Smooth Multi-Sided Blending of bi-2 Splines

Kęstutis Karčiauskas

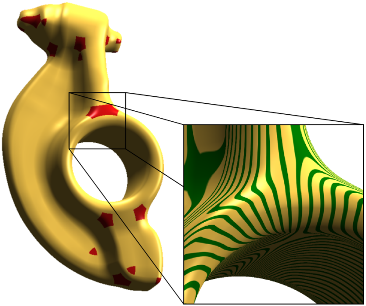
Vilnius University

Jörg Peters

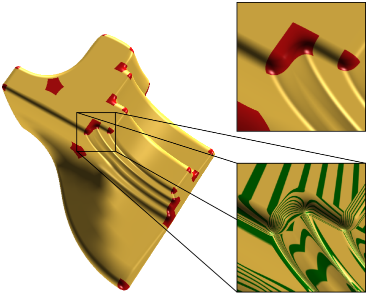
University of Florida

Quad models converted to CAD-compatible splines

gold = C^1 bi-2 splines; red = G^1 bi-3;



(a) rocker arm



(b) fan disk

- Continuity of normals often suffices (+ highlight lines well-distributed)
- Low degree preferable (fewer oscillations, lower downstream cost, ...)
- Matched G^k constructions yield C^k iso-geometric elements [P2013]

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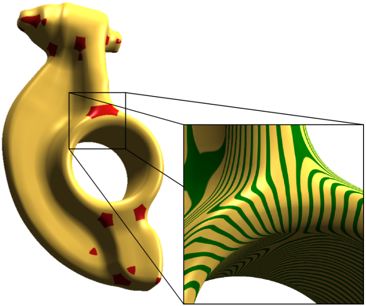

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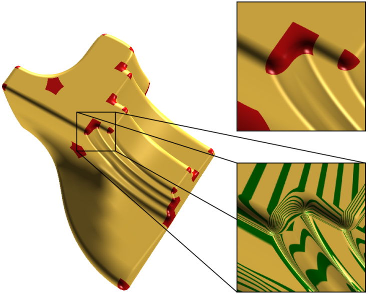

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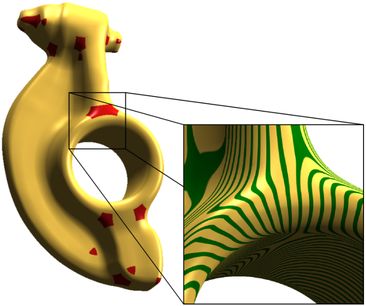

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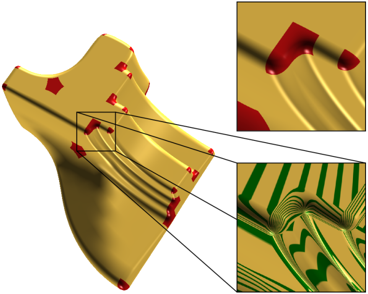

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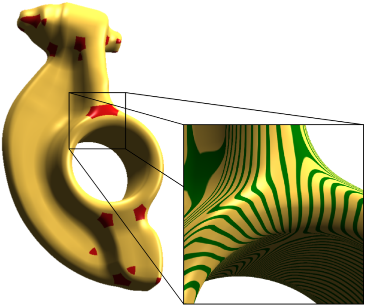

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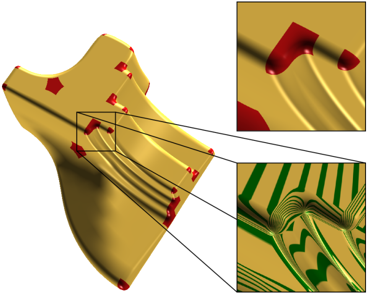

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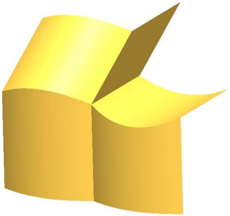
Joining and capping a collection of bi-2 spline surfaces



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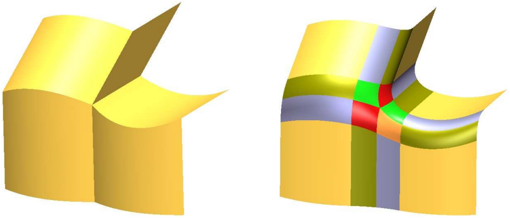

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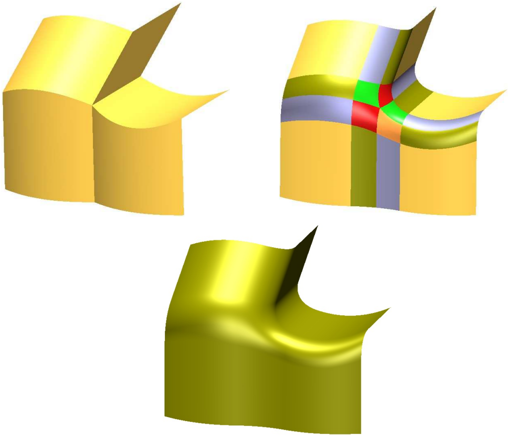

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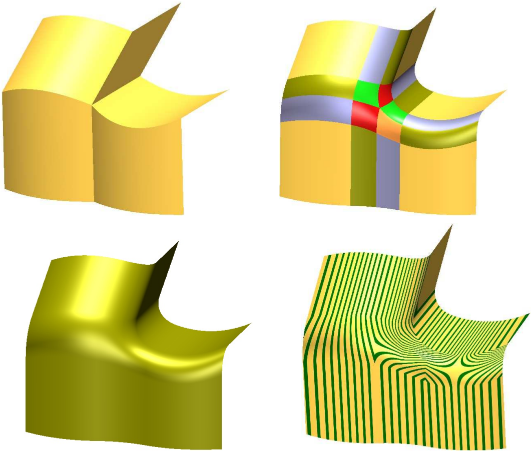

Outline

1. Why not classical 1980s, 1990s solutions?
2. Multi-sided blends: unified input, geometric continuity
3. Construction Highlights
4. Bi-3 caps when $n = 3, 5$
5. More examples and comparisons

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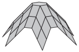
1980s, 1990s solutions



(a) input

(b) Doo-Sabin (DS)

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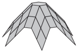

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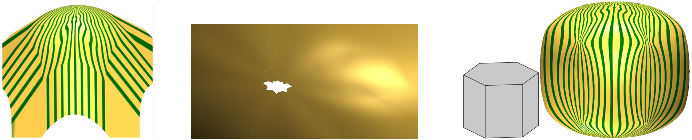

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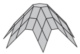

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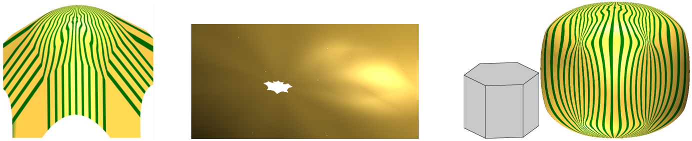

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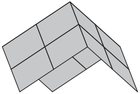
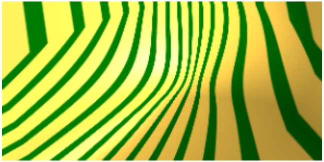


(c) input



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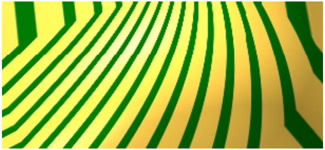

(e) our cap

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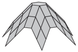

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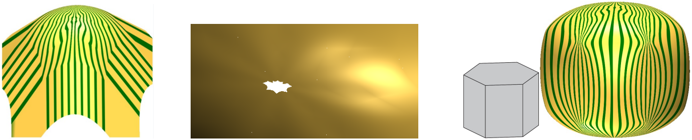

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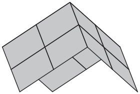

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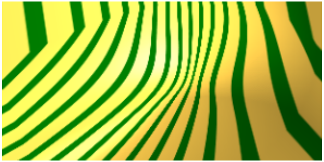

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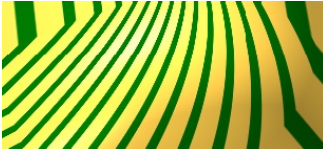
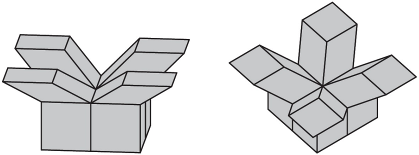


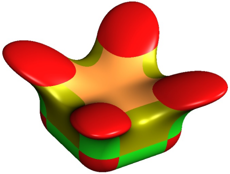
(e) our cap

singular constructions, rational (normalized) constructions, simplex splines, manifold splines, 3-sided patches,. . . not adopted by industry

New ingredients – 1990’s vs 2014: parameterization



quad mesh with nodes of valences 3,4,6,8



SMI 2014

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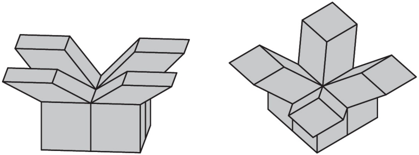

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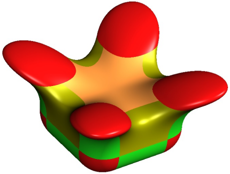

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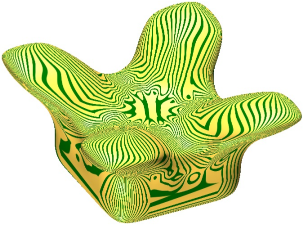

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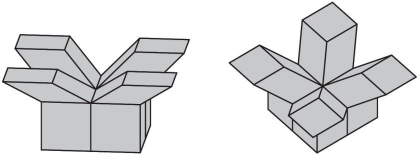

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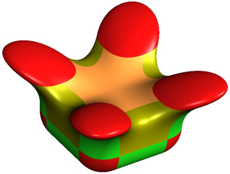

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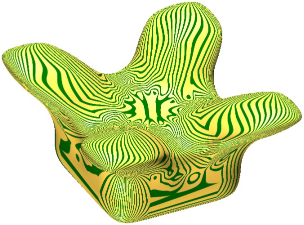

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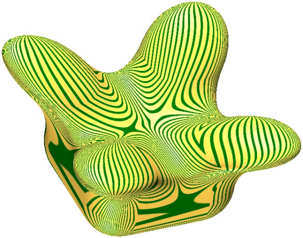
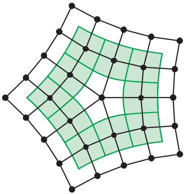


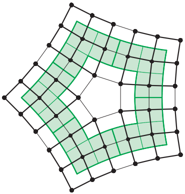
SMI 2014

Outline

Unified Input

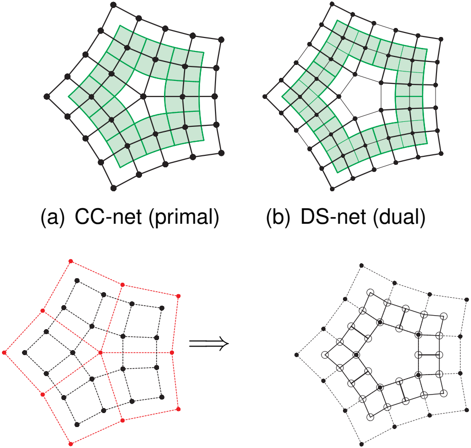


(a) CC-net (primal)



(b) DS-net (dual)

Unified Input



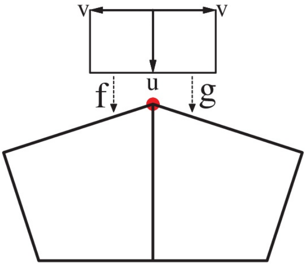
(a) CC-net (primal) (b) DS-net (dual)

(c) virtual refinement (d) tensor-border **b**

Border = ring of position and derivative data in BB-form.

Geometric continuity

$G^1: f_v(u,0) + g_v(u,0) = b(u) f_u(u,0)$



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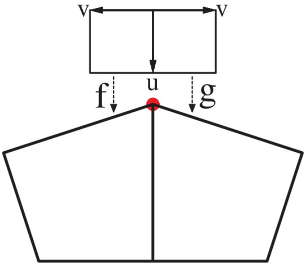

- (1990's) $b(u) := 2\cos\frac{2\pi}{n}(1-u)^2 \Rightarrow$
  input Hermite data is matched directly (C^1) $\Rightarrow$ low quality.

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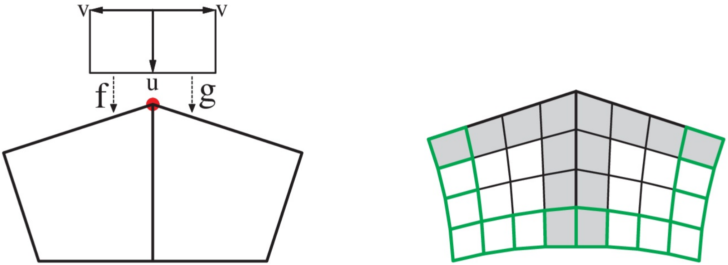

- (1990's) $b(u) := 2\cos\frac{2\pi}{n}(1-u)^2 \Rightarrow$
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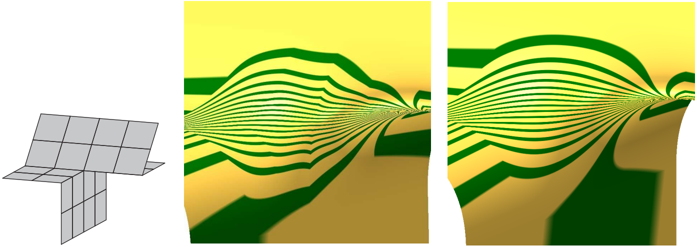
- bi-4 capping
  (2014) $b(u) := 2\cos\frac{2\pi}{n}(1-u) \Rightarrow$
  input reparameterized to make green compatible with inter-sector.

Outline

1. The positive effect of border reparameterization



(a) input a,b,c (b) **b**, C^1, bi-4 (c) our cap

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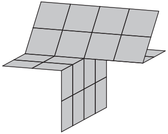

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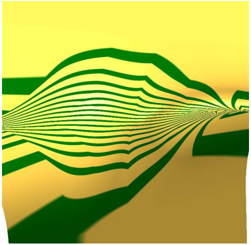
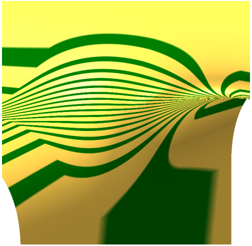
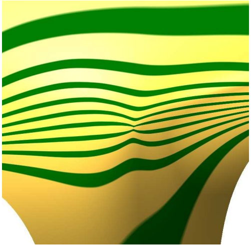


(b) **b**, C^1, bi-4

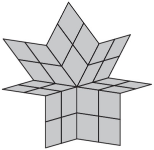


(c) our cap



(d) Catmull-Clark

2. Curvature continuity at the extraordinary point



$n = 7$ CC-net

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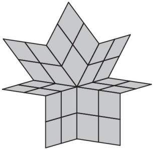

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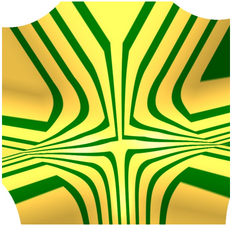
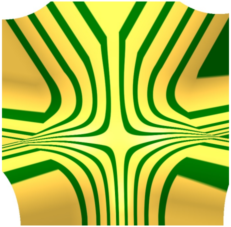


bi-4, G^1 eop



bi-4, G^2 eop

3. Functionals – but only after careful parameterization!

$$\mathcal{F}_m f := \int_0^1 \int_0^1 \sum_{i,j\geq 0}^{i+j=m} \frac{m!}{i!j!}(\partial_s^i f \partial_t^j f)^2, \quad m\text{-jet} \quad \mathcal{F}_\kappa^* f \quad := \int_0^1 \int_0^1 (\partial_s^\kappa f)^2 + (\partial_t^\kappa f)^2$$

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Fix central point then minimize. Low $n < 7 \rightarrow \mathcal{F}_3$, High $n > 6 \rightarrow \mathcal{F}_4$.

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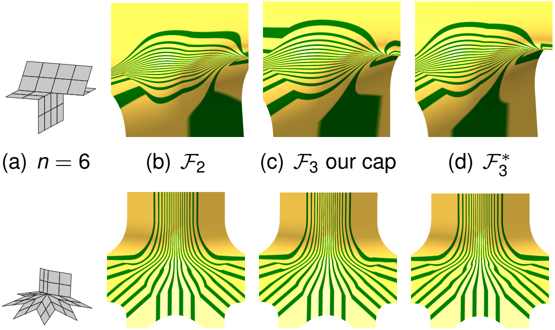
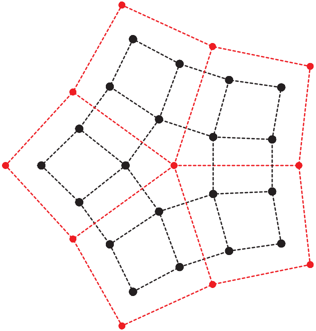


(a) $n = 6$ (b) $\mathcal{F}_2$ (c) $\mathcal{F}_3$ our cap (d) $\mathcal{F}_3^*$

(e) $n = 8$ (f) $\mathcal{F}_3$ (g) $\mathcal{F}_4$ our cap (h) $\mathcal{F}_3^*$

Implementation via generating functions

Tabulate 4 generating functions (3 if primal)



Assemble patch covering sector s

$$\text{patch}_{ij}^{s} := \sum_{k=0}^{n-1} \sum_{m=1}^{4} \text{table}_{ij}^{k,m} \text{net}_{m}^{s-k}. \quad (1)$$

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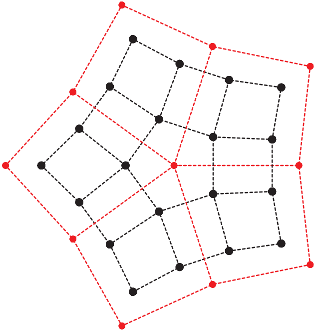

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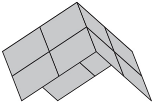
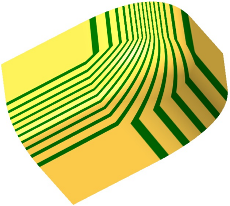
Outline

Bi-3 cap when $n = 3$

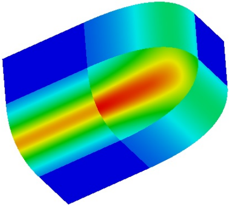
Theorem For $n = 3$ any smooth piecewise polynomial cap, satisfying symmetric G^1 constraints is curvature continuous at the central point.



CC-net



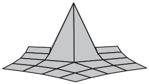
highlights



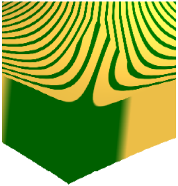
mean curvature

Bi-3 cap when $n = 5$

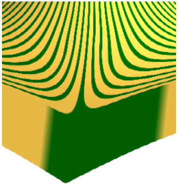
One patch per sector!



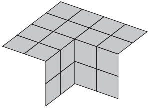
(a) $n = 5$



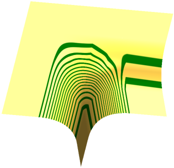
(b) $\mathcal{F}_3$



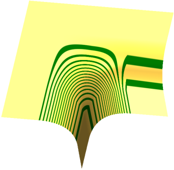
(c) $\mathcal{F}_5$ to set central point



(d) $n = 5$



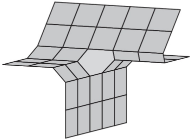
(e) Gregory-Zhou



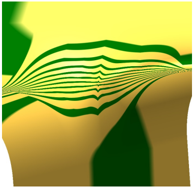
(f) our cap

Outline

Beams joining and subdivision

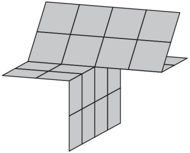


DS-net

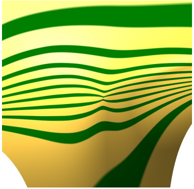


DS

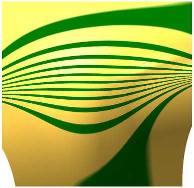
augmented DS



CC-net

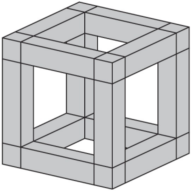


CC

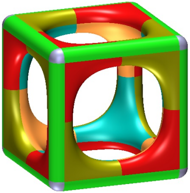


bi-4

Modeling with multi-sided patches

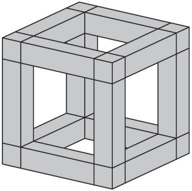


quad mesh, n=3,4,5,6



regular bi-2 + caps

Modeling with multi-sided patches



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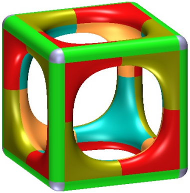
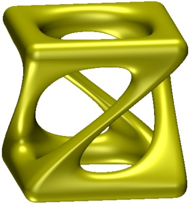
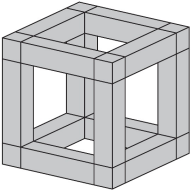


regular bi-2 + caps



'rotation'

Modeling with multi-sided patches



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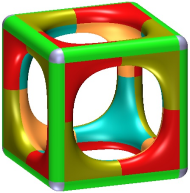

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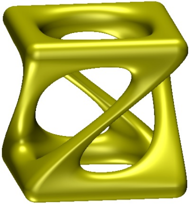
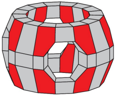
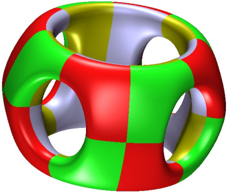


'rotation'

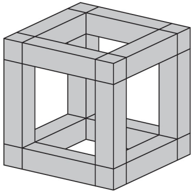


5-sided +
4-sided faces

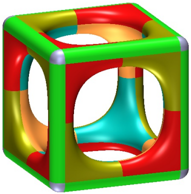


bi-3 surface

Modeling with multi-sided patches

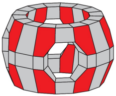


quad mesh, n=3,4,5,6



regular bi-2 + caps

'rotation'



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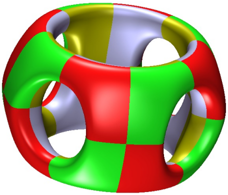
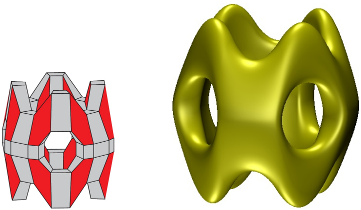


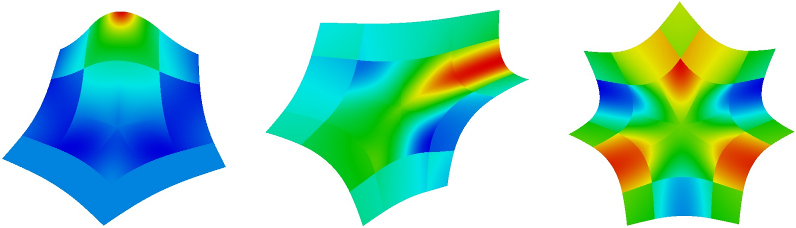
bi-3 surface



modification

Multi-patch caps naturally fill a bi-2 C^1 complex

Mean curvature



$n = 5$ $n = 6$ $n = 7$

Conclusion

- G^1 with well-distributed highlight lines – sufficient for inner surfaces or mechanical parts.
- Degree bi-4 (default); bi-3 when $n = 3, 5$.
  (Alternatively bi-3 for all n using a 2×2 split.)
- Immediate boundary reparameterization!
- Curvature continuity at the extraordinary point.
- Minimize functionals – but only after careful parameterization!
- $\Rightarrow$ cap behaves like one patch:

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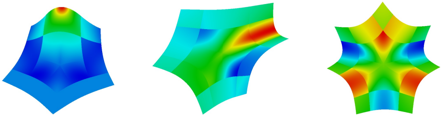

Questions?