Smooth Parametric Surface Construction

Jörg Peters

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joint work with K Karčiauskas, A. Myles, U Reif, …

Smooth Parametric Surface Construction

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Polar Structure
- Bi-3 Spline
- Modeling
- Polar refinement
- General mesh refinement
- NURBS Constructions

Fast Parallel Surface Construction

High-quality Surfaces
- What is a (high-quality) multi-sided blend?
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- Quality vs Degree Trade off
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.. in partes tres

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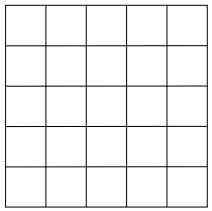

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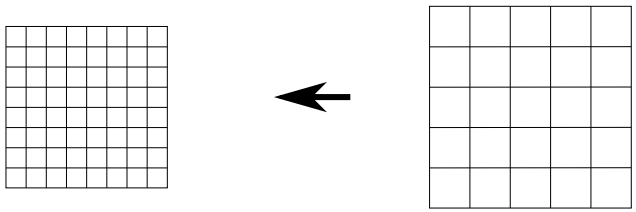

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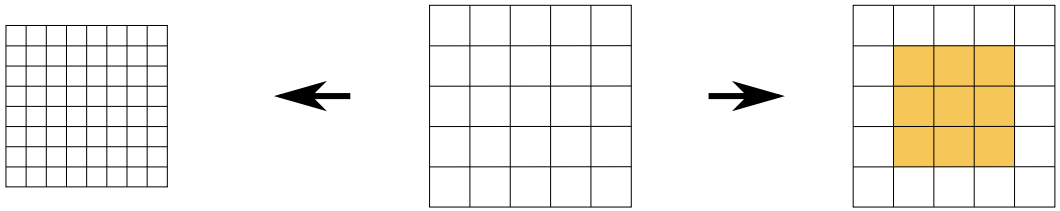

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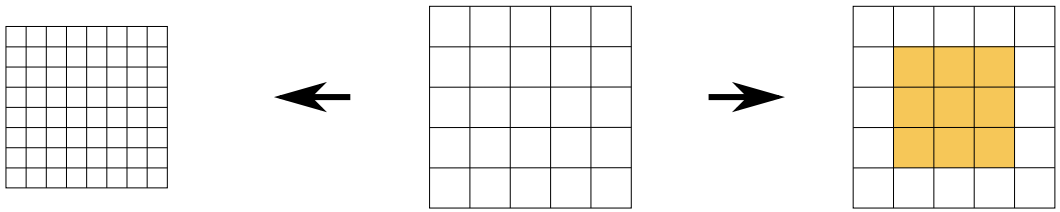

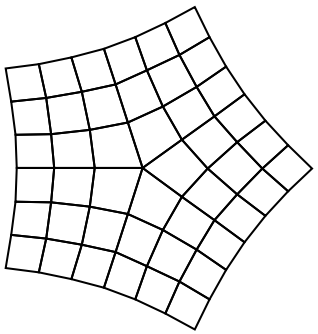

Valence $\neq 4$

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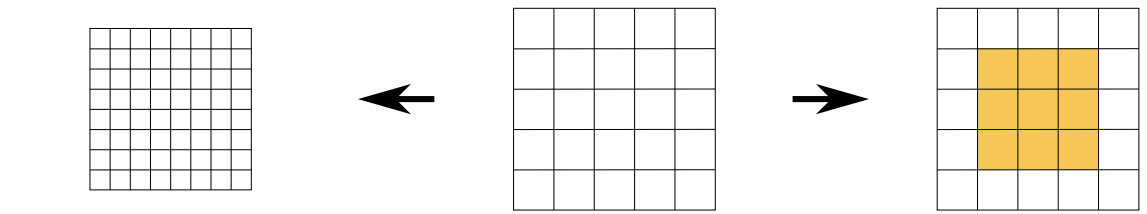

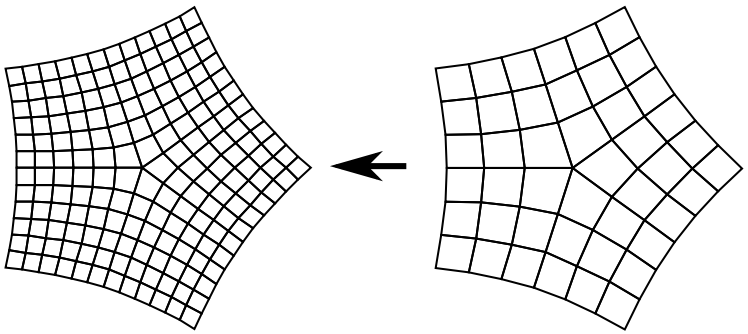

Catmull-Clark subdivision

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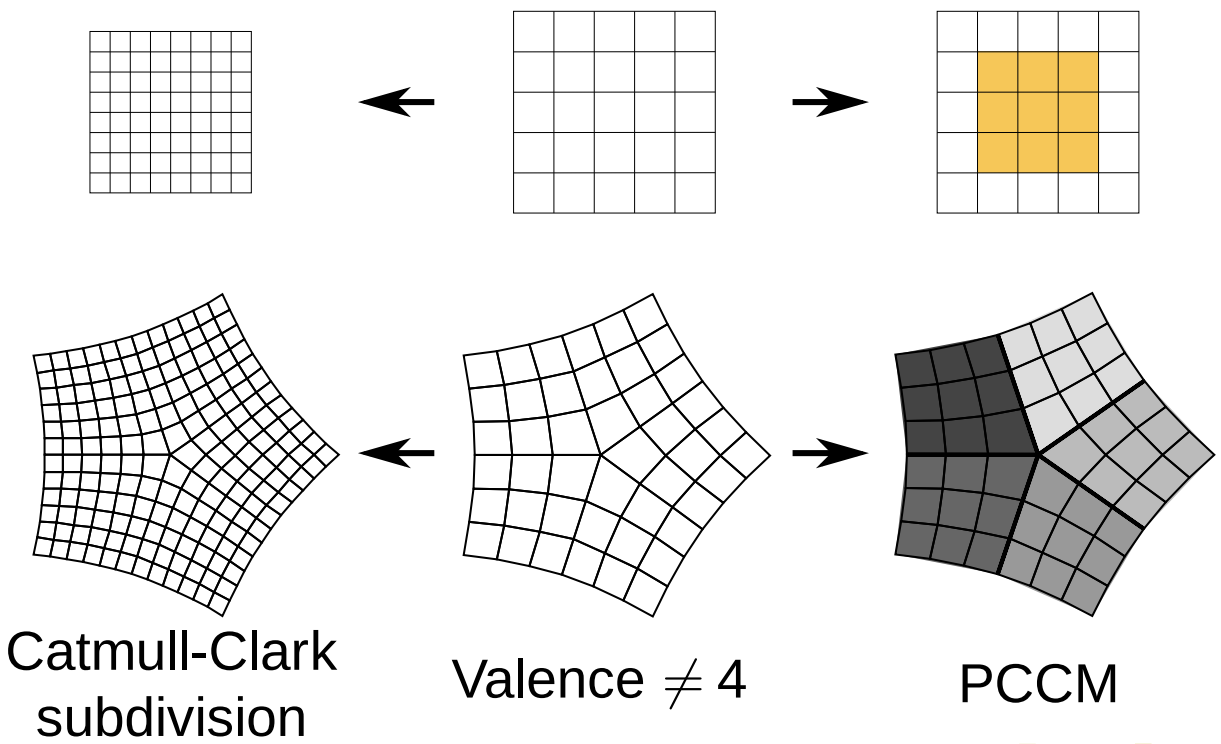

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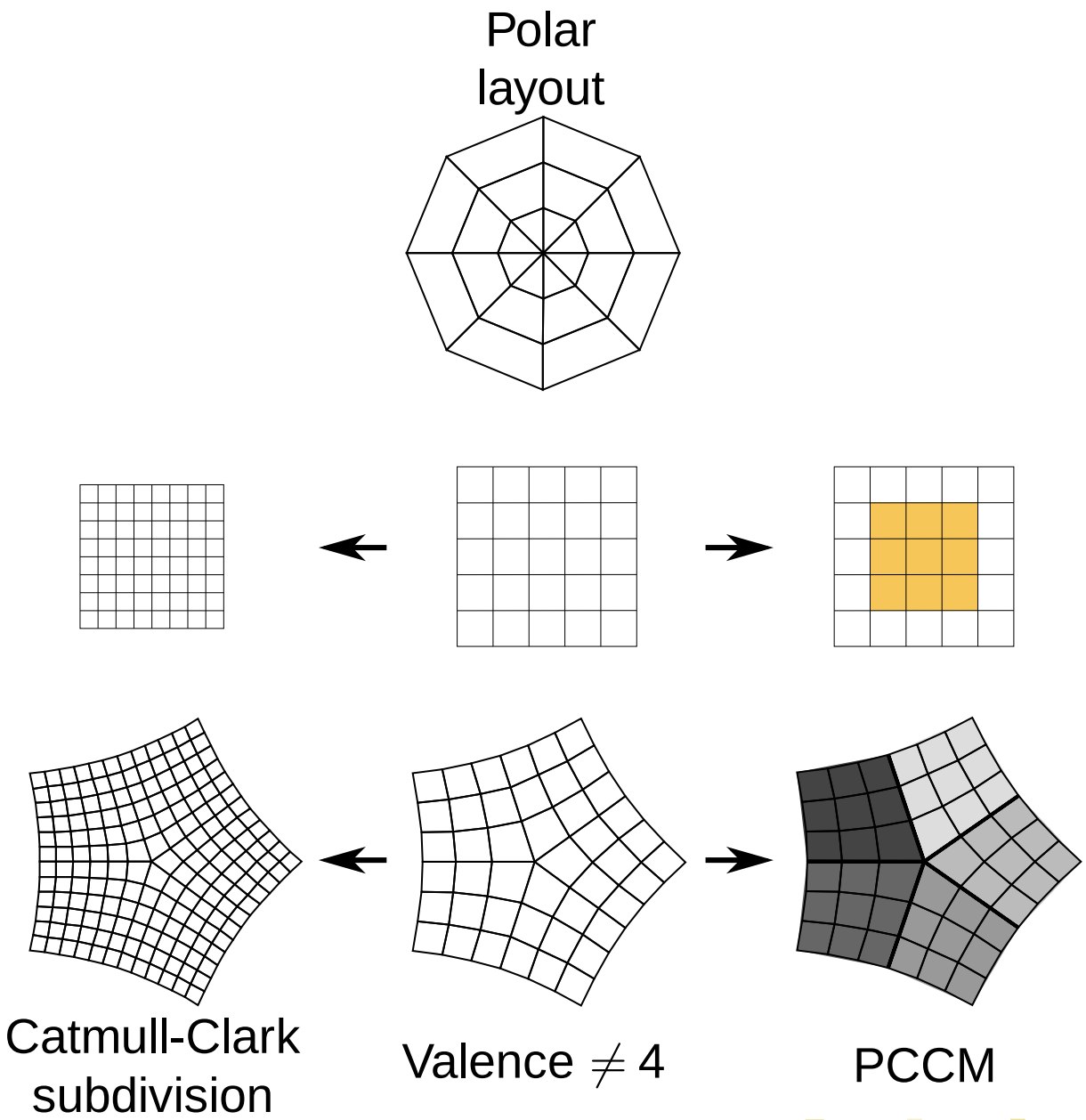

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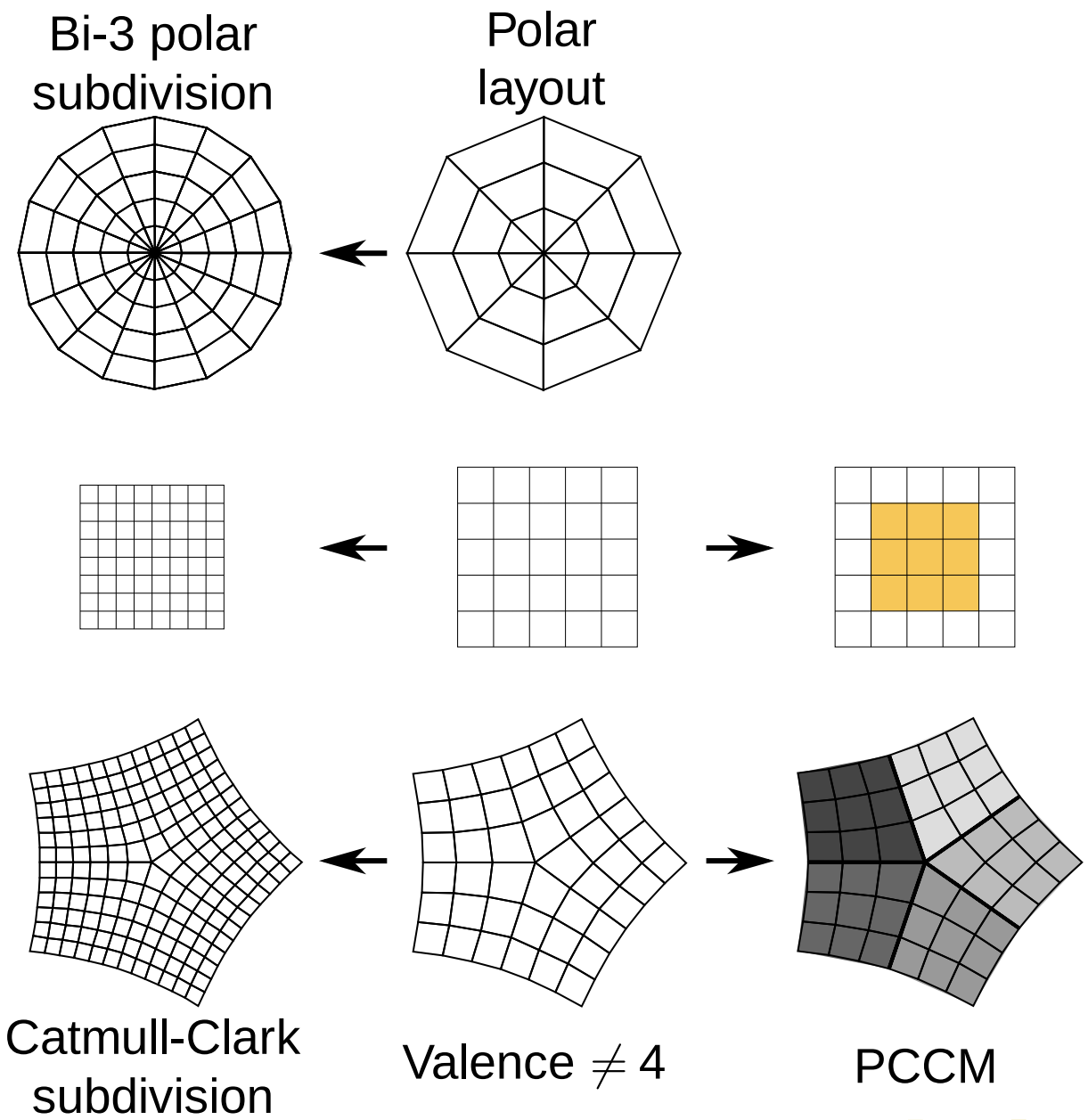

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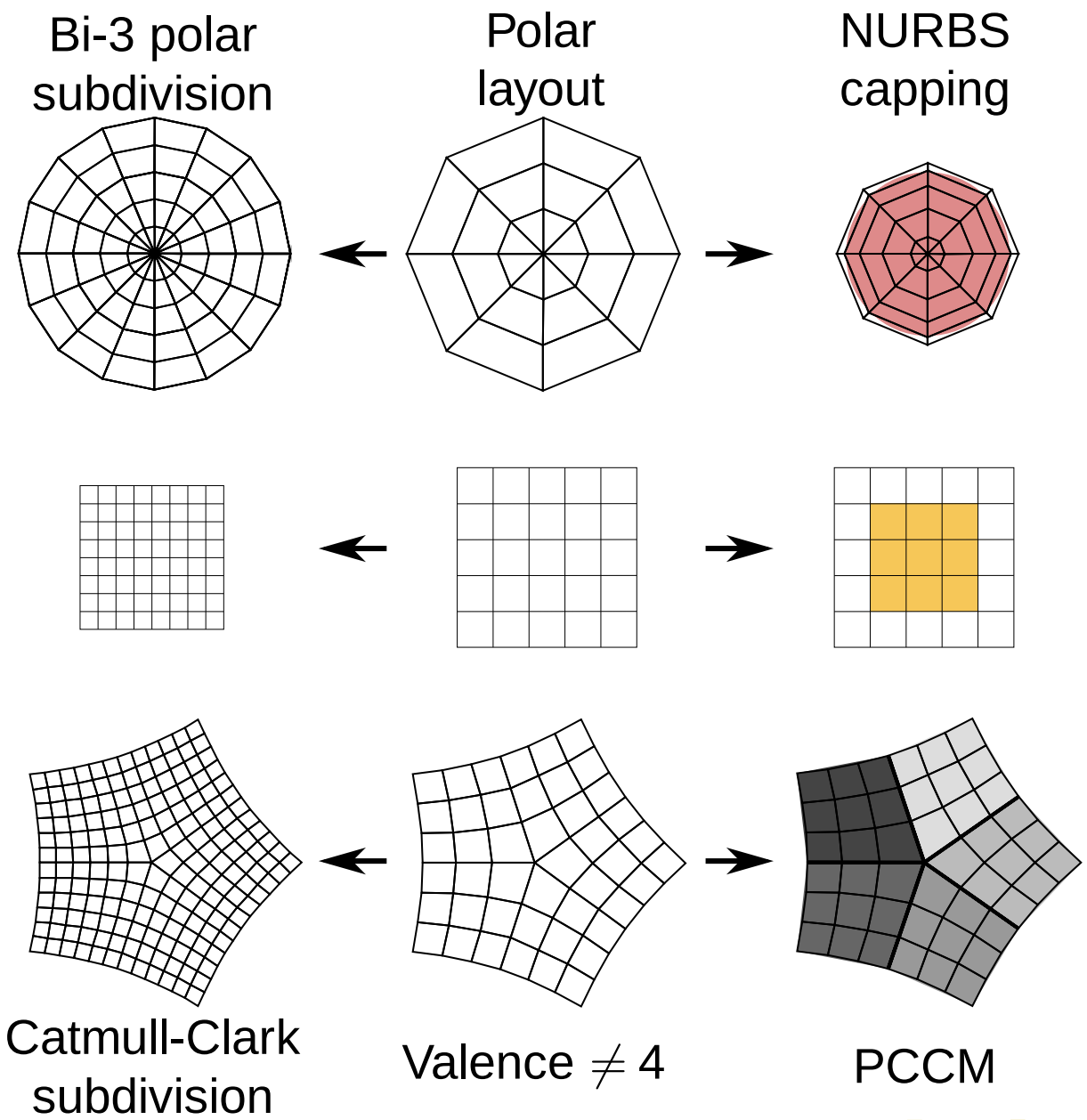

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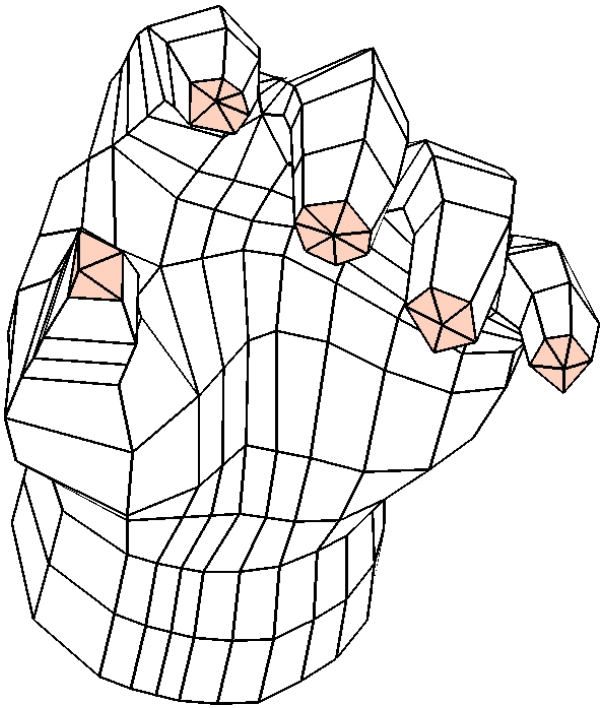
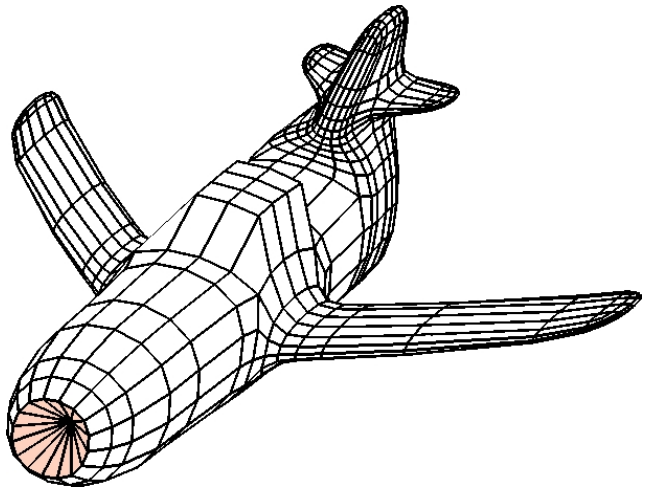
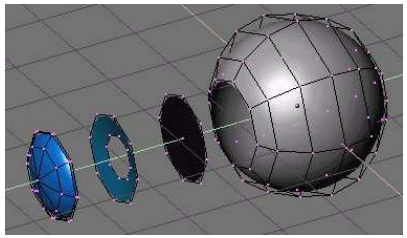
Polar structures appear naturally







Eye courtesy of ”Blender: Noob to Pro”

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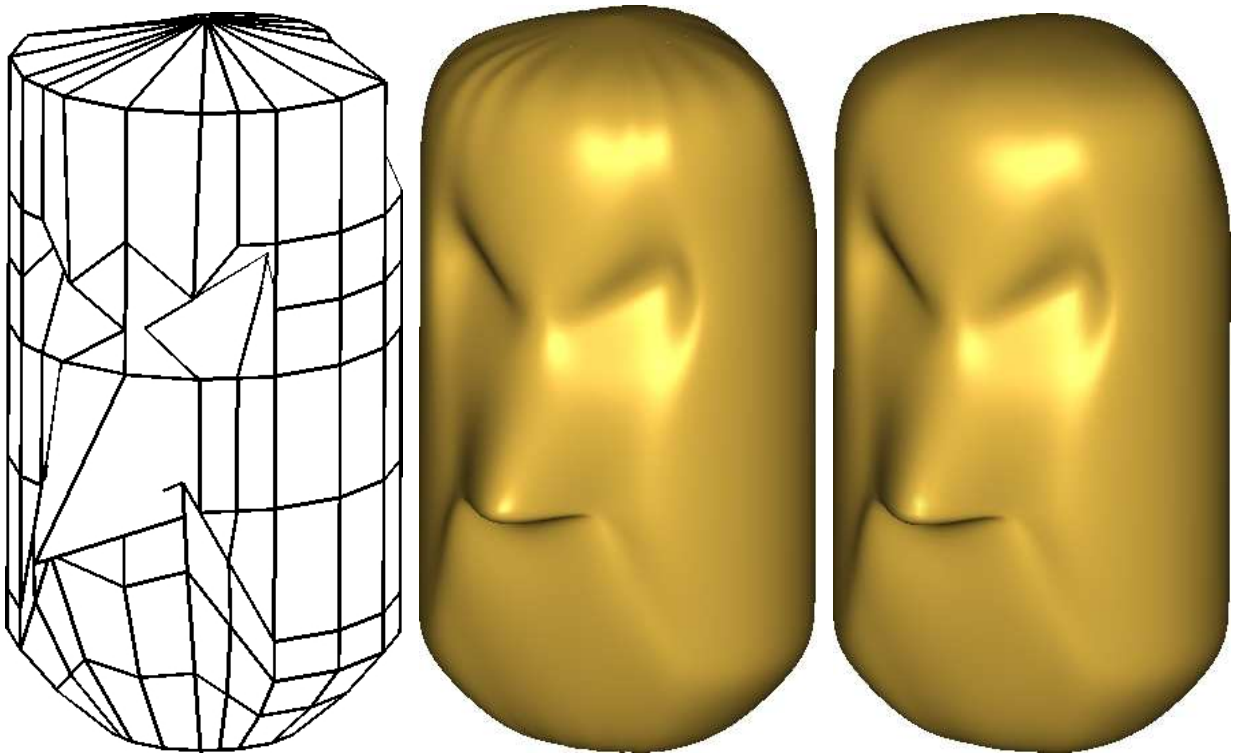
Remove those unsightly wrinkles



