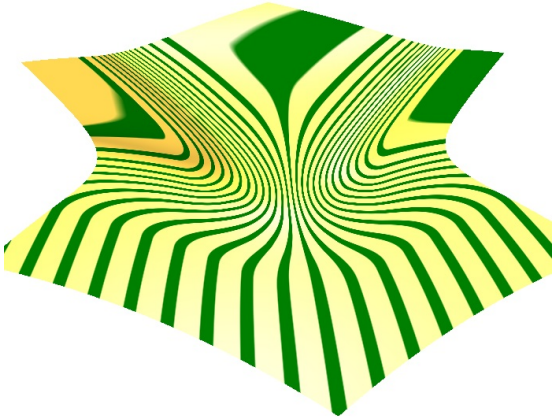
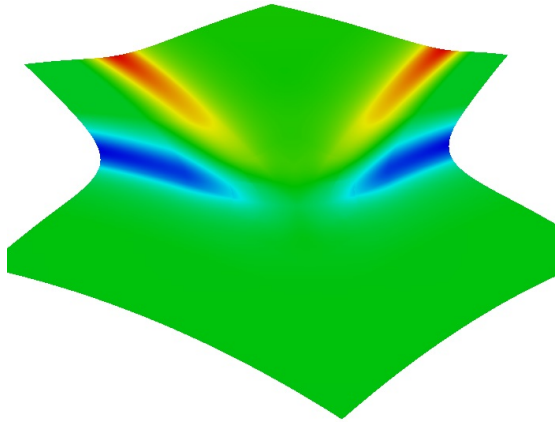


highlight shading



mean curvature