Catmull-Clark Bi-3 Polar

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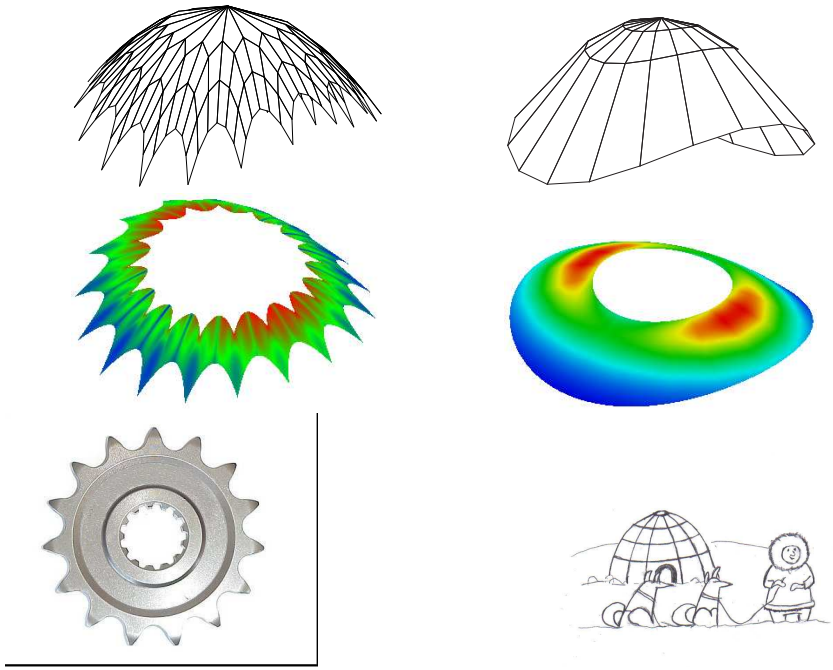
Sprocket vs Polar



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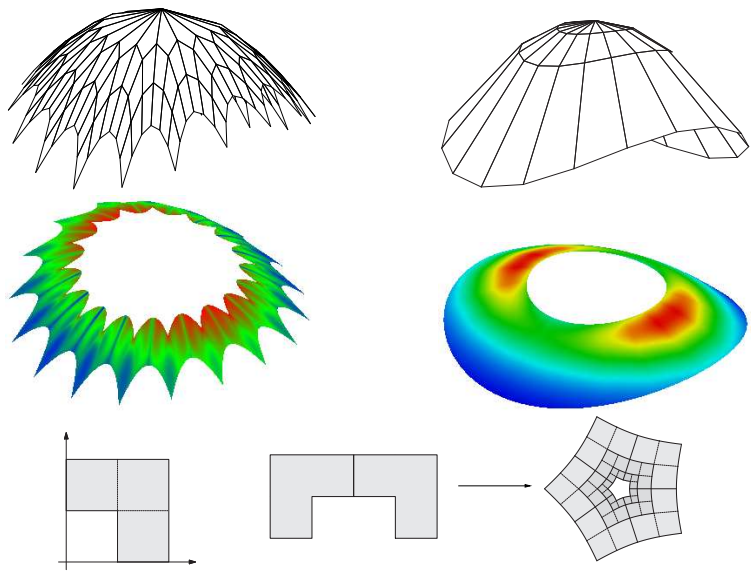

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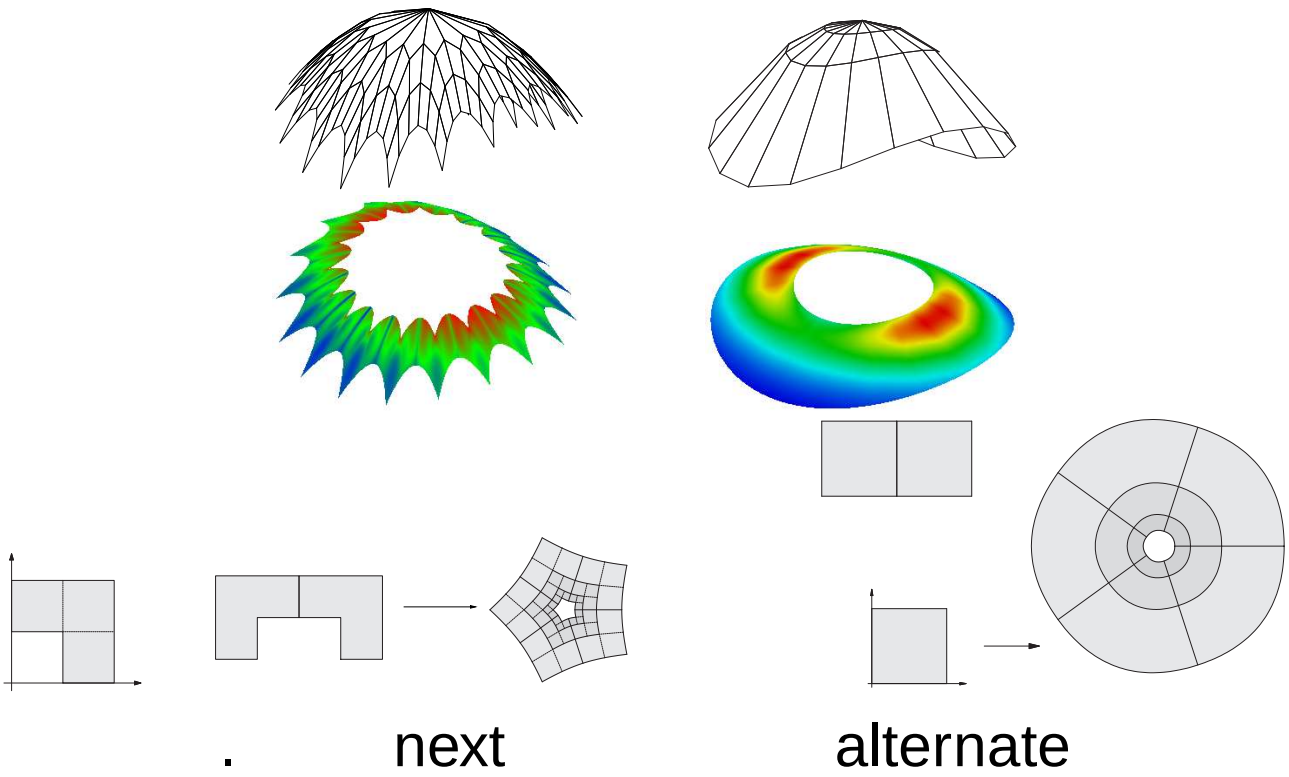

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Modeling challenge

Pre-polar (CC only)

1. Align control mesh along features.

2. Use only quads.

3. Keep valence low.

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Modeling challenge

With polar

1. Align control mesh along features.
2. Use quads *and* polar structures.
3. Keep quad-mesh valence low. High polar valence OK!

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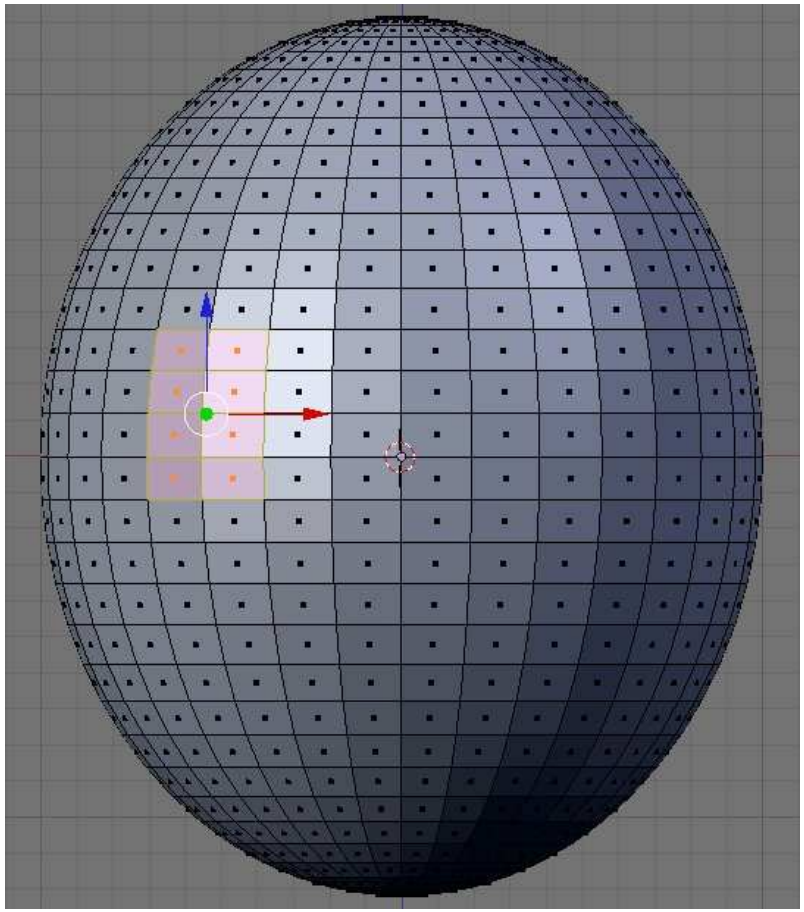
Model a face using polar structures and multi-sided (in blender)



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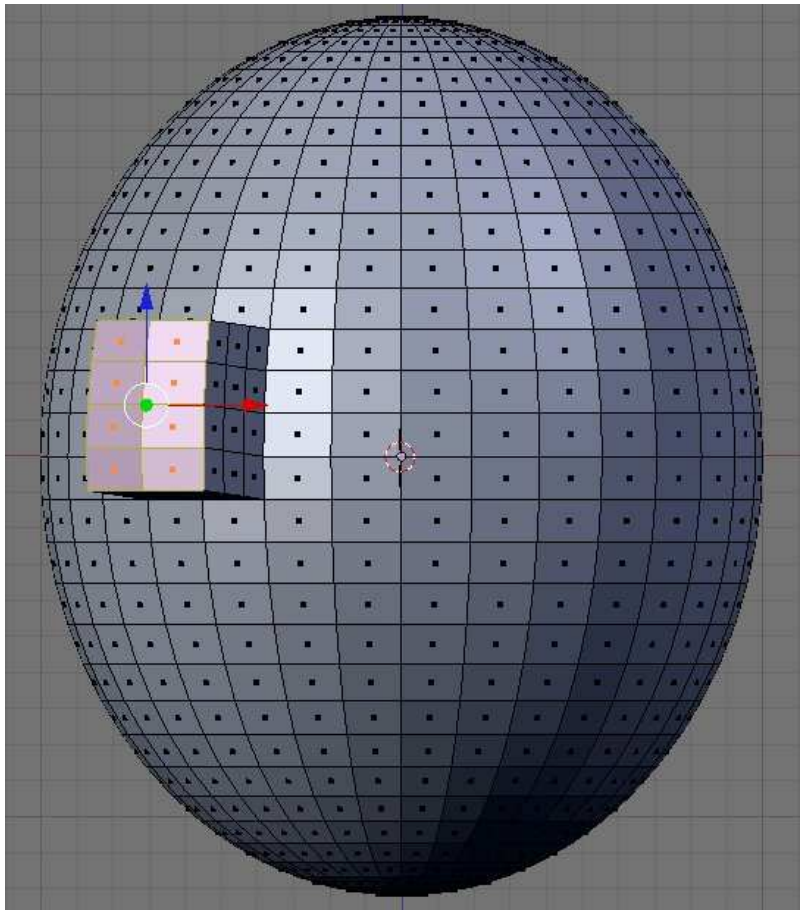

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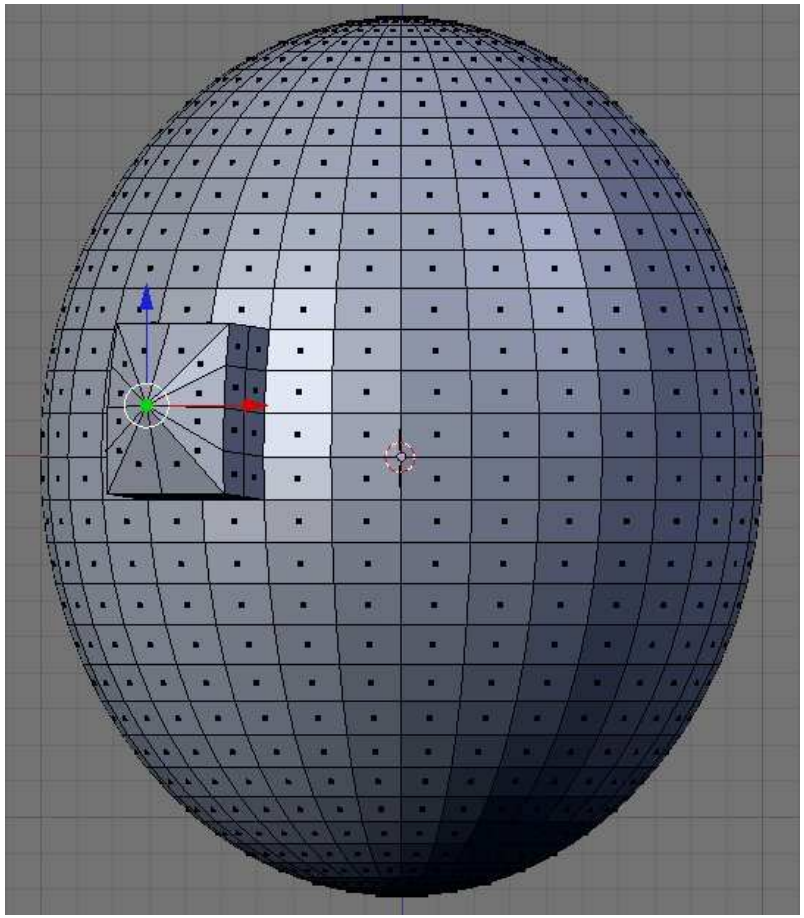

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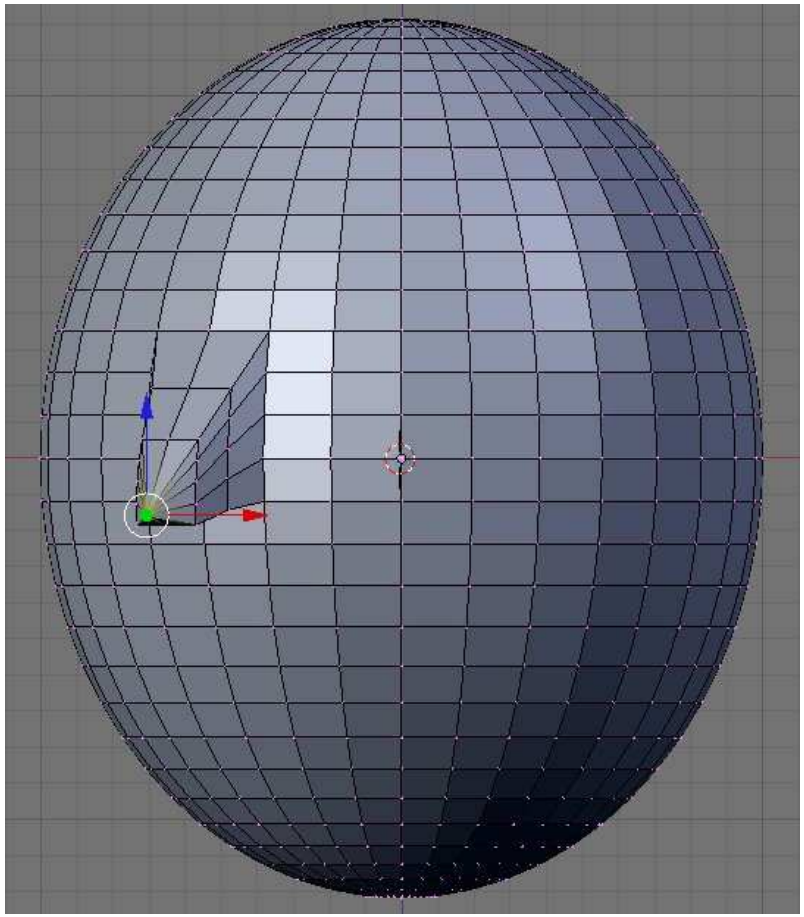

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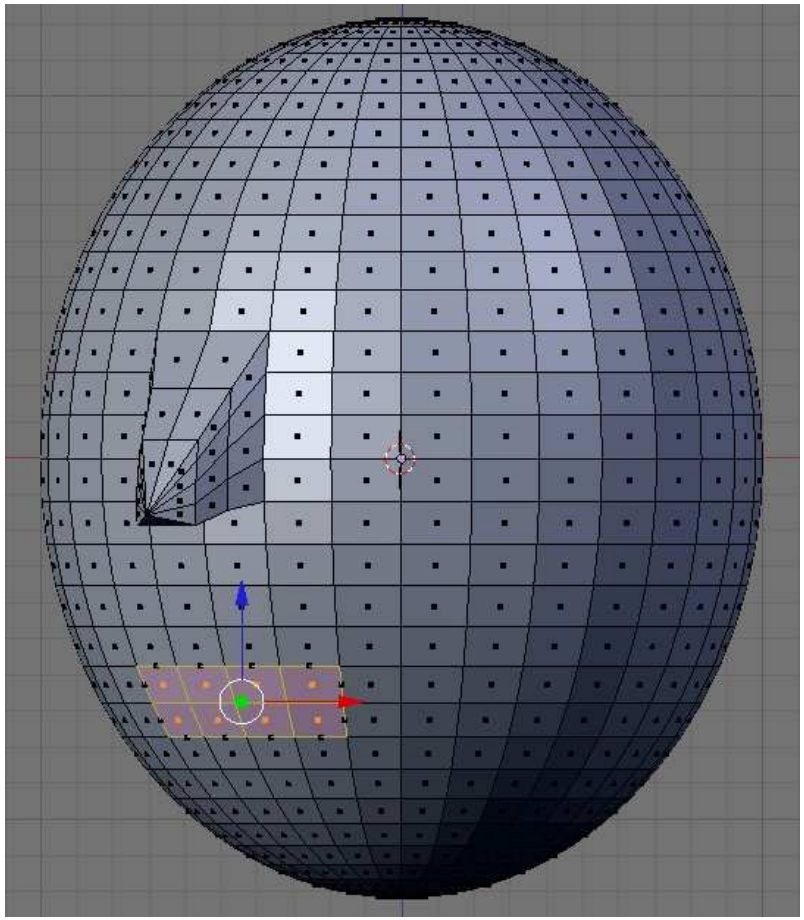

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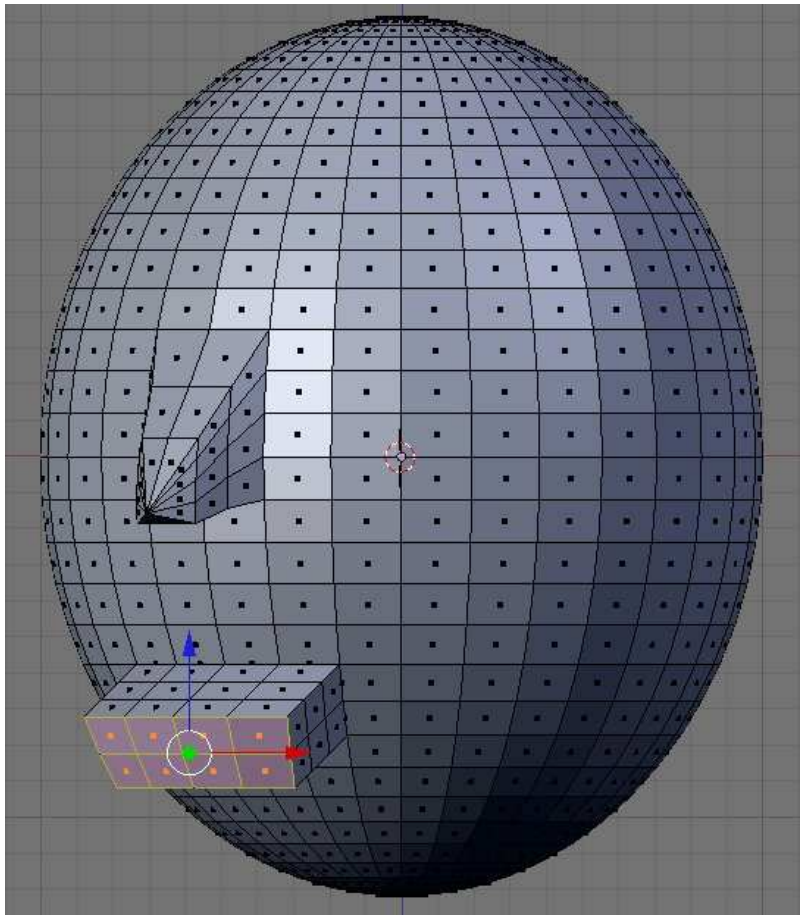

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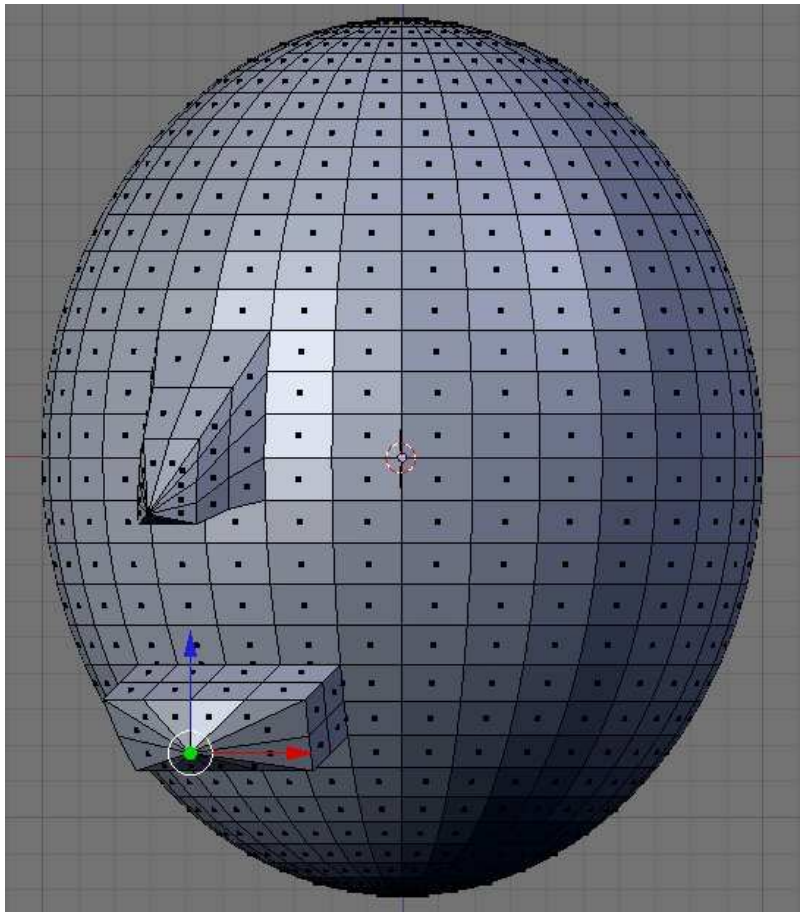

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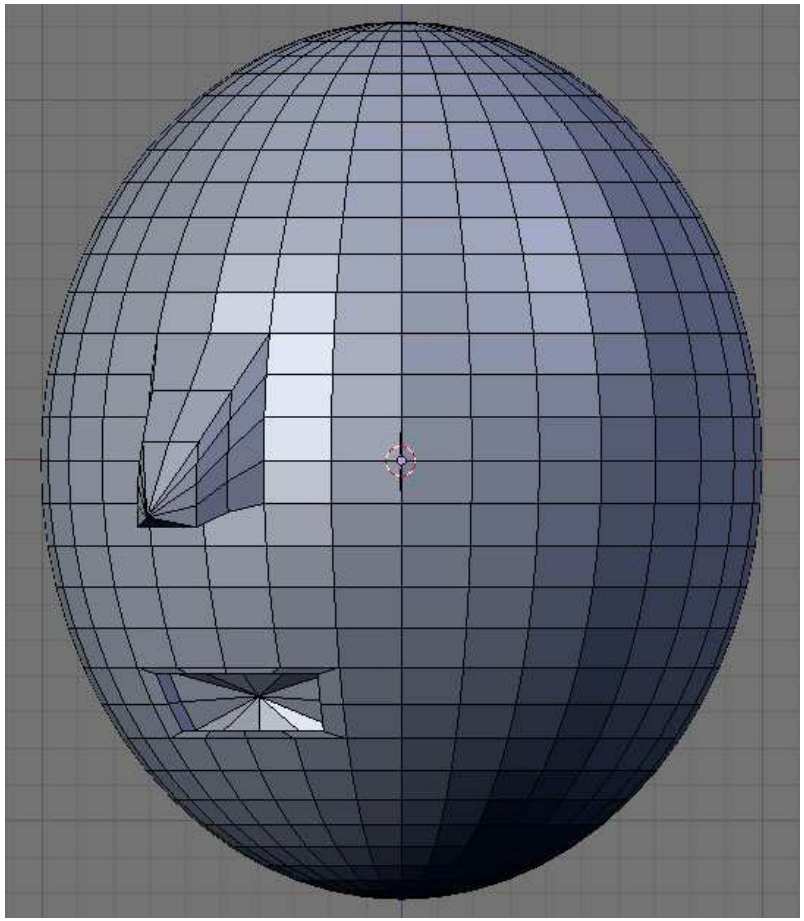

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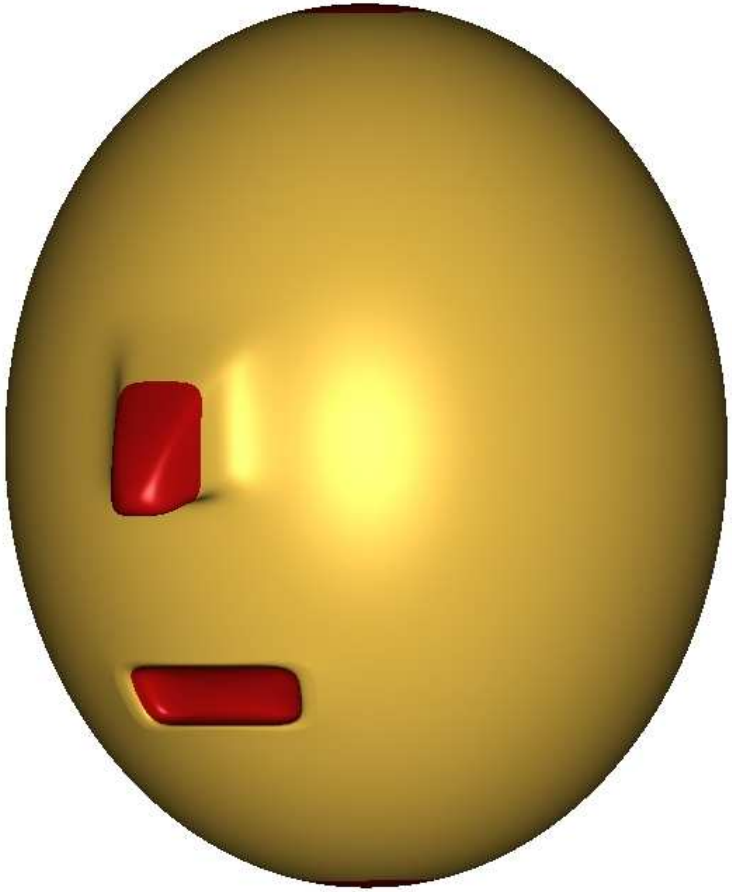

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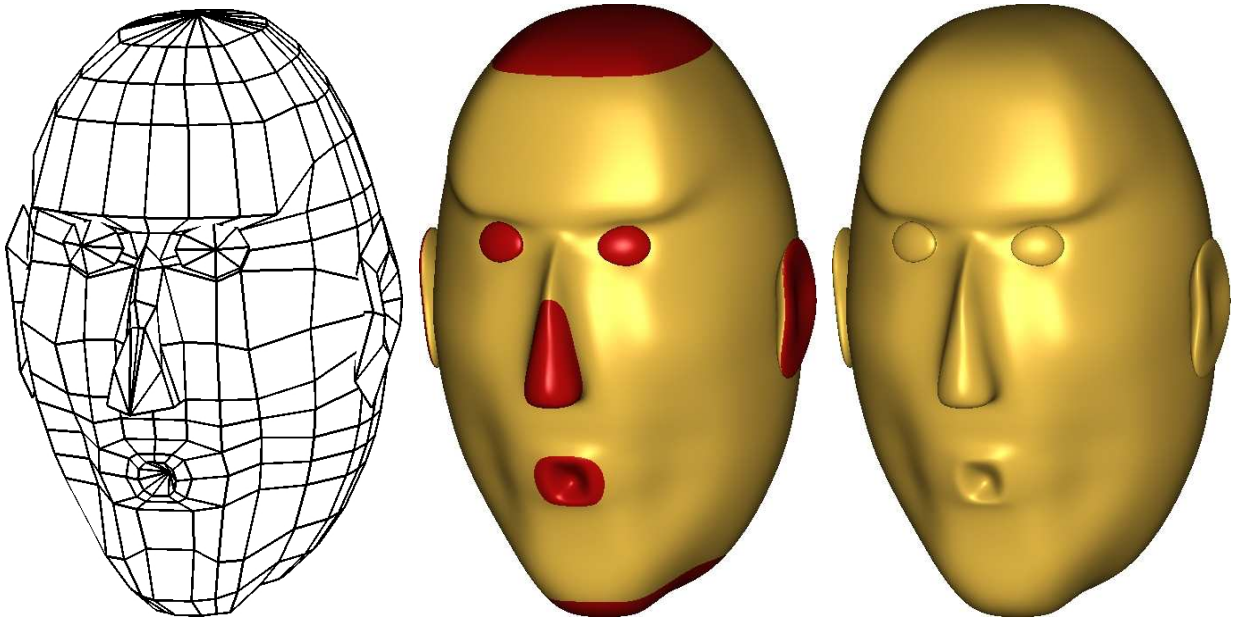
Modeling with polar connectivity



1. keeps the Catmull-Clark valence low,
2. shifts high-valence connectivity to polar structures, and
3. orients the control lines along model features (e.g. mouth).

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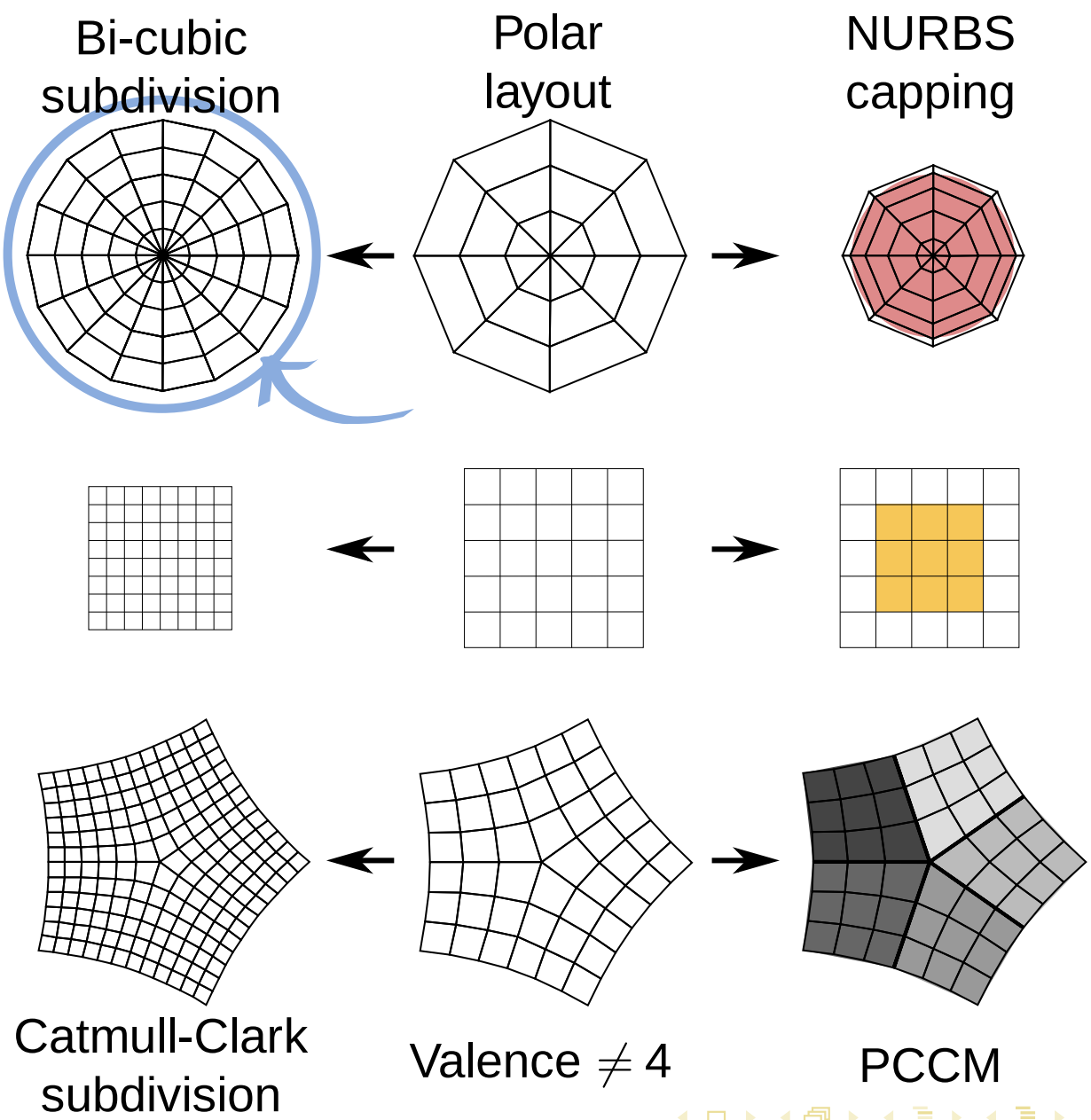
Mesh Refinement



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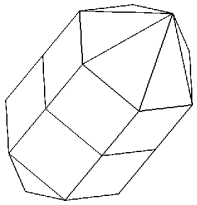

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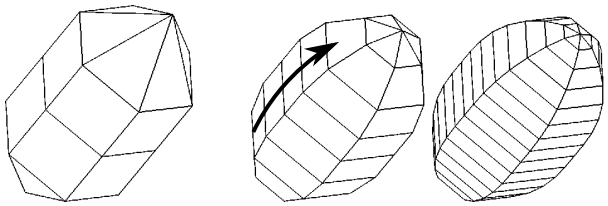
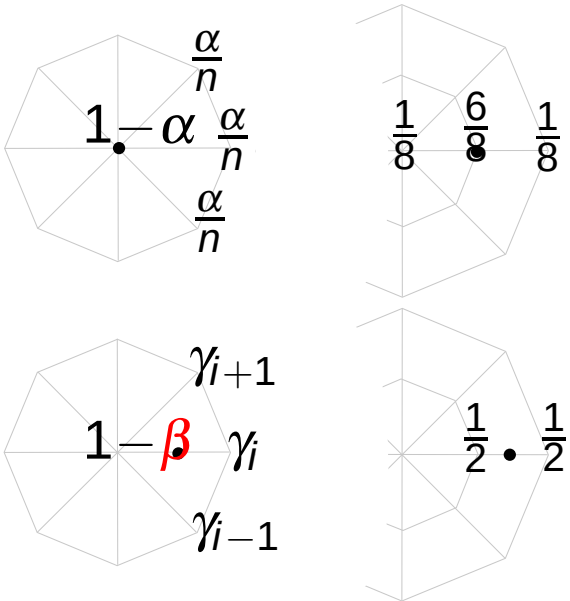
Polar refinement



radial



radial

$$\alpha := \beta - \frac{1}{4}, \quad \beta := \frac{5}{8},$$
$$c_n^k := \cos\left(\frac{2\pi k}{n}\right),$$
$$\gamma_k := \frac{1}{n}\left(\beta - \frac{1}{2} + \frac{5}{8}c_n^k + (c_n^k)^2 + \frac{1}{2}(c_n^k)^3\right)$$

Smooth Parametric Surface Construction

Jörg Peters

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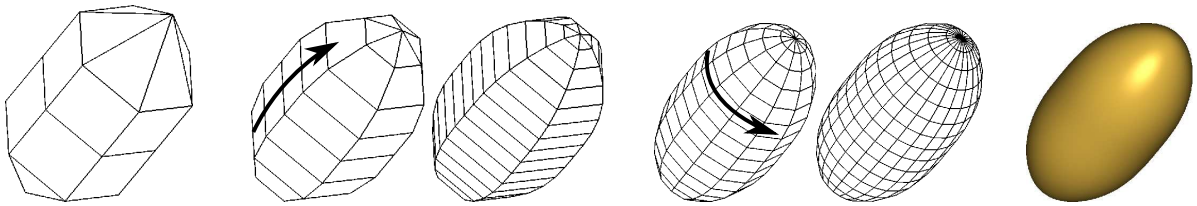
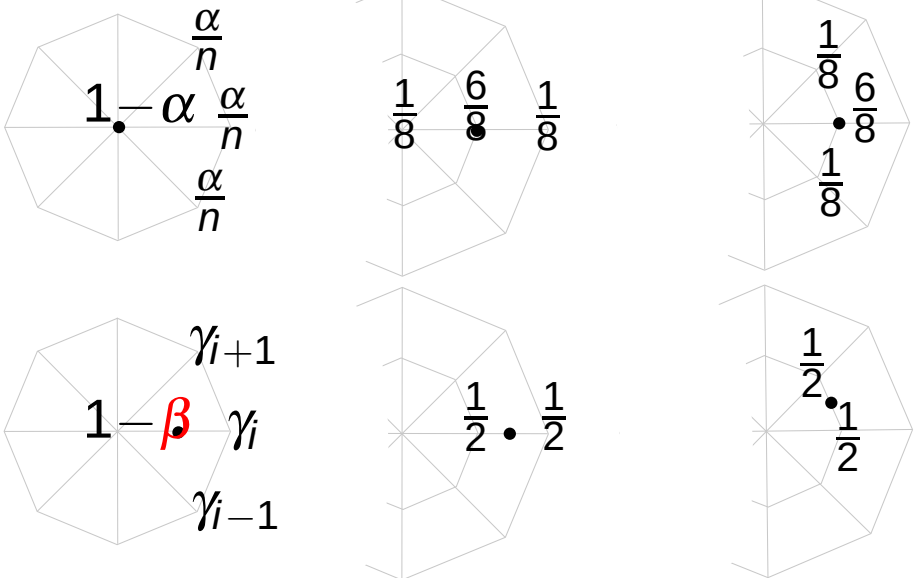


radial circular



radial circular

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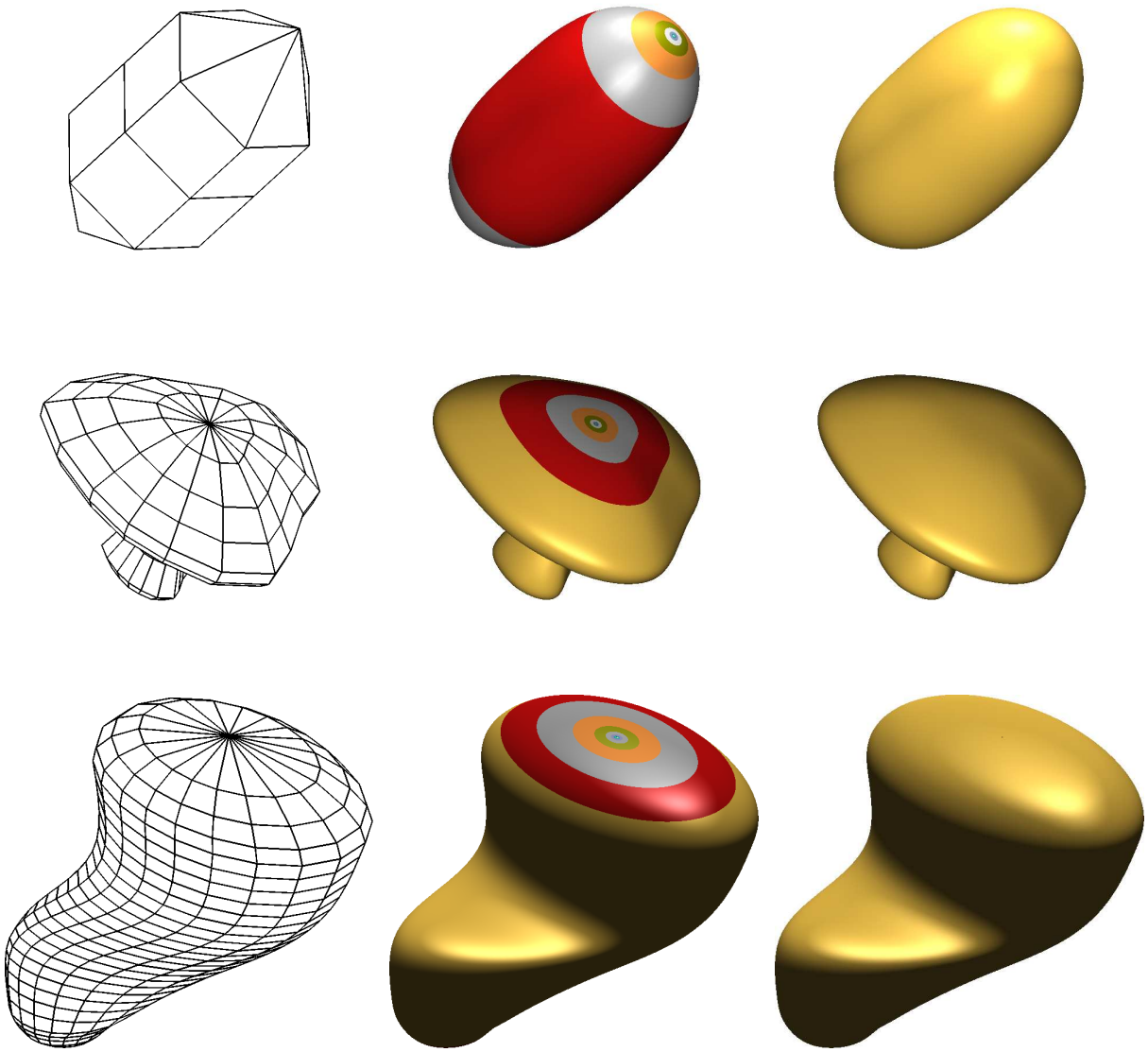
Purely Polar



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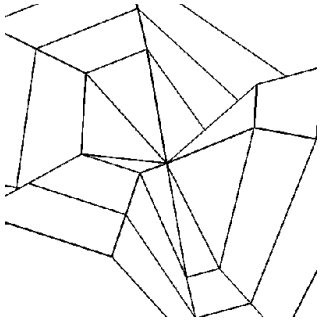
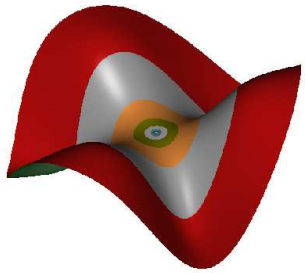
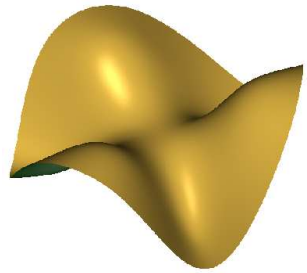
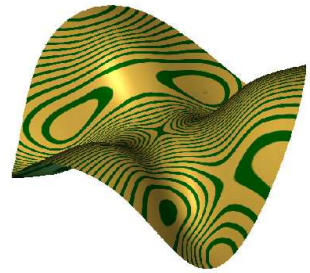

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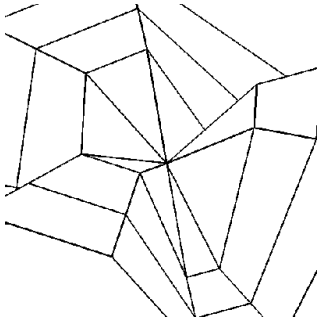
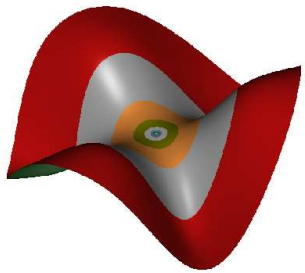
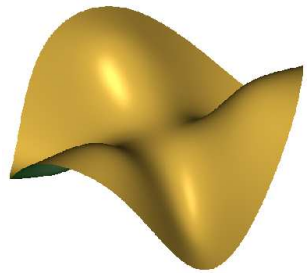
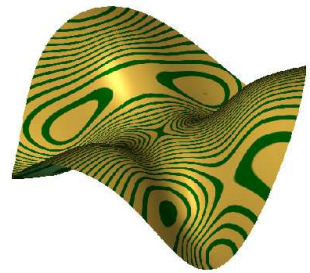

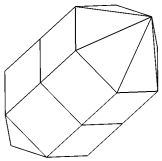
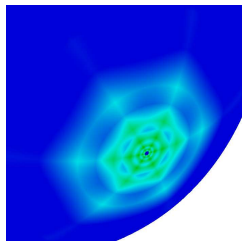
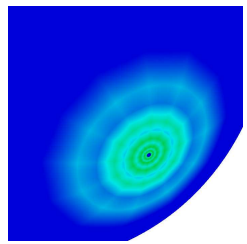
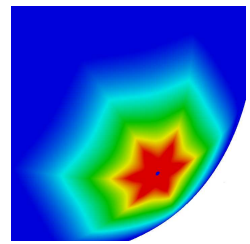
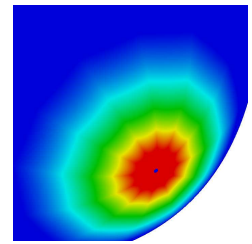

control net

$n = 6$ $\beta = \frac{5}{8}$

$n = 12$ $\beta = \frac{5}{8}$

$n = 6$ $\beta = \frac{1}{2}$

$n = 12$ $\beta = \frac{1}{2}$

Smooth Parametric Surface Construction

Jörg Peters

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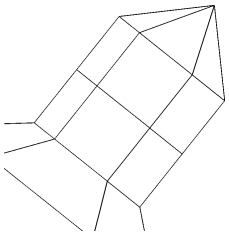
Bi-3 Polar complements Catmull-Clark



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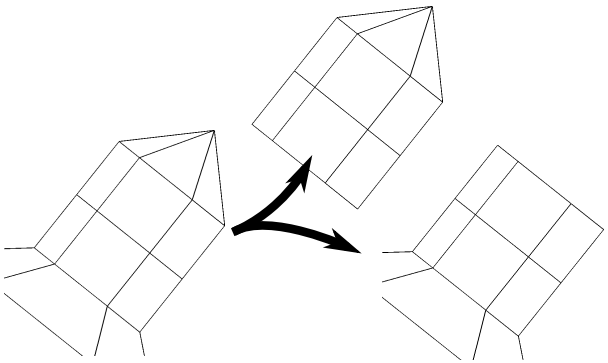

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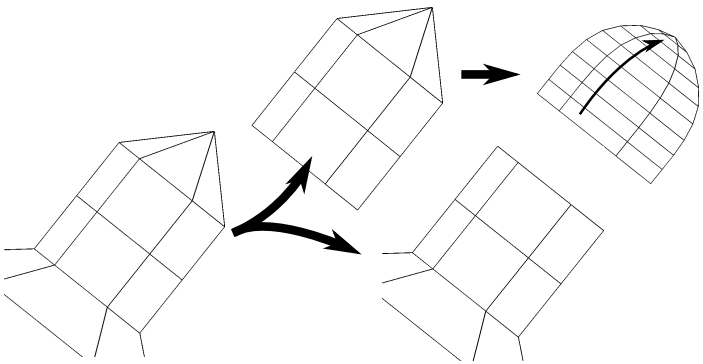

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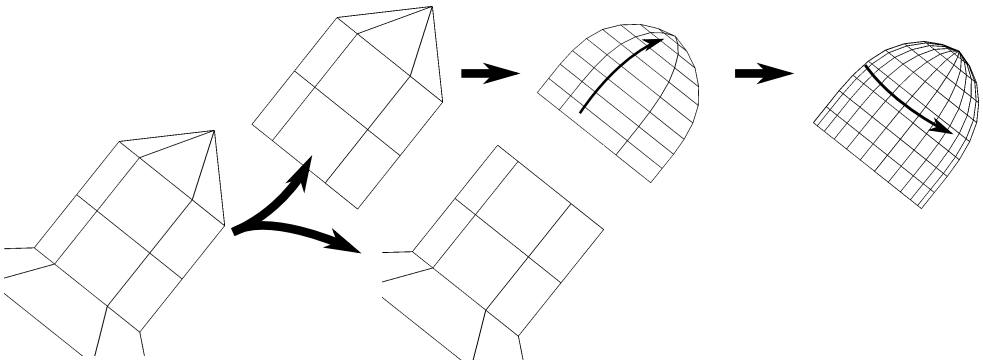

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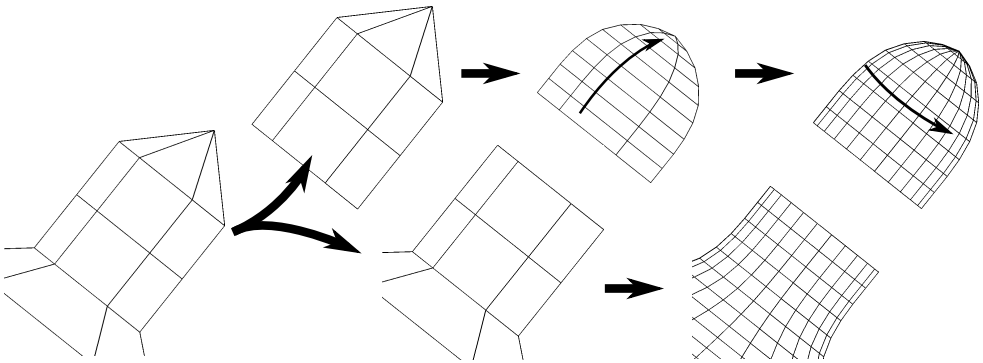

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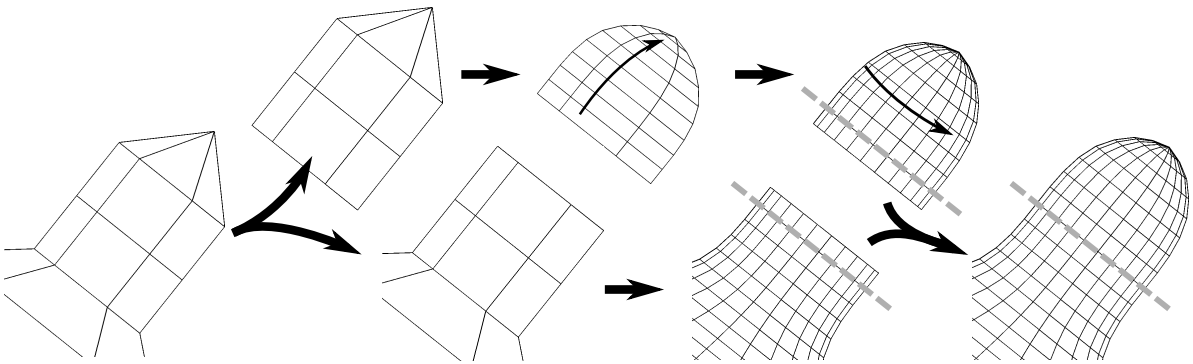

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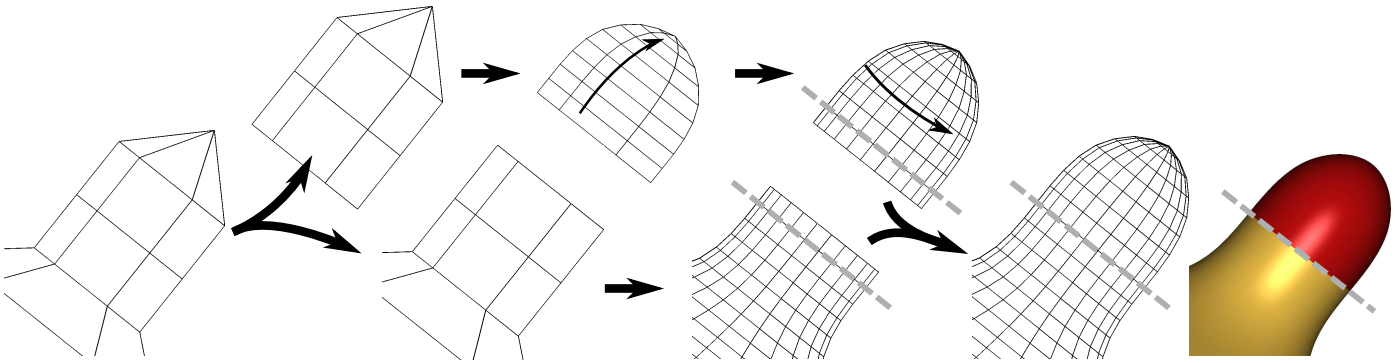

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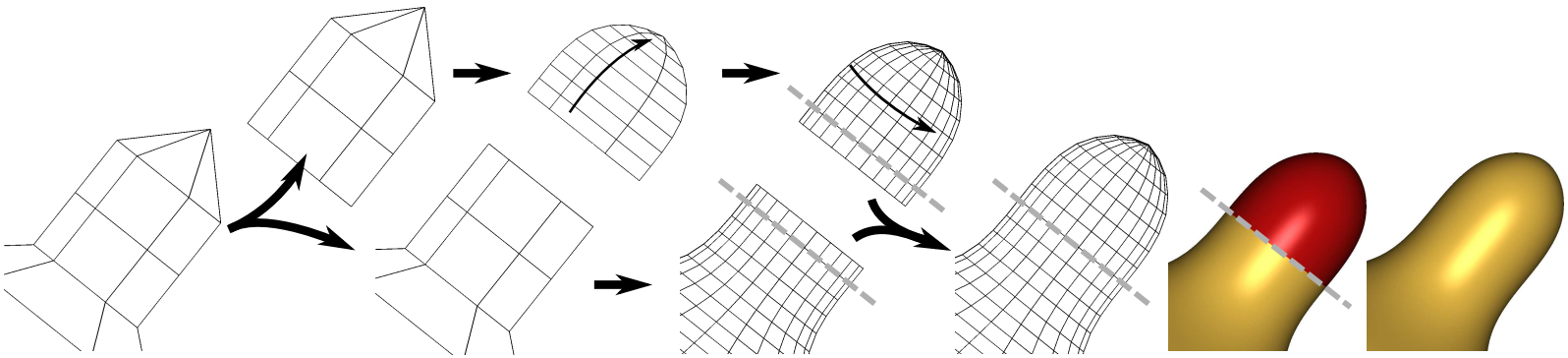

$\Rightarrow$ C^1 with bounded curvature at the polar limit point.

- Verified using standard analysis tools from subdivision theory.

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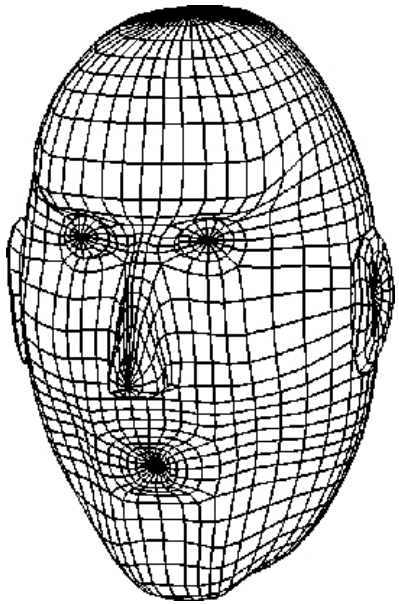
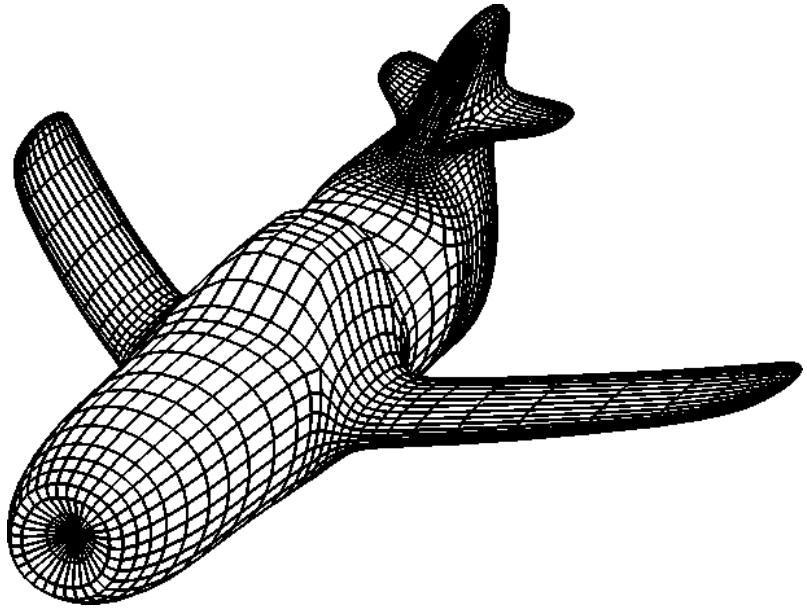
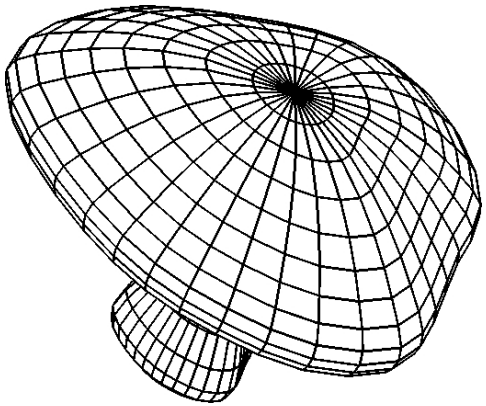
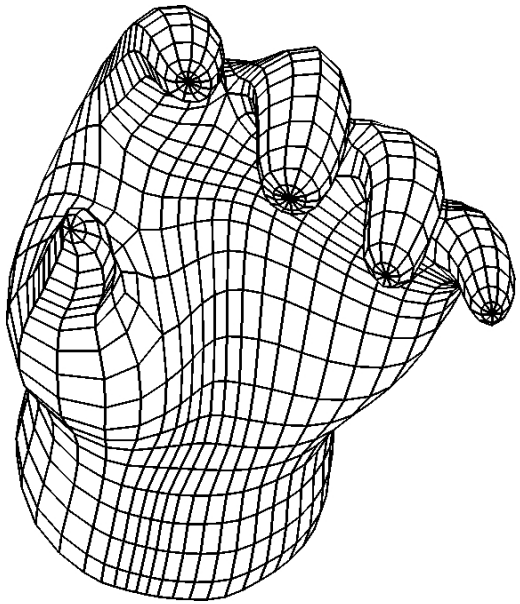
Refined Meshes









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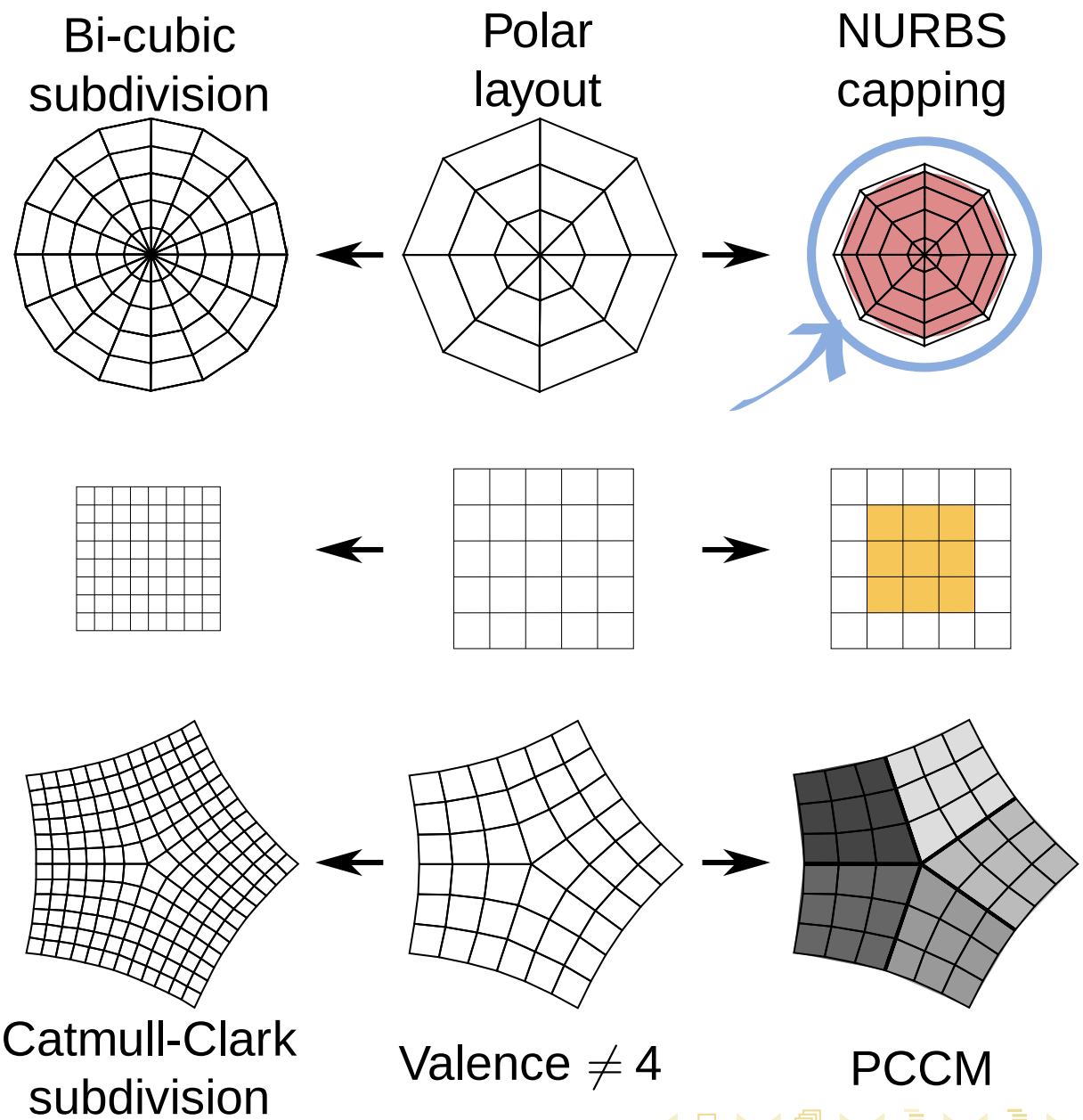

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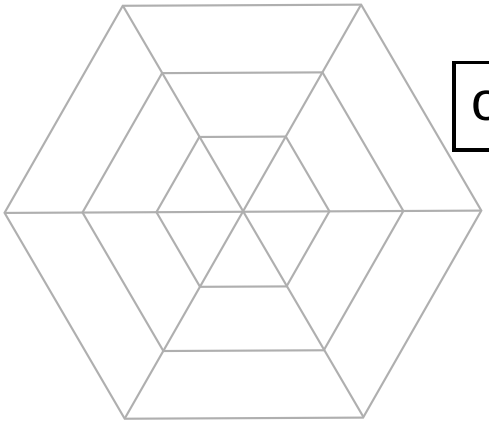
Polar structures can be C^1 capped by a single NURBS patch



original control mesh

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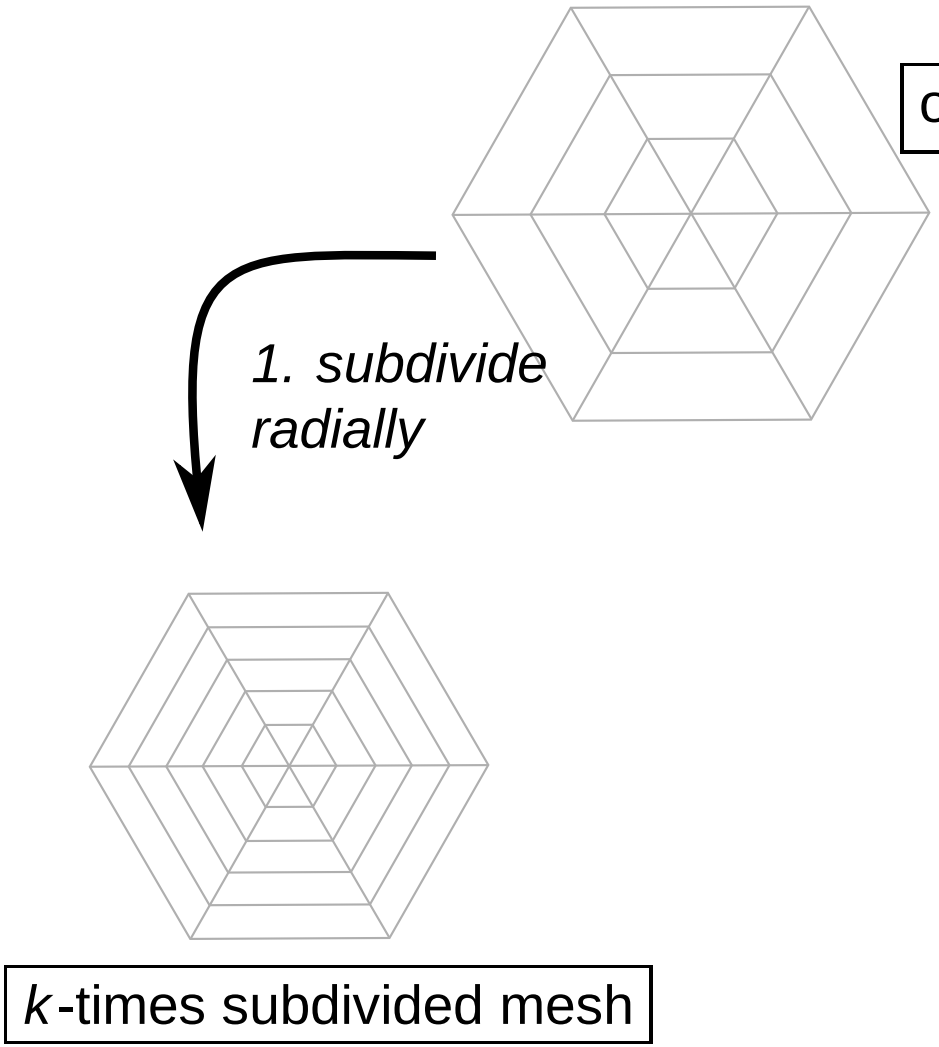

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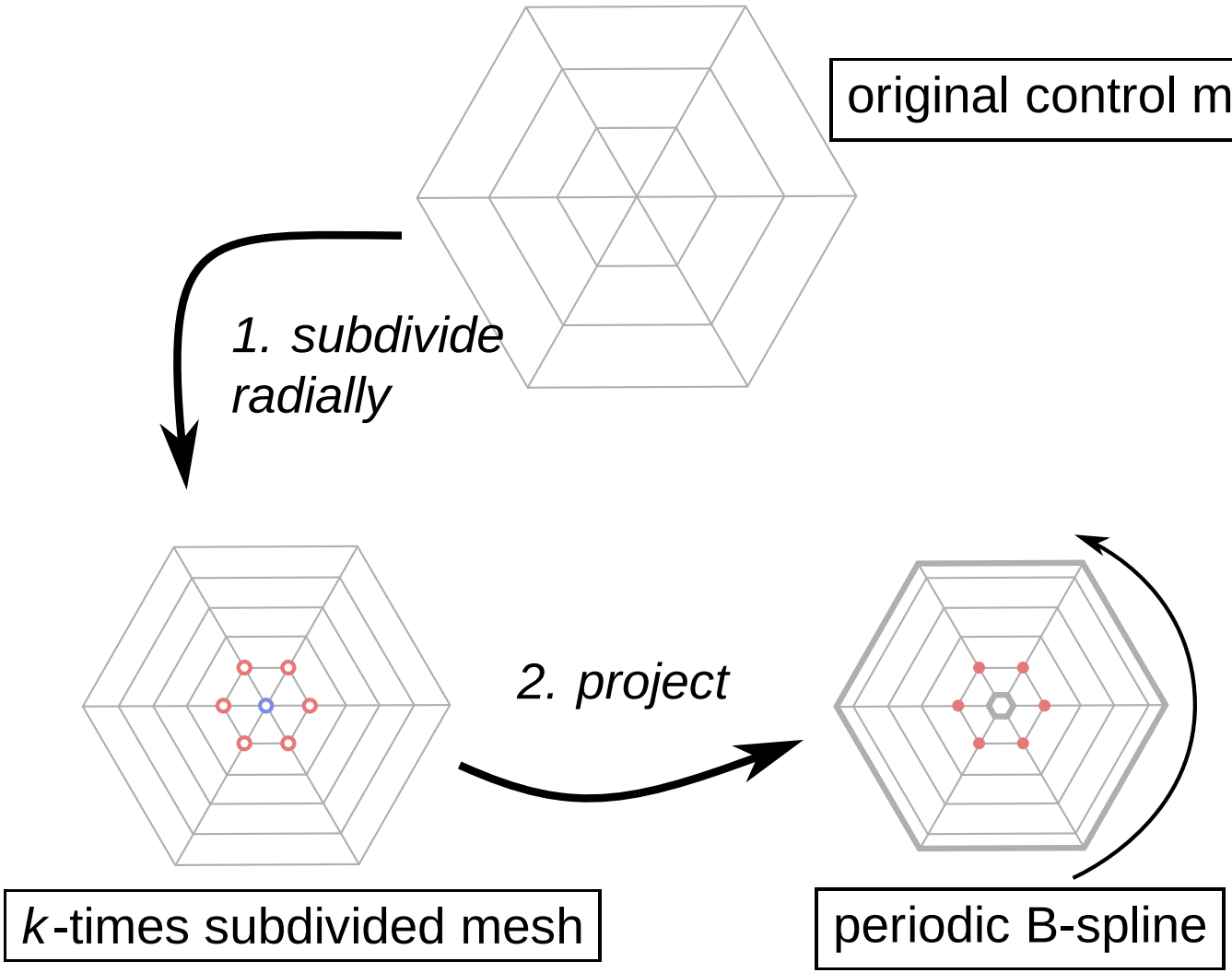

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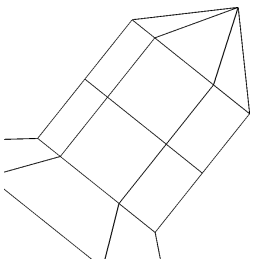

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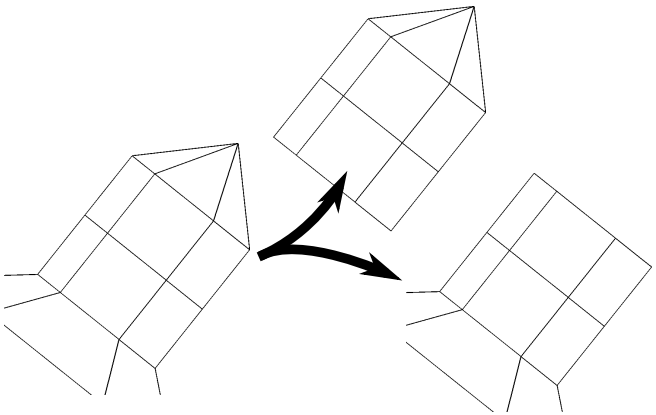

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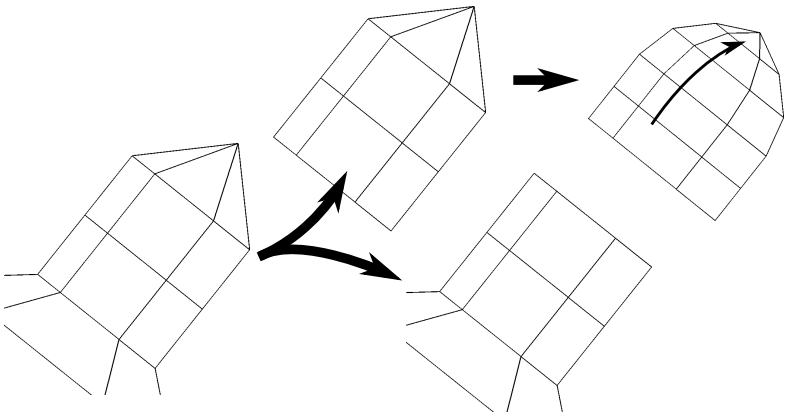

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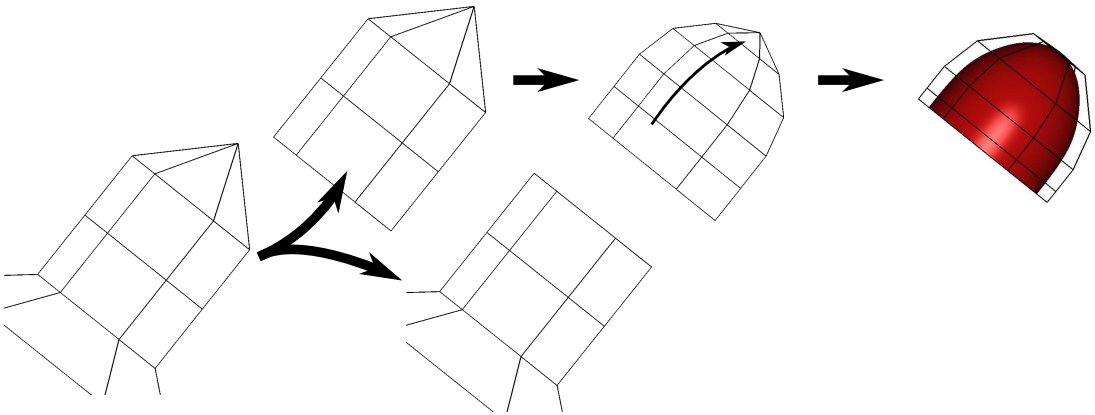
Capping Polar with a single NURBS patch



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Fast Parallel Surface Construction

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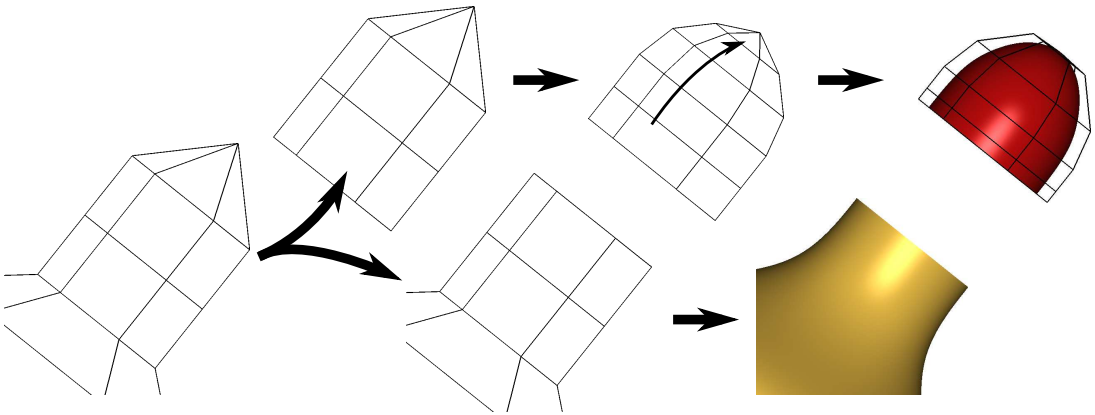

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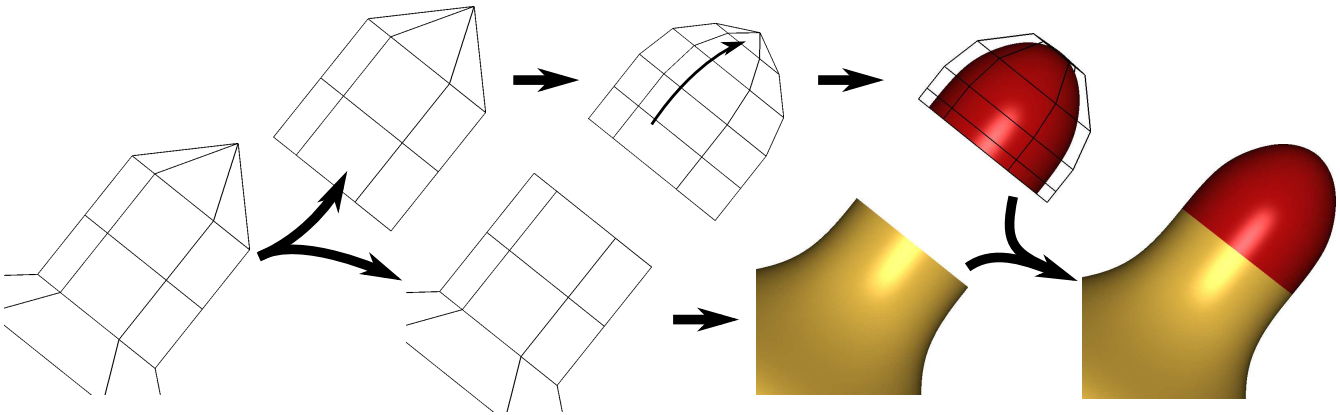

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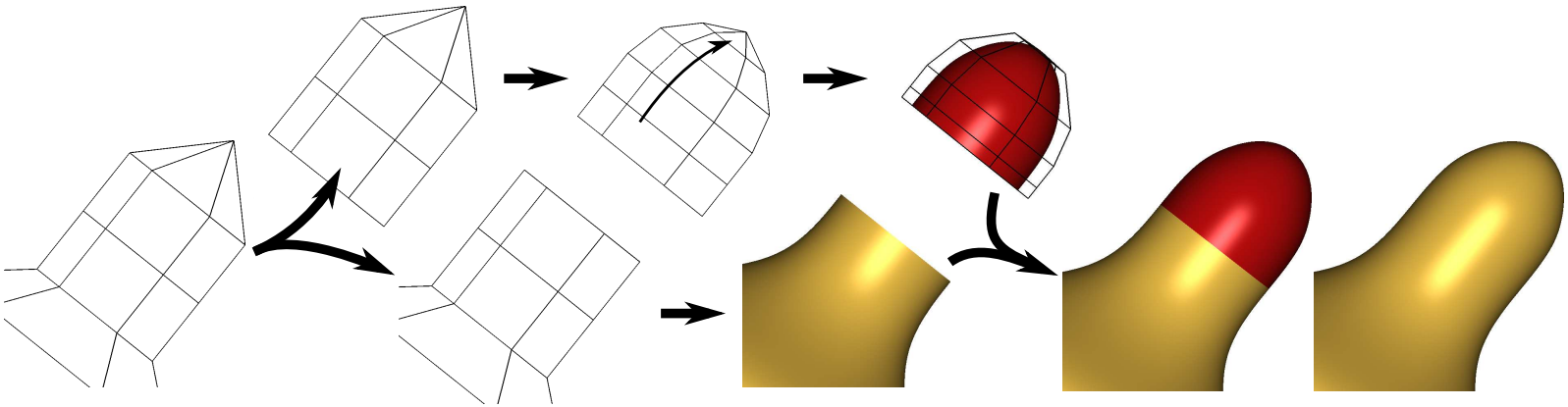

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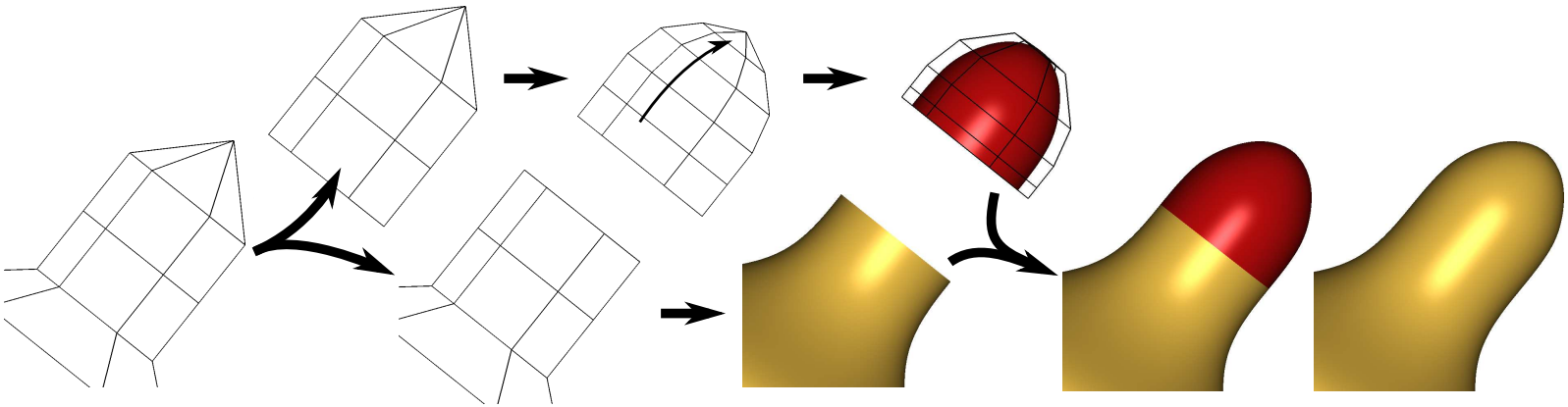

$\Rightarrow$ C^1 with bounded curvature at the polar limit point.

- Singular parametrization analyzed by auxiliary subdivision scheme!

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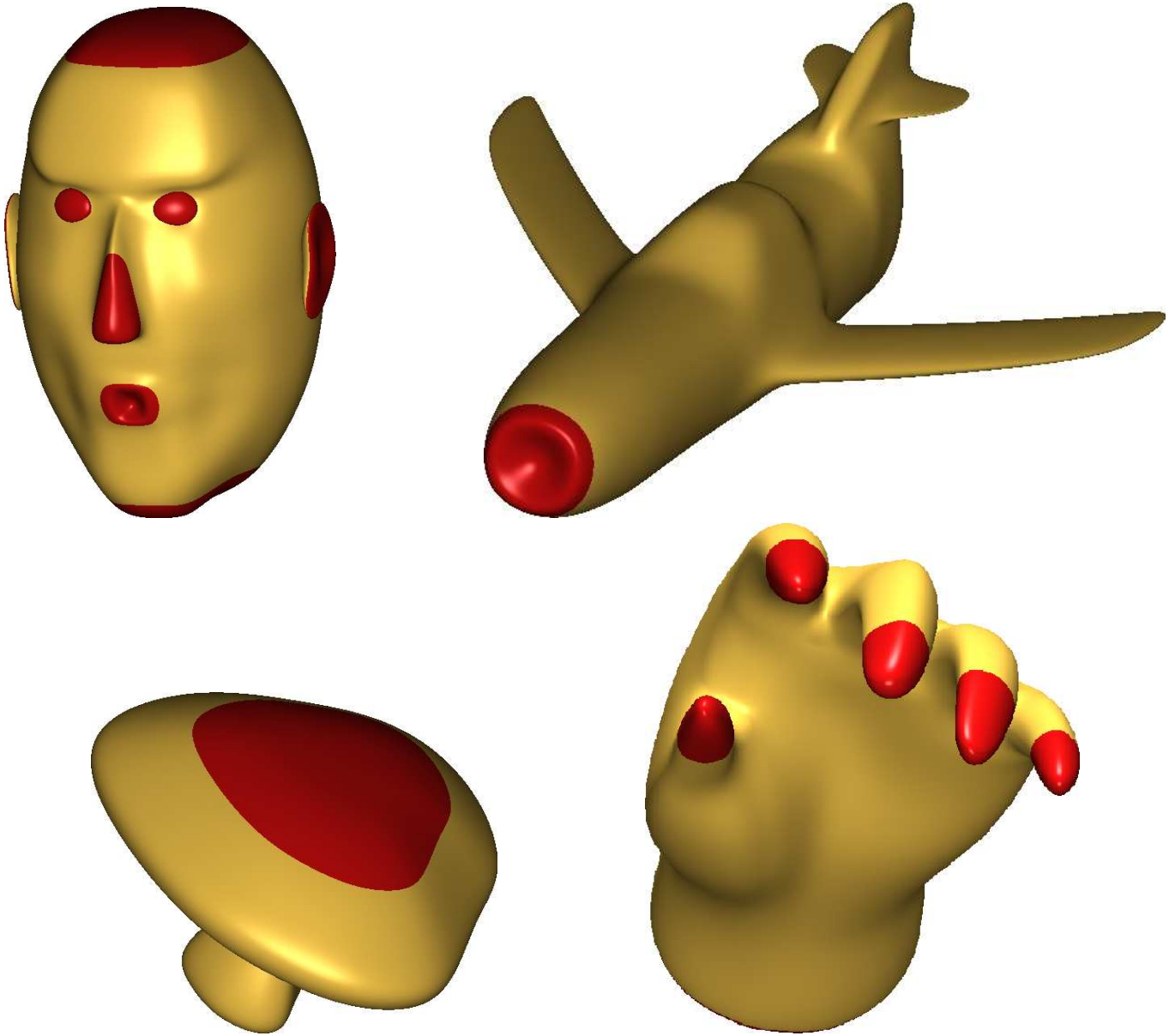
Spline Models



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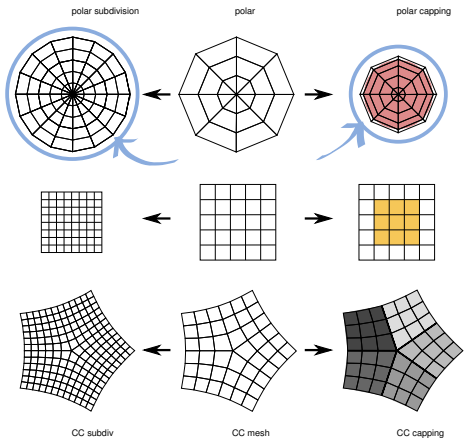
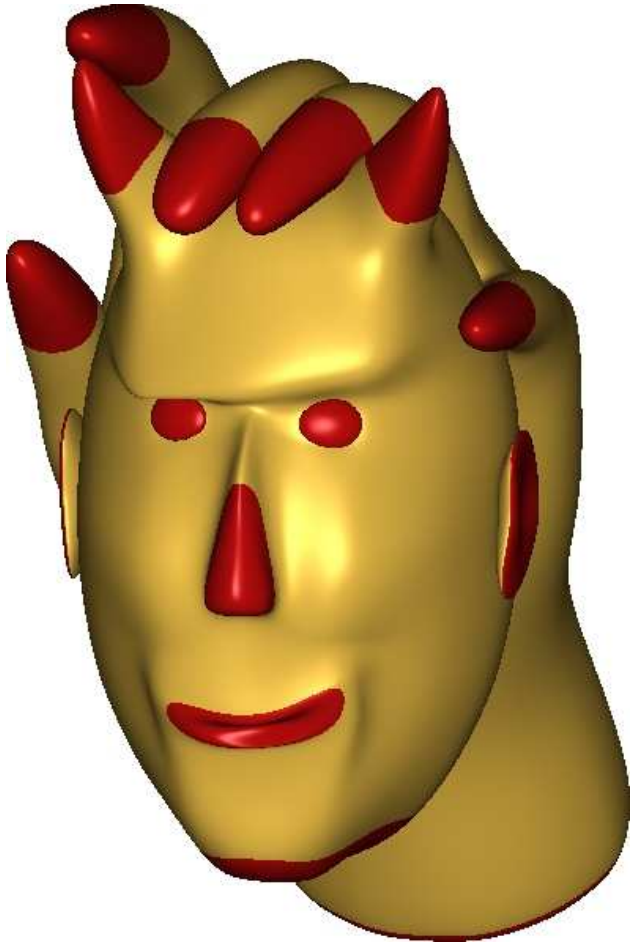
Modeling with polar connectivity





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.. in partes tres

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- Animation/Simulation: surfaces constructed from sparse meshes or ‘skeletons’ on the fly; discarded at $>$ 30 frames per second.
  Not A-class surface design!
- Graphics Processing Unit GPU: : $\geq$128 single instruction multiple data streams.
- Mesh mutation on graphics hardware [Shiue, Peters 2003]: co-processor for (simple, parallel) algorithmic tasks on meshes

Smooth Parametric Surface Construction

Jörg Peters

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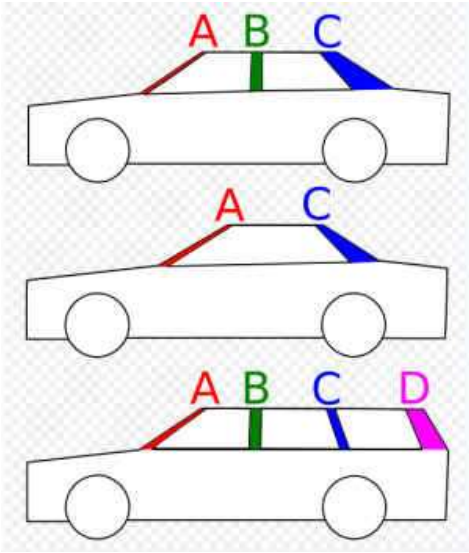

C-pillar

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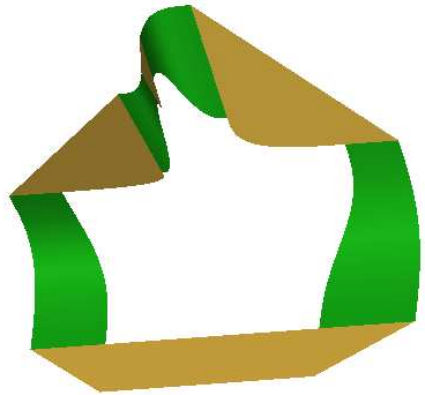

Primary surface, pairwise blend, multi-sided blend

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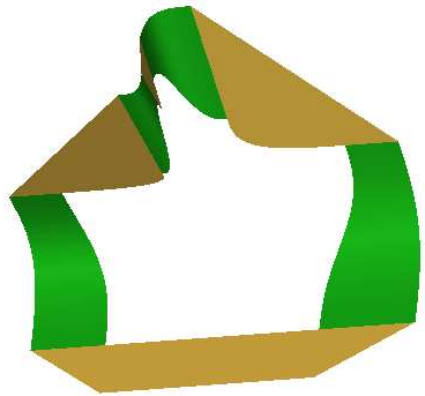

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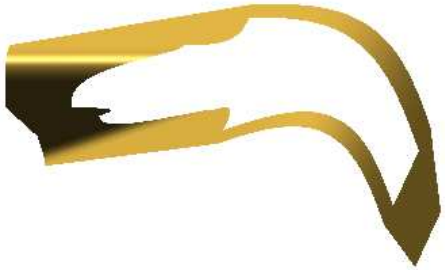
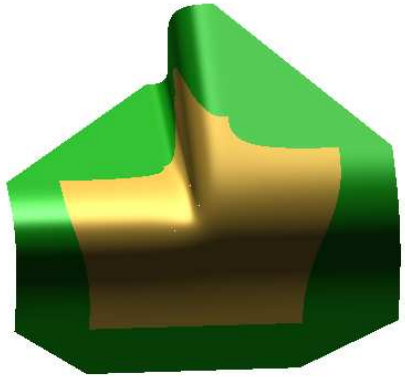
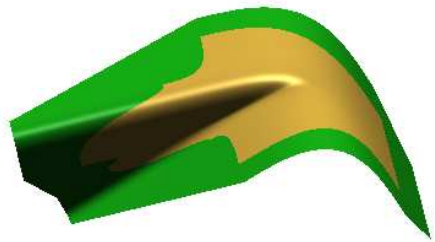

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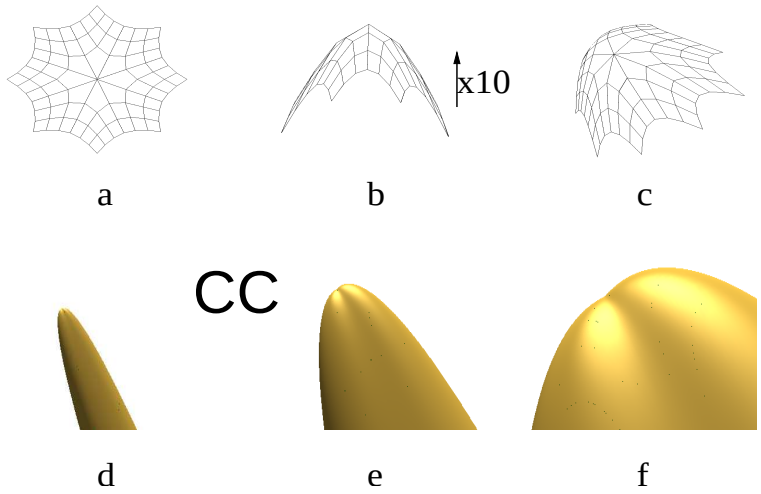

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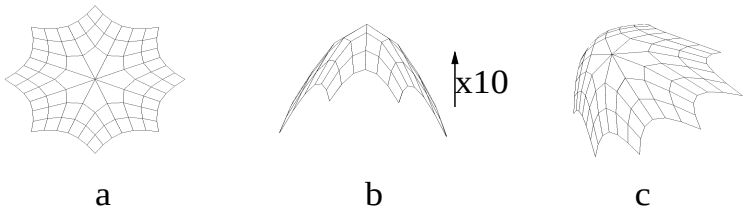
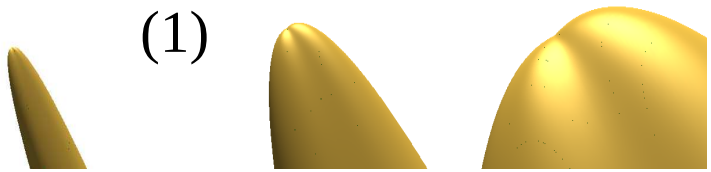


a b c

(1)



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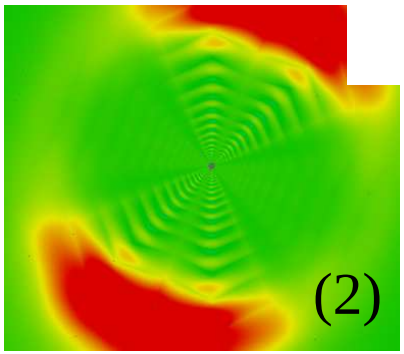

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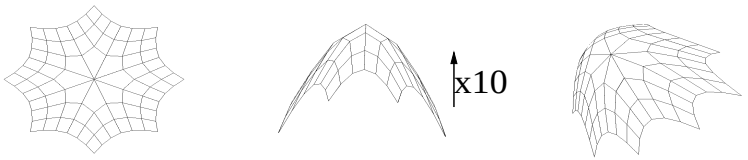

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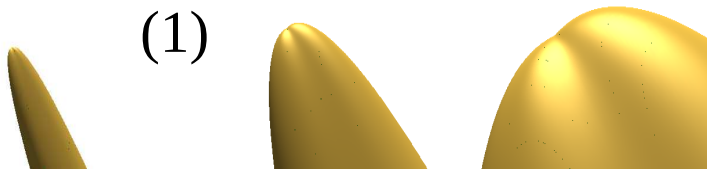

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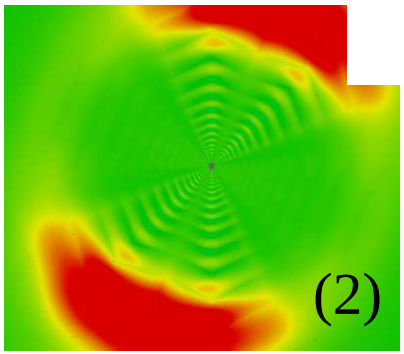

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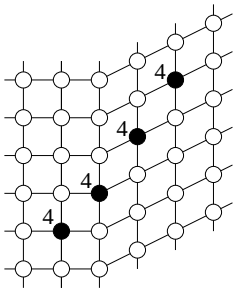

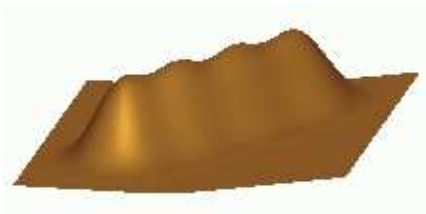

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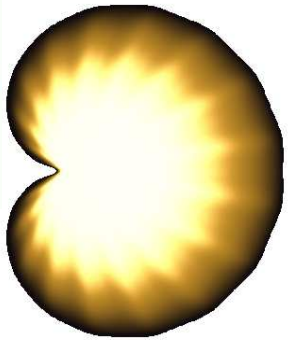

(4)

Smooth Parametric Surface Construction

Jörg Peters

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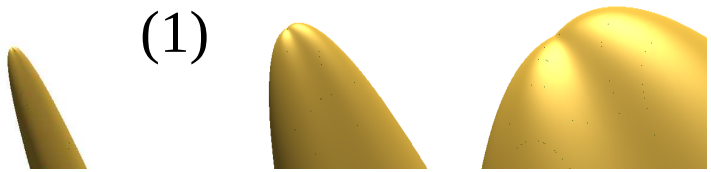

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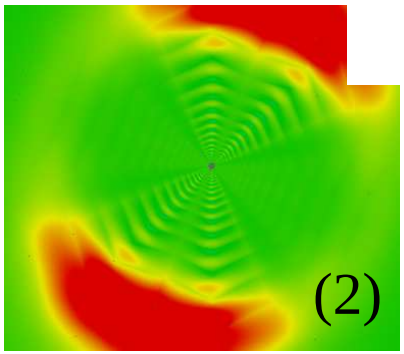

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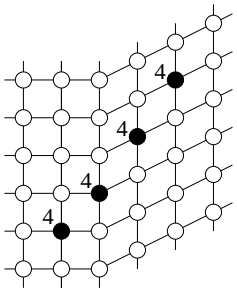

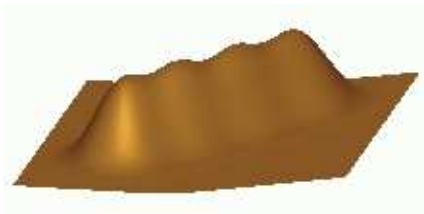

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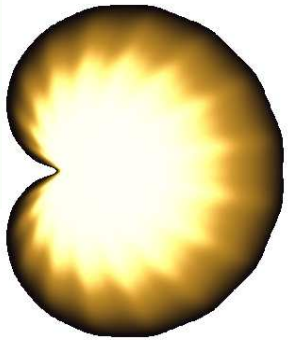

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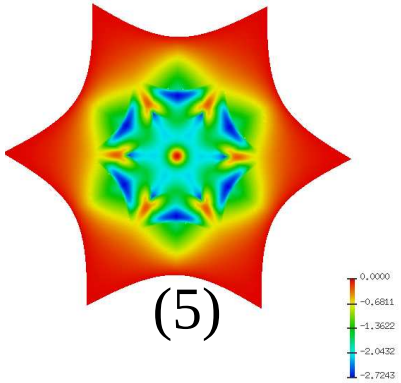

(5)

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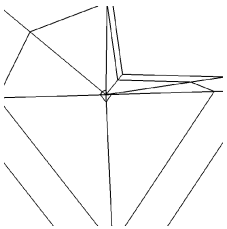
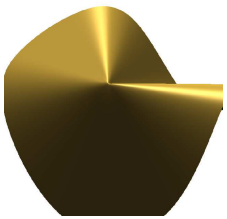
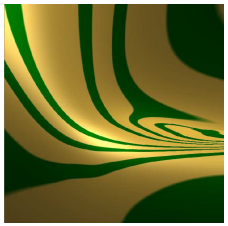
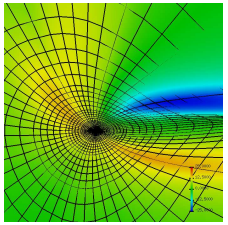
High-quality surface blend: working definition

Given input data to be matched,

a multi-sided surface blend is of high quality if

- it does not create curvature features not implied by the input data
- Secondary characterizations:
  - preserves feature lines
  - minimizes curvature variation.
  - parameter lines ?

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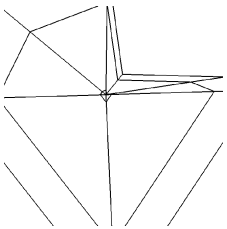
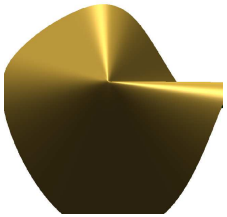
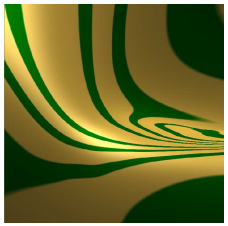
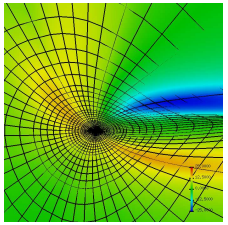

Smooth Parametric Surface Construction

Jörg Peters

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High-quality surface blend: working definition

Given input data to be matched,
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- it does not create curvature features not implied by the input data
- Secondary characterizations:
  - preserves feature lines
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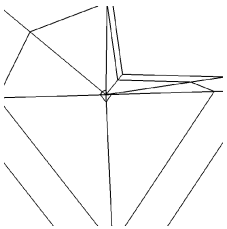
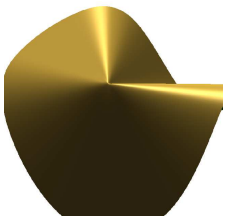
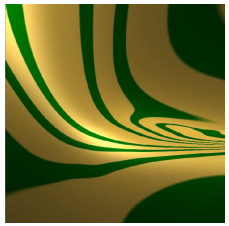
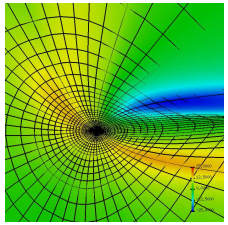
  - ? parameter lines ?

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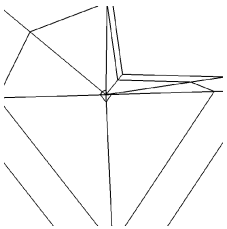
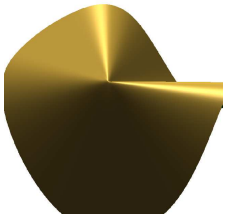
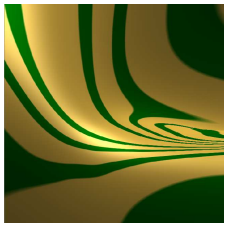
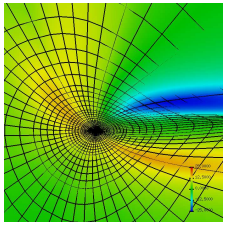

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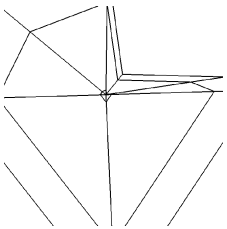
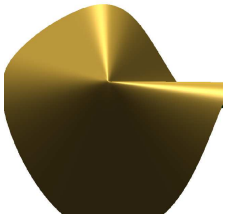
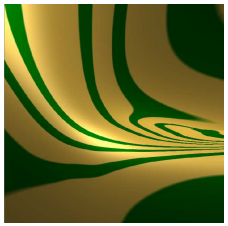
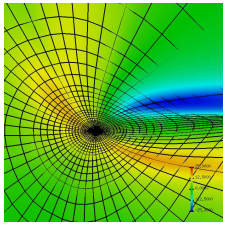

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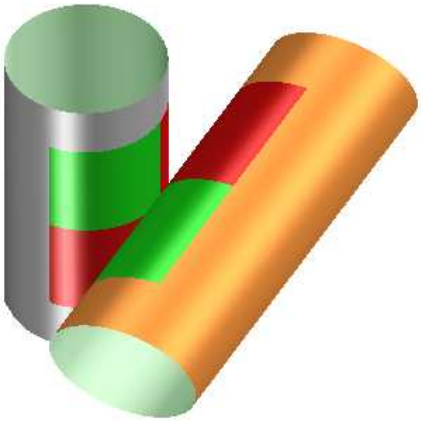
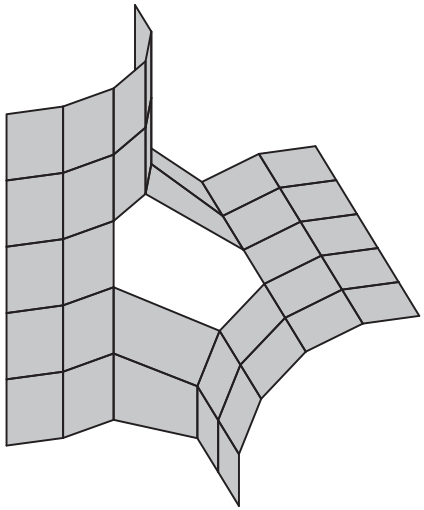
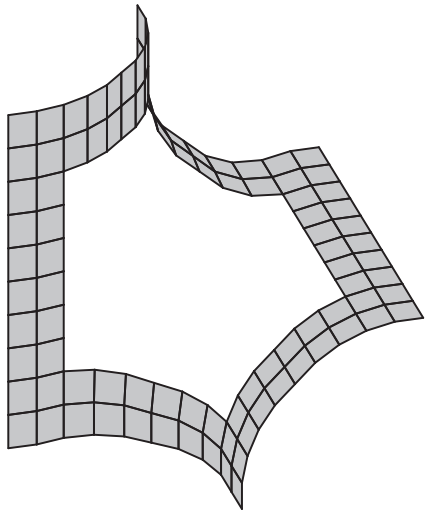


cylinders



spline net



depth 2

tensor-border

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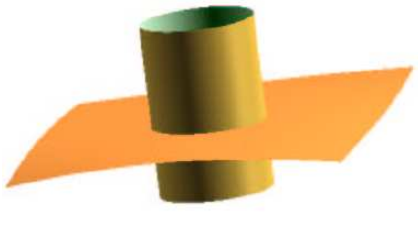
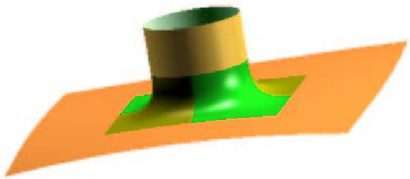
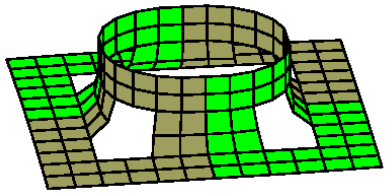
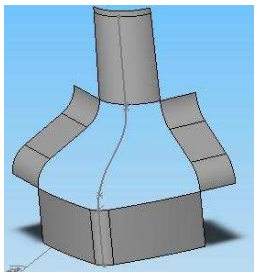
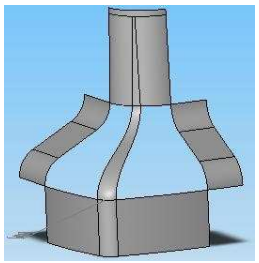




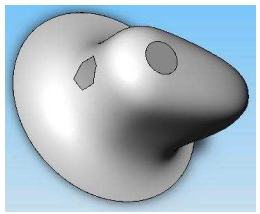
Blends

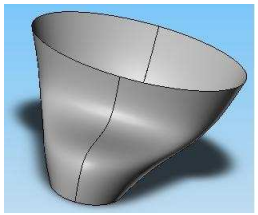






Design Modif





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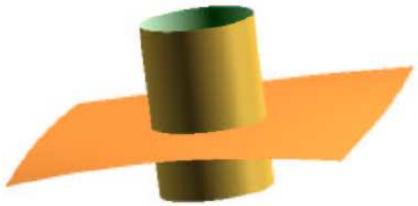
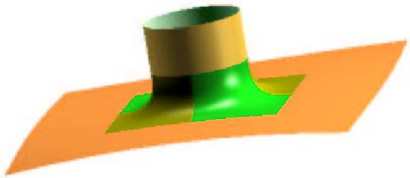
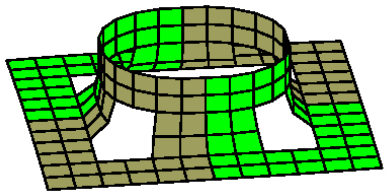
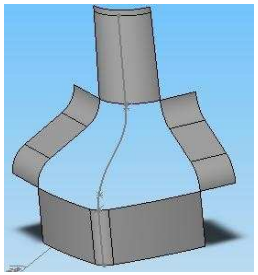
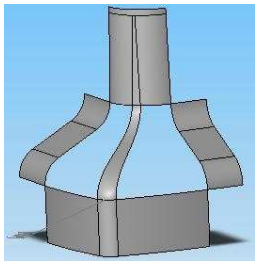



Blends



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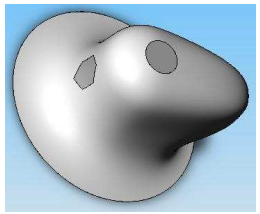
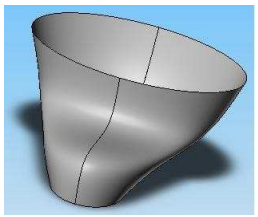
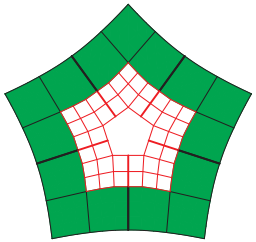
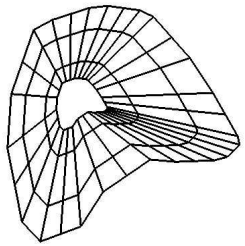
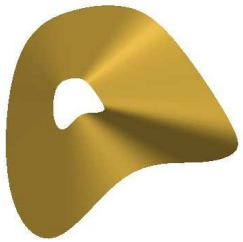





Subdivision

Smooth Parametric Surface Construction

Jörg Peters

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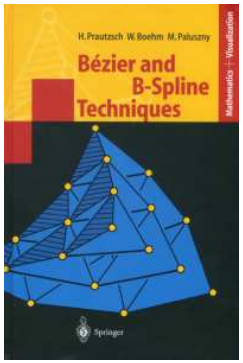
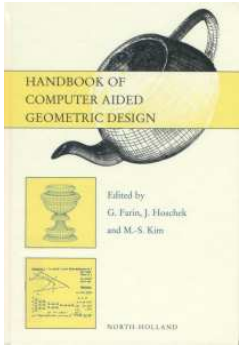
Construction: Smoothness not a challenge

G^k: Matching derivatives after reparameterization





G^2 Polynomial [Hahn&Gregory 1988,9], [Ye 1997], [Prautzsch 1997 &Umlauf 2000], [Reif 98], [Gregory&Zhou 1999], [Peters 2002], [Loop et al 2004,8], [Karciauskas& Peters 2004,6], ...
Rational [Loop& DeRose 1995] [Grimm 1997], [Cotrina et al 2000, 2007], ...
non-rational [Ying 2004],...

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- linear functionals: $\min \int \sum (\partial_{ij} \mathbf{x})^2$

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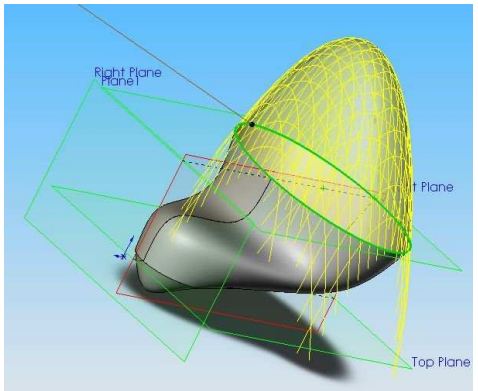
- Use shape hints (guide surface)



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Construction by rings

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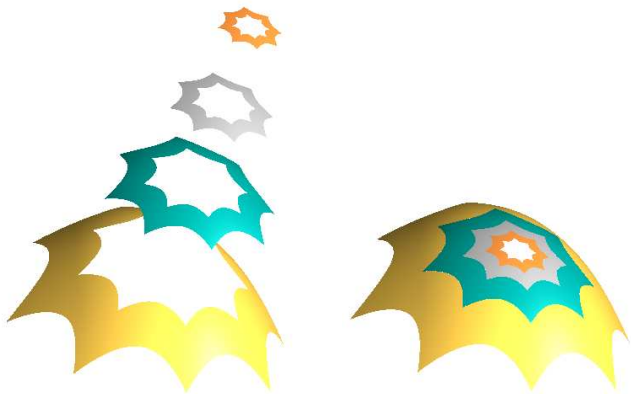

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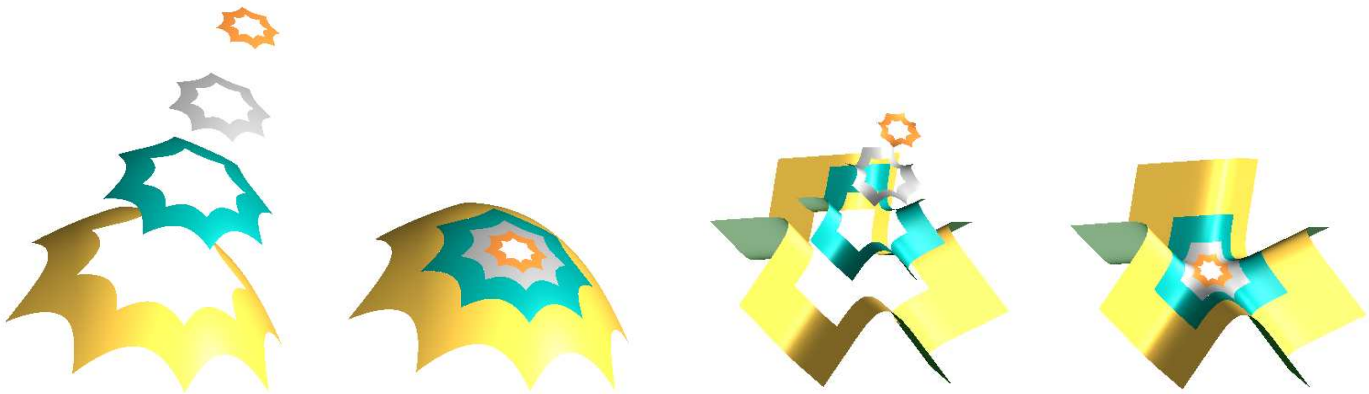

Polar Structure
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Fast Parallel Surface Construction

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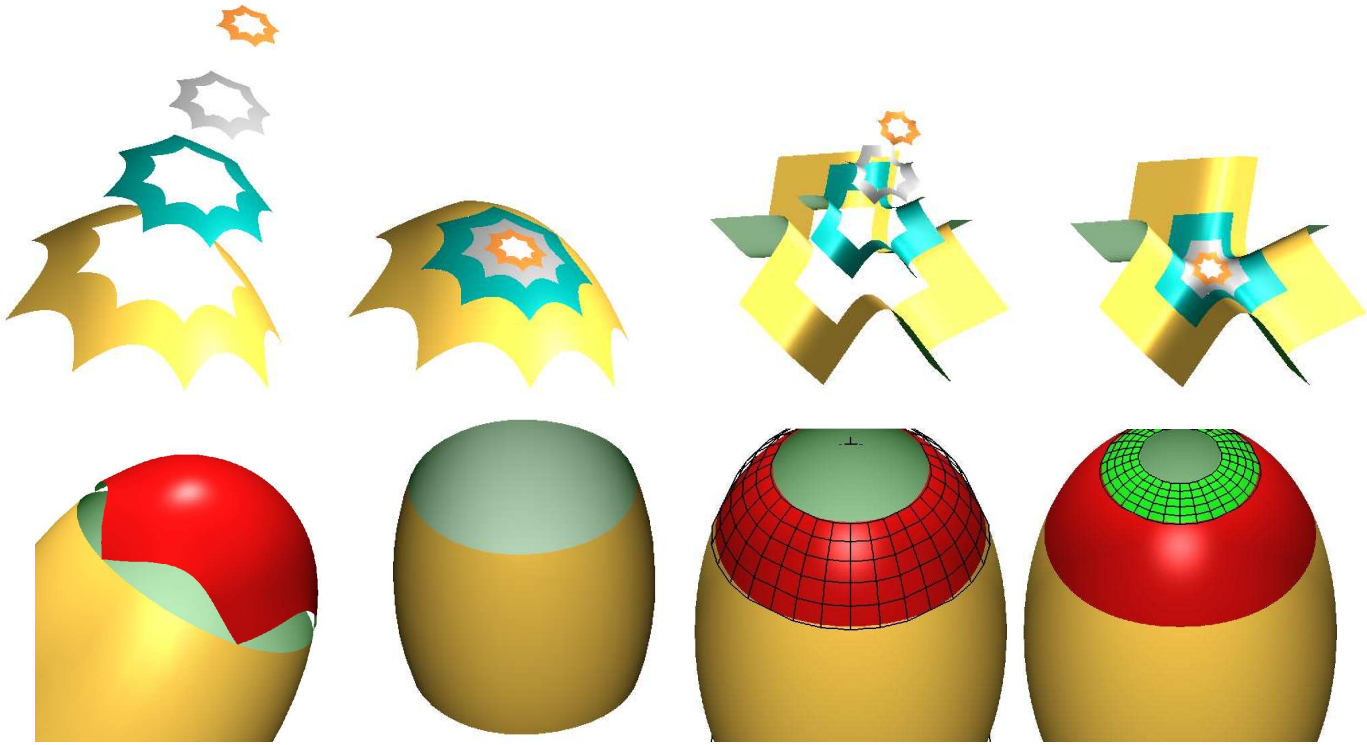
Construction by rings



guide **g**

Subdivision?

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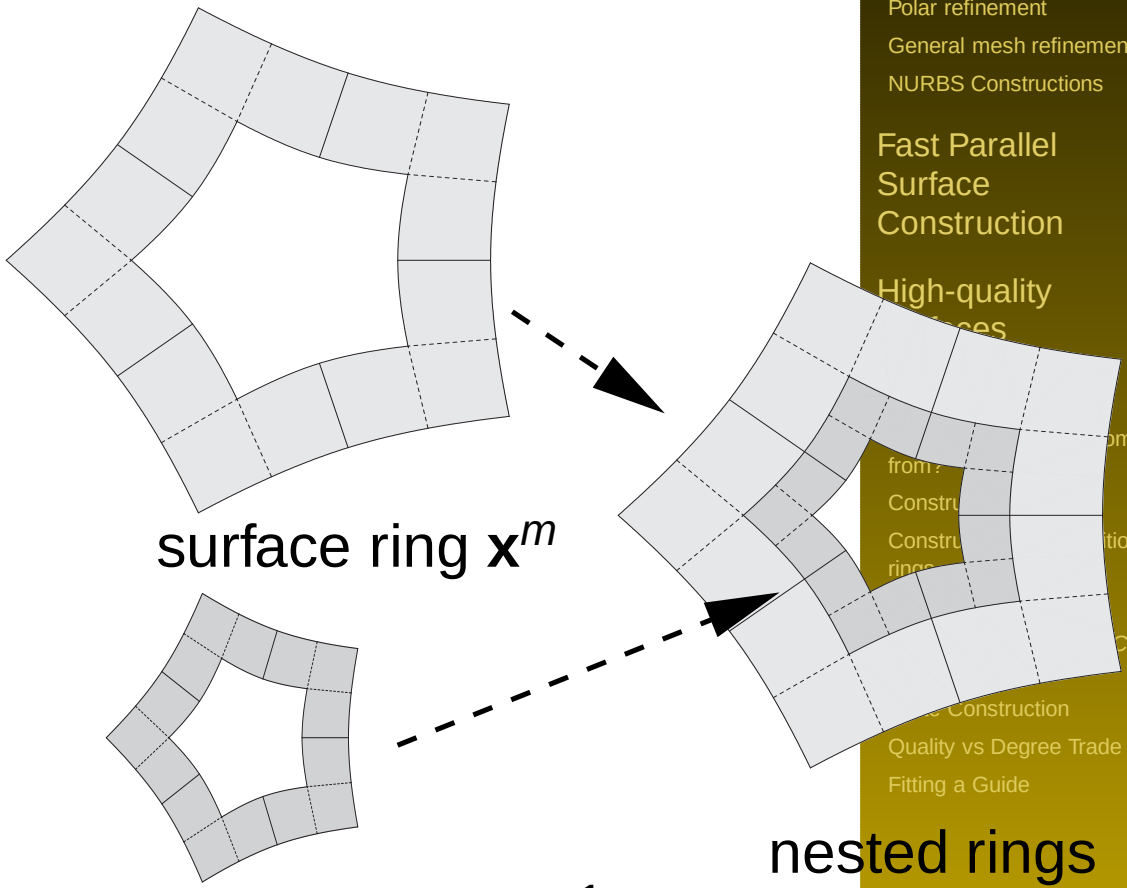
Subdivision

guide $\mathbf{g}$ $\mathbb{R}^2 \to \mathbb{R}^3$ and
concentric tesselation map ρ $(\Gamma \subsetneq \mathbb{R}^2) \times n \to \mathbb{R}^3$:

control net $A^m\mathbf{c}$
or
$H(\mathbf{g} \circ \lambda^m \rho)$ generating system

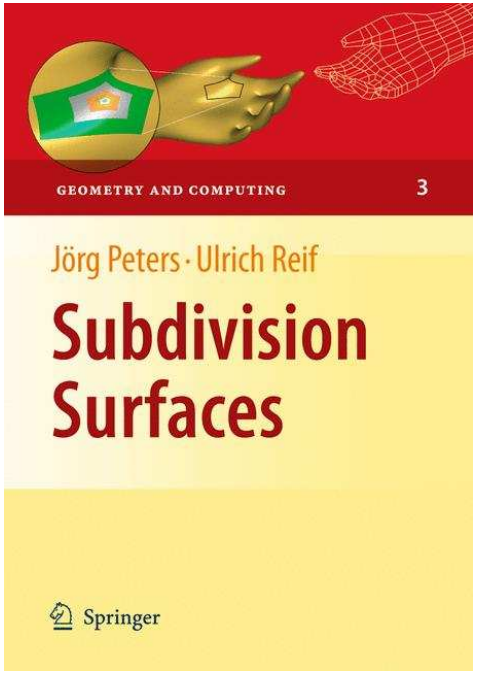
refinement

control net $A^{m+1}\mathbf{c}$
or
$H(\mathbf{g} \circ \lambda^{m+1} \rho)$ generating system



Subdivision

General Theory:



Ch 8

Guide determines eigenvectors (with correct eigenvalues) to yield a C^2 limit surface

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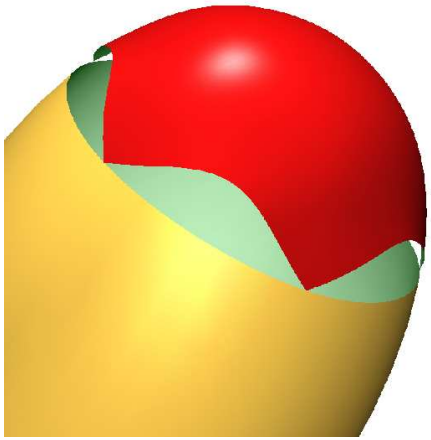
Example construction: C^2 polar subdivision



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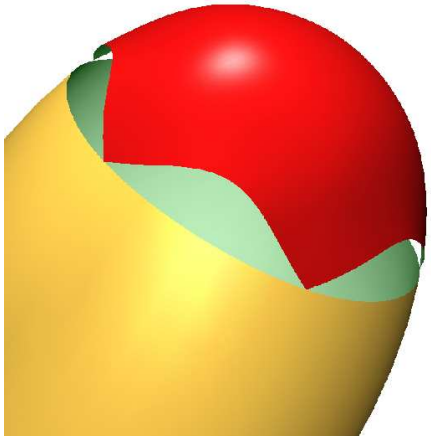
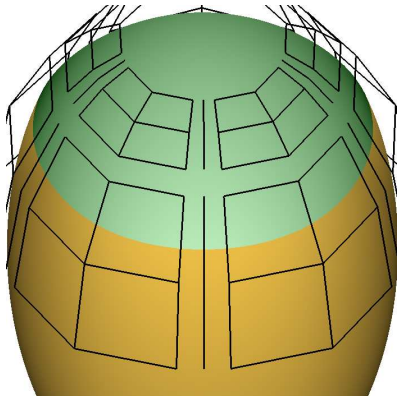
Example construction: C^2 polar subdivision

map 2×2-jet to 2×2-jet [Myles et al, SGP 2006]





$$\begin{matrix} f & \partial_s f & \partial_s^2 f \\ \partial_t f & \partial_s \partial_t f & \partial_s^2 \partial_t f \\ \partial_t^2 f & \partial_s \partial_t^2 f & \partial_s^2 \partial_t^2 f \end{matrix},$$

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Example construction: C^2 polar subdivision

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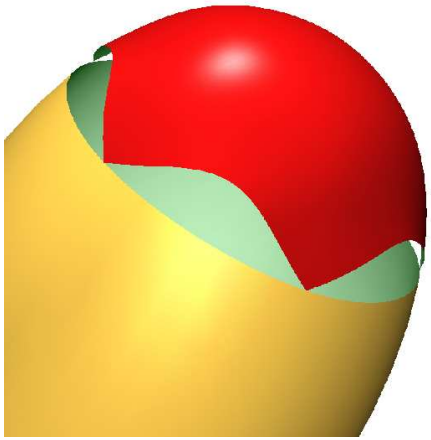
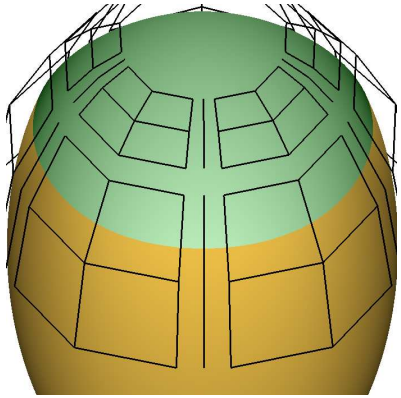
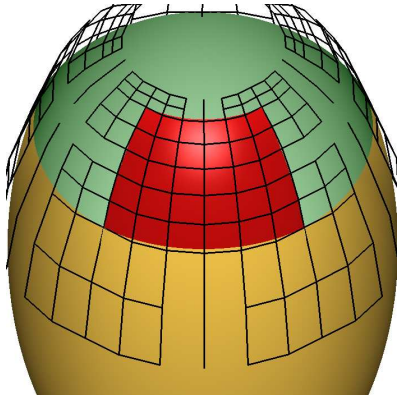

Smooth Parametric Surface Construction

Jörg Peters

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Example construction: C^2 polar subdivision

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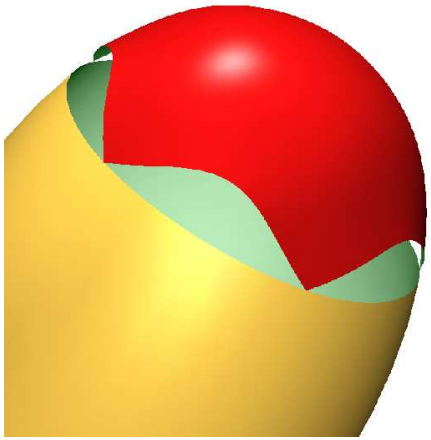
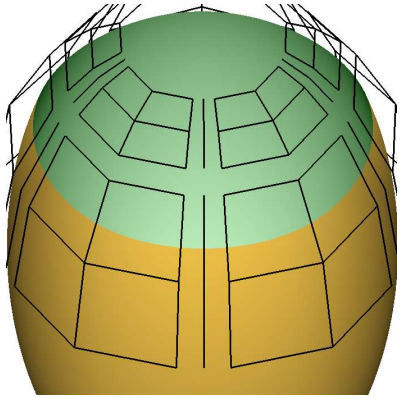
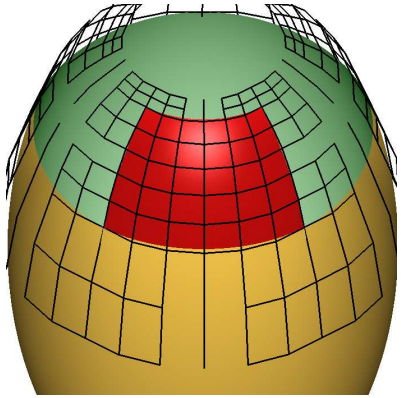
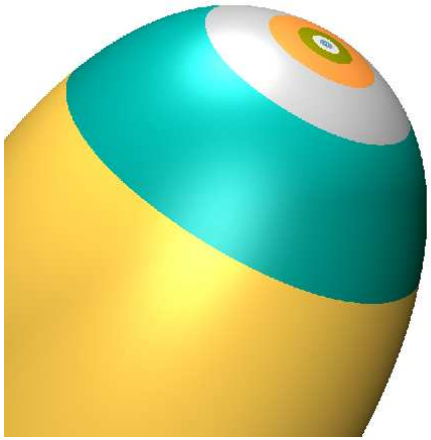

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Example construction: C^2 polar subdivision

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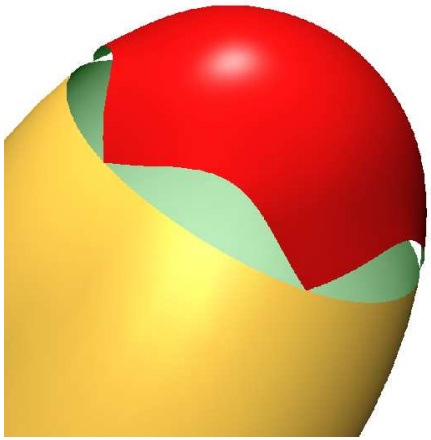
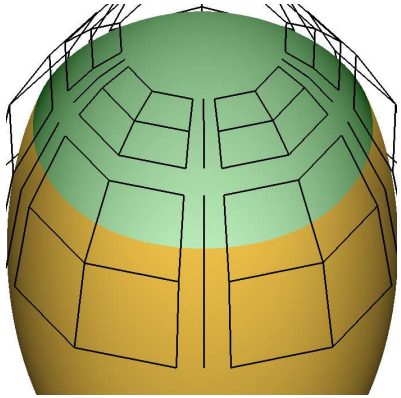
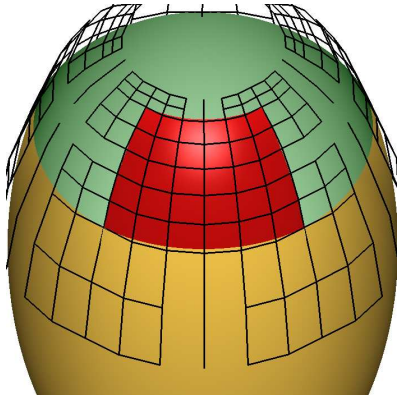
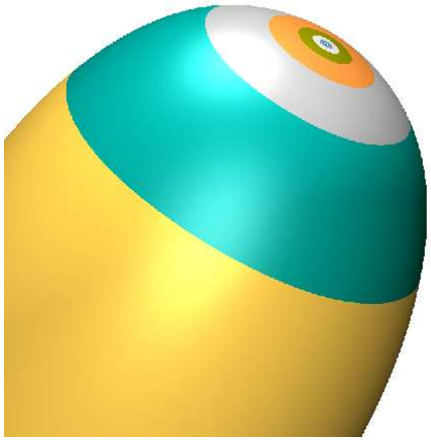
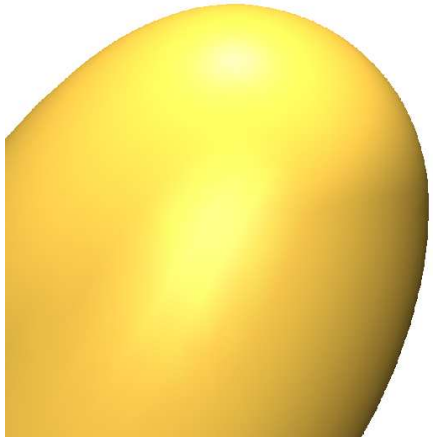

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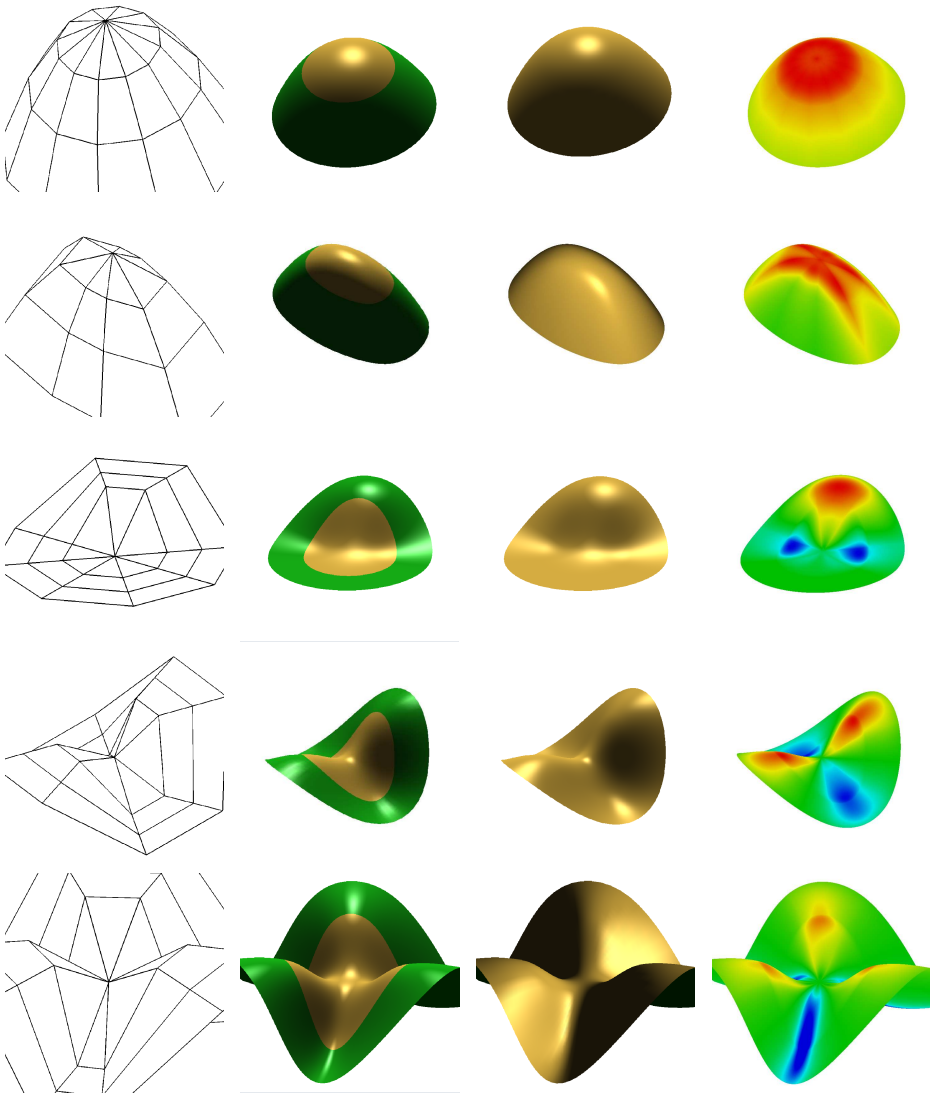
Polar C^2 Jet Subdivion test gallery



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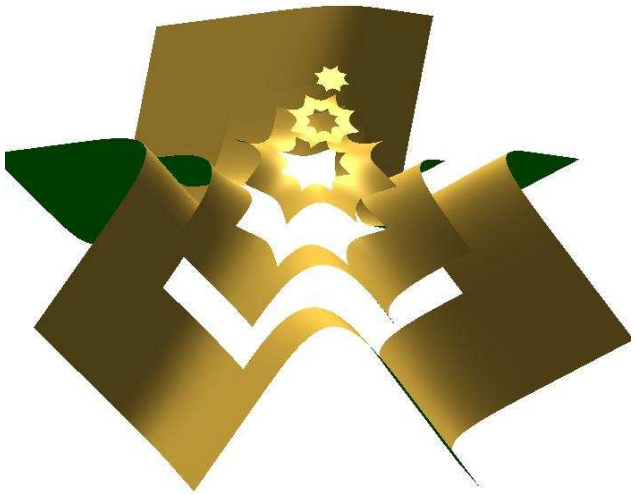
Finite Construction



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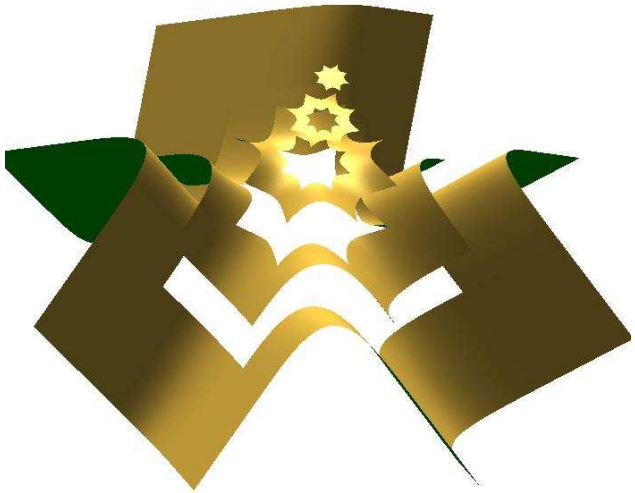

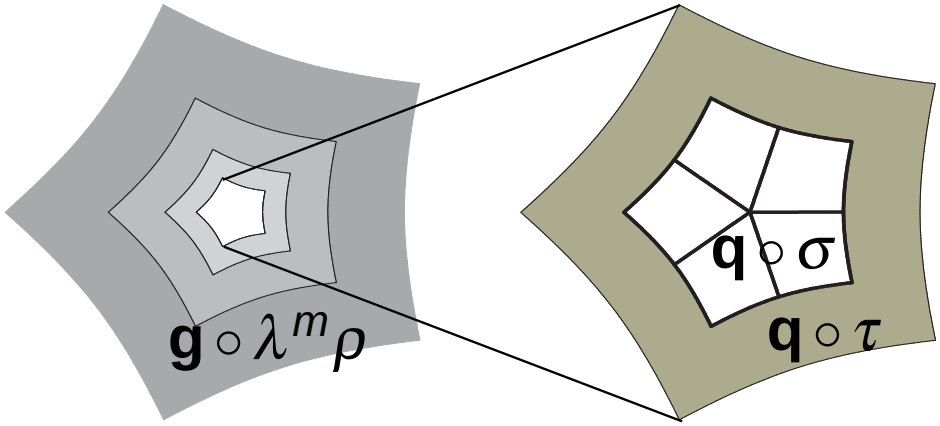

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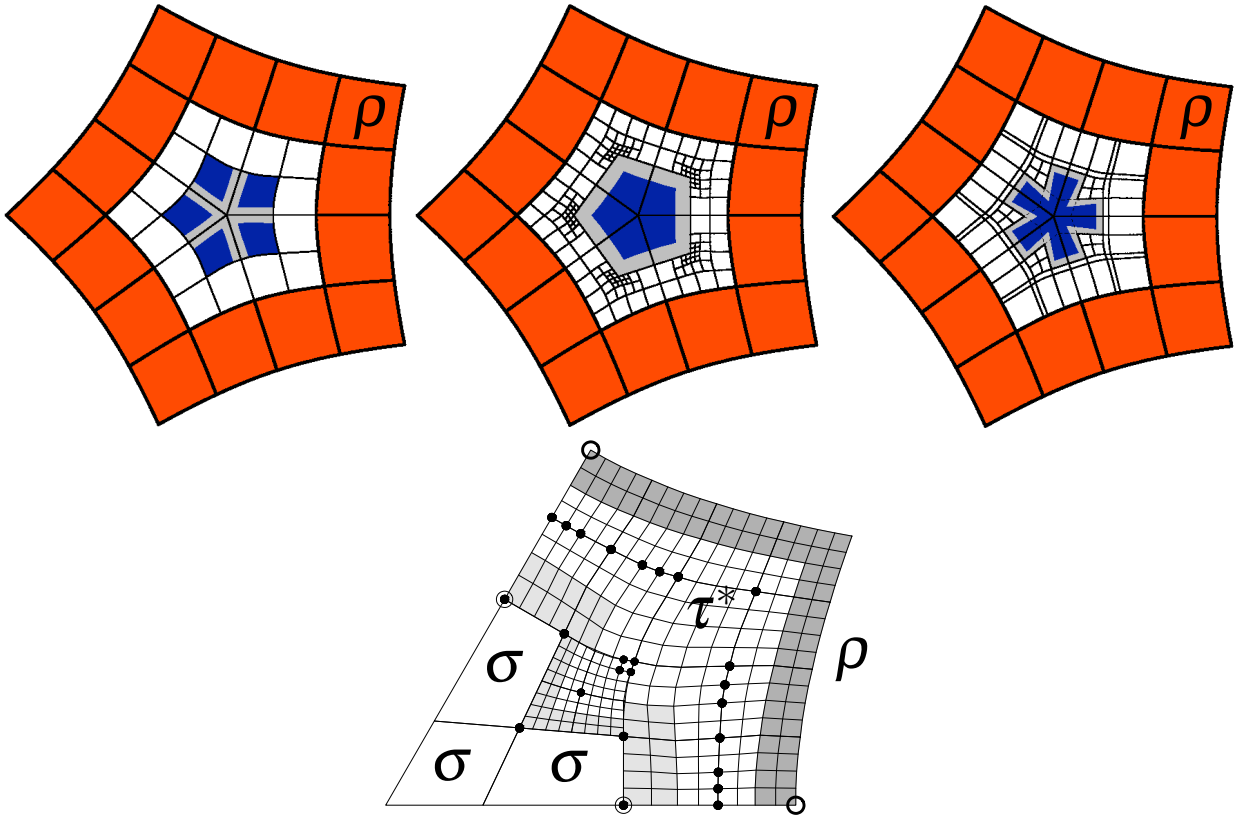
Three finite constructions . [Karciauskas Peters 2006-7]



guide stabilizes domain transition

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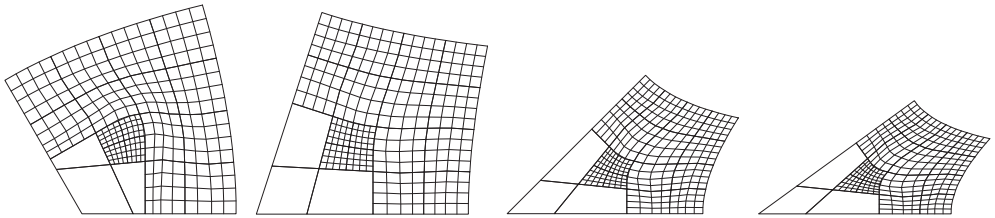
Finite patch construction



BB-net of ρ

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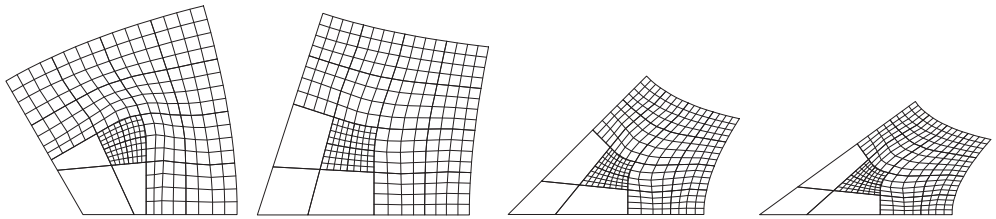

BB-net of ρ

Joining torus pieces with curvature continuity

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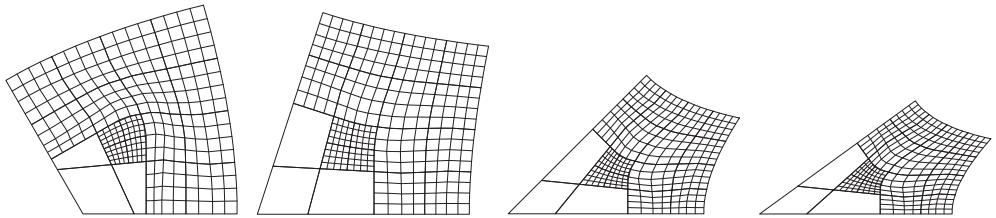

BB-net of ρ

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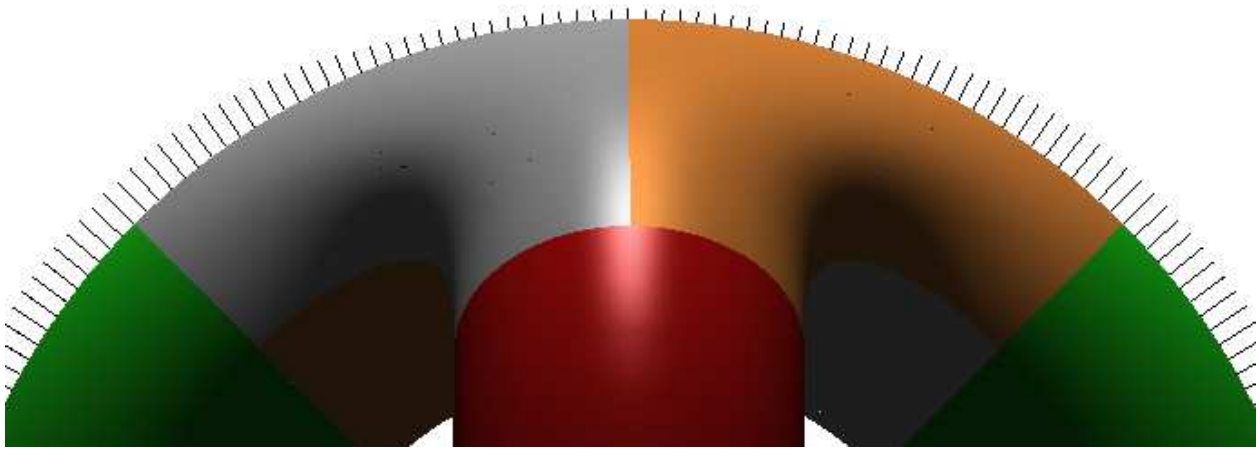

Curvature Comb (normals scaled by Gauss curvature attached to surface points)

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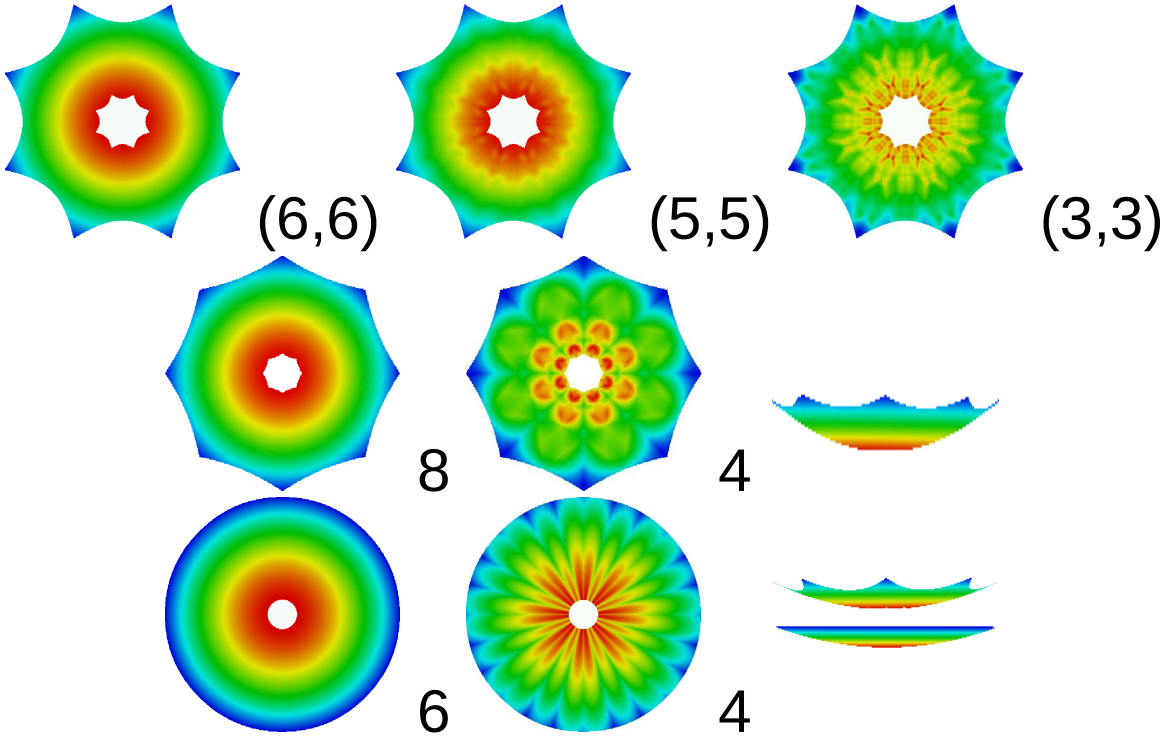
Quality vs Degree Trade off – Subdivision

Guide: (x, y, x^2+y^2) Surface:



Smooth Parametric Surface Construction

Jörg Peters

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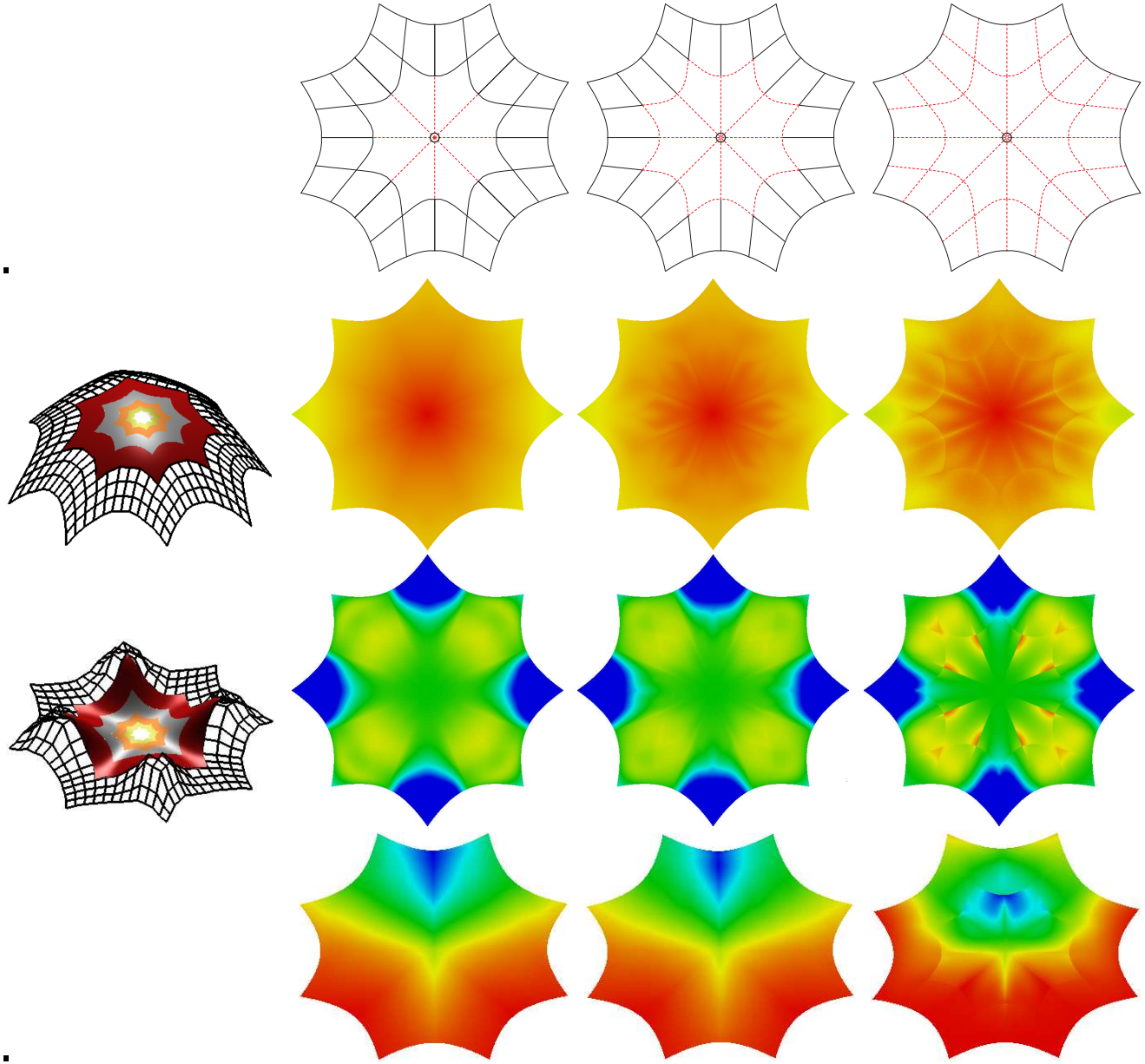
Quality vs Degree Trade off – Finite



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More details

- SGP 2006: C^2 Polar Jet Subdivision
- CAGD 24 (2): Concentric Tesselation Maps and Curvature Continuous Guided Surfaces
- CAGD 25 (2): On the curvature of guided surfaces
- CAGD ??: Guided Spline Surfaces
- Subdivision Surfaces, Springer

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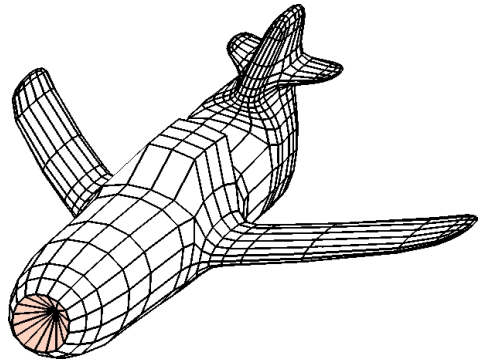
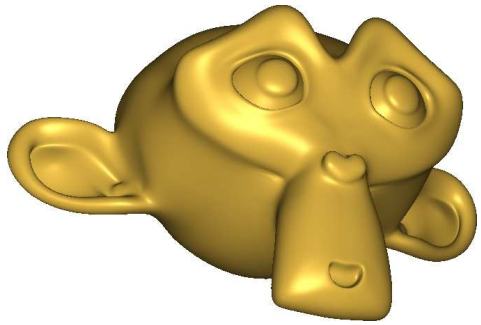
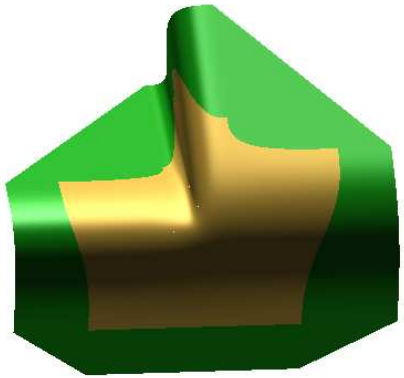
Q&A Time

- Polar Surface Structure





- Fast Parallel Surface Construction



- High-quality Surfaces

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Backup Slides

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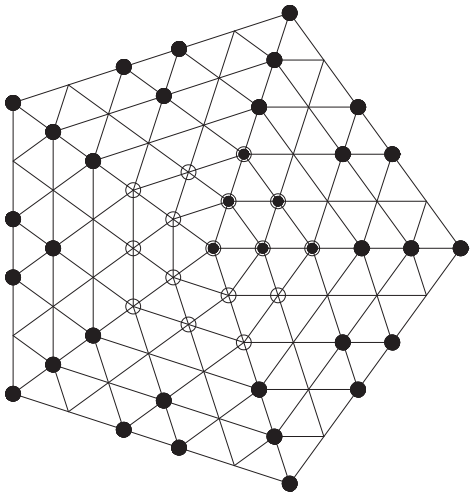
A piecewise polynomial C^2 Guide

(Fitting a Guide if the Designer does not)



C^1 and C^2 constraints:

- center quadratic polynomial
- circulant system

$$\mathbf{g}^{j-1}_{d-3,2,1} + 4c\mathbf{g}^{j}_{d-3,2,1} + \mathbf{g}^{j+1}_{d-3,2,1} = 2R_j$$

Smooth Parametric Surface Construction

Jörg Peters

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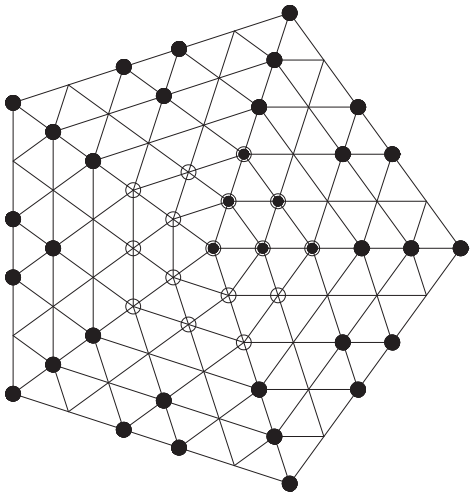

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- center quadratic polynomial
- circulant system

$$\mathbf{g}^{j-1}_{d-3,2,1} + 4c\mathbf{g}^{j}_{d-3,2,1} + \mathbf{g}^{j+1}_{d-3,2,1} = 2R_j$$

Smooth Parametric Surface Construction

Jörg Peters

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- General mesh refinement
- NURBS Constructions

Fast Parallel Surface Construction

High-quality Surfaces
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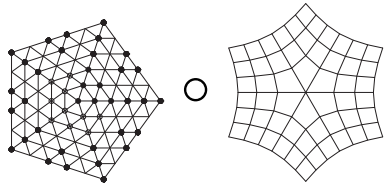
A piecewise polynomial C^2 Guide

$$\min_{\text{free}\,\mathbf{g}_{ijk}} \| H\Big(\quad \circ \quad \Big) - H\Big(\quad \Big) \|$$



. $\triangle \times n \to \mathbb{R}^3 \quad \Gamma \times n \to \mathbb{R}^2 \quad \Gamma \times n \to \mathbb{R}^3$

. guide ct-map tensor-border

H – Hermite (Jet) Sampling

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Guide: piecewise of moderate degree

- Single polynomial ([Prautzsch 1997], [Reif 1998], [Ying:2004], [Levin 06])

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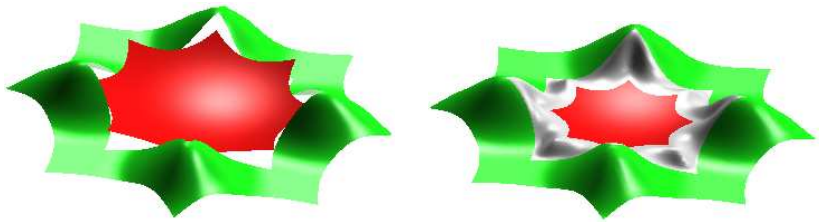

cubic

-

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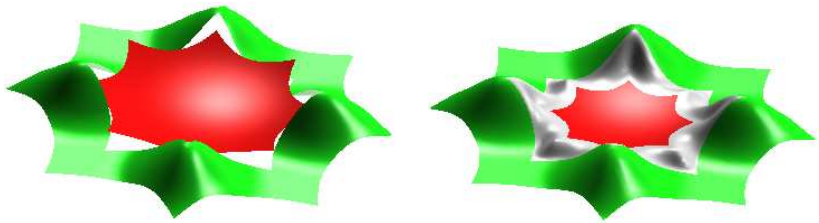
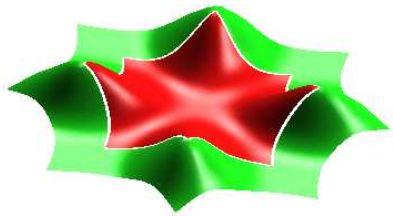


cubic

- better: piecewise polynomial



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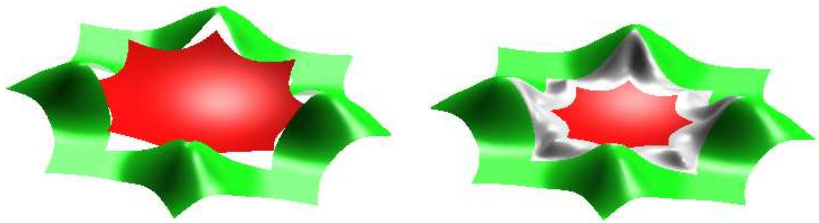

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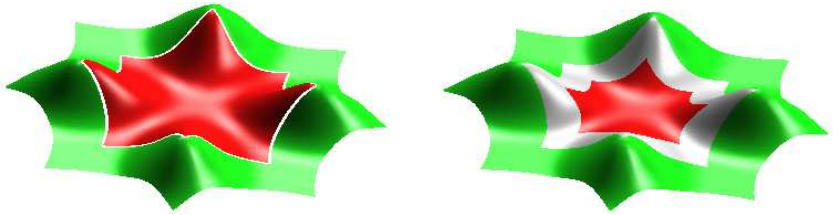

pw 5

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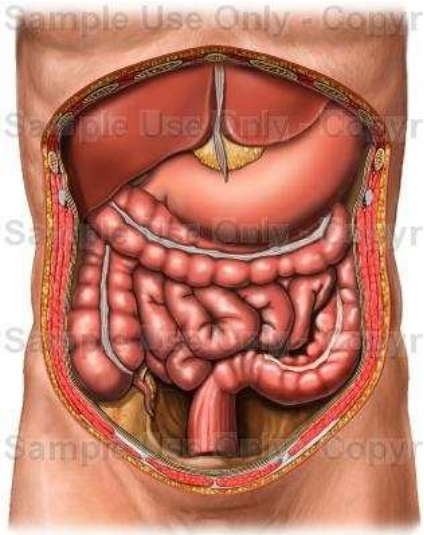
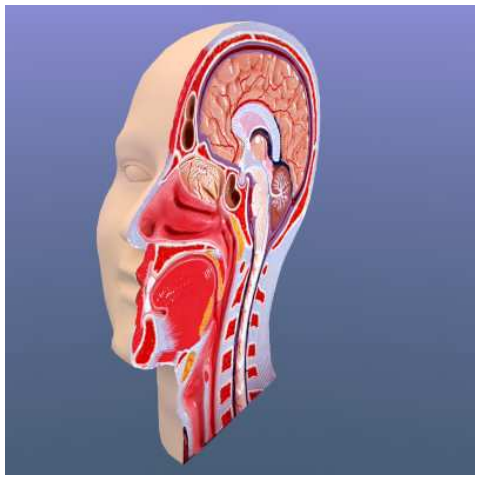
guide + constraints





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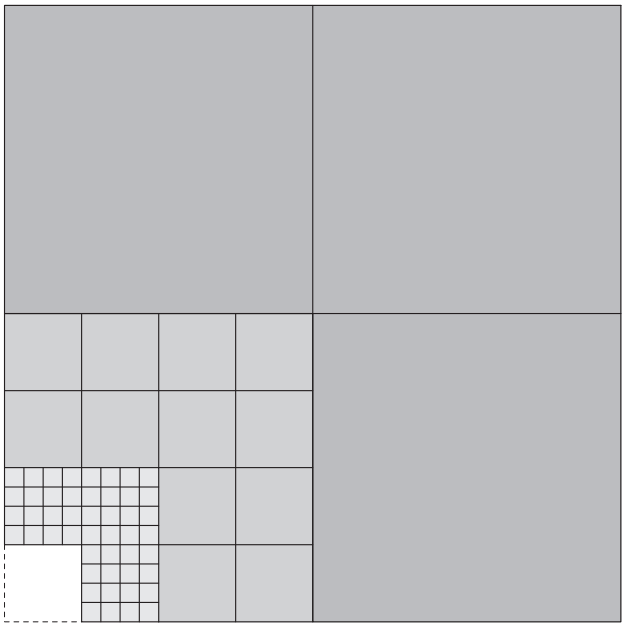
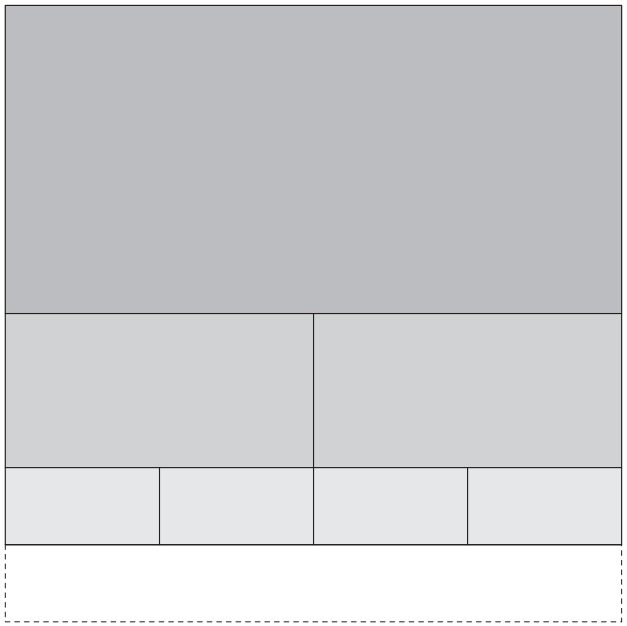
Structure of the accelerated bicubic subdivision





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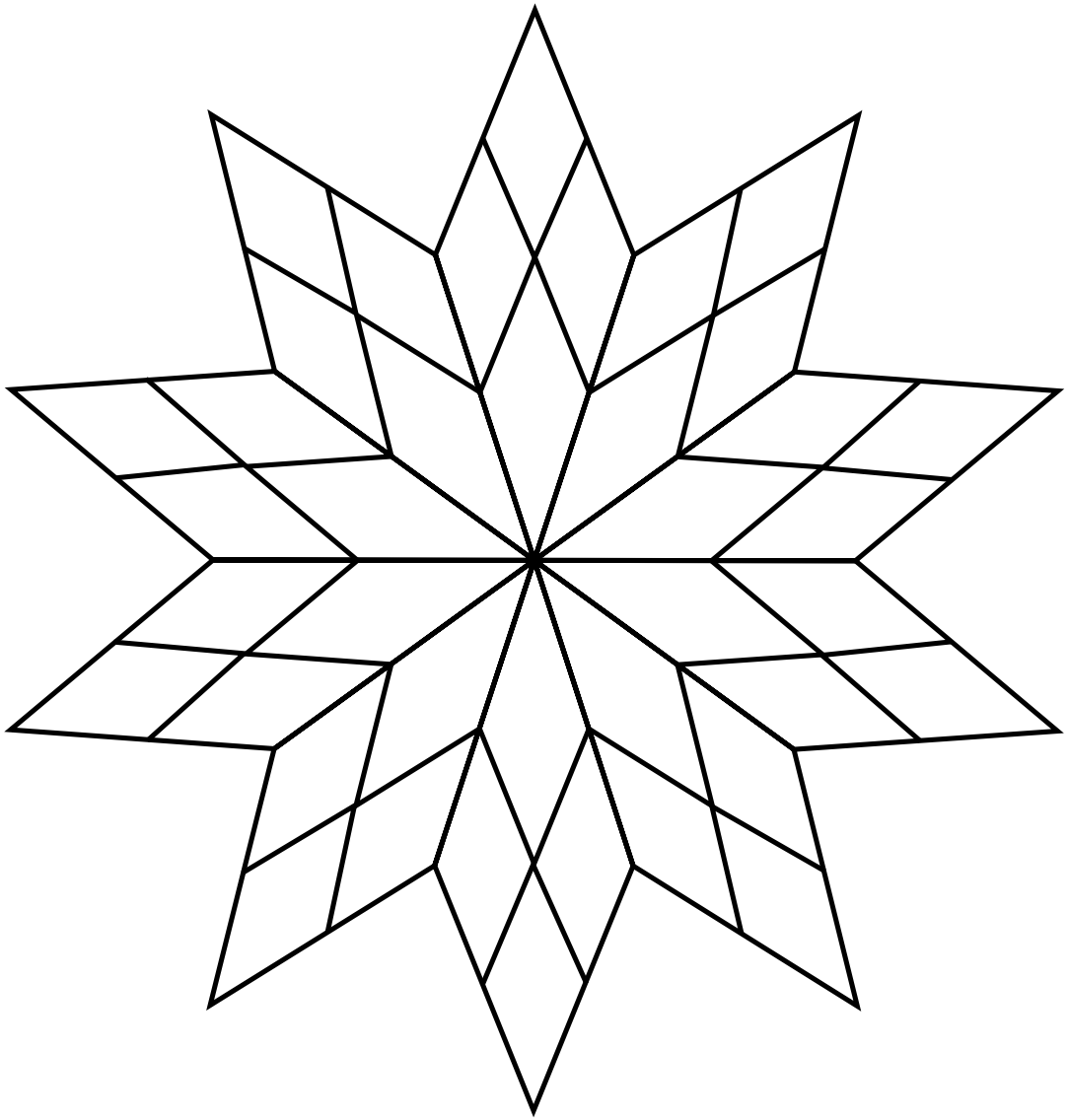
High-valent Catmull-Clark layout → polar layout



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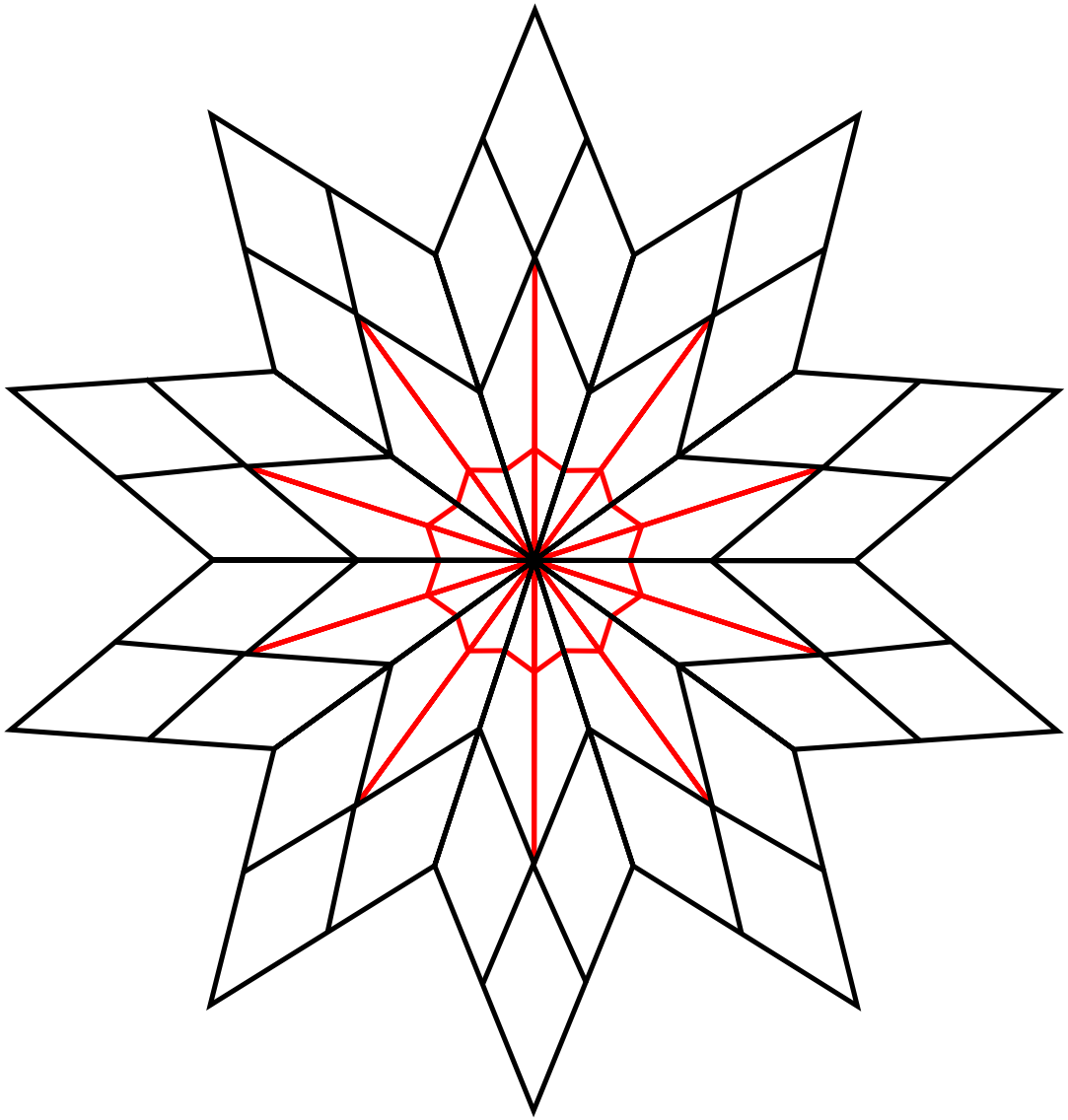

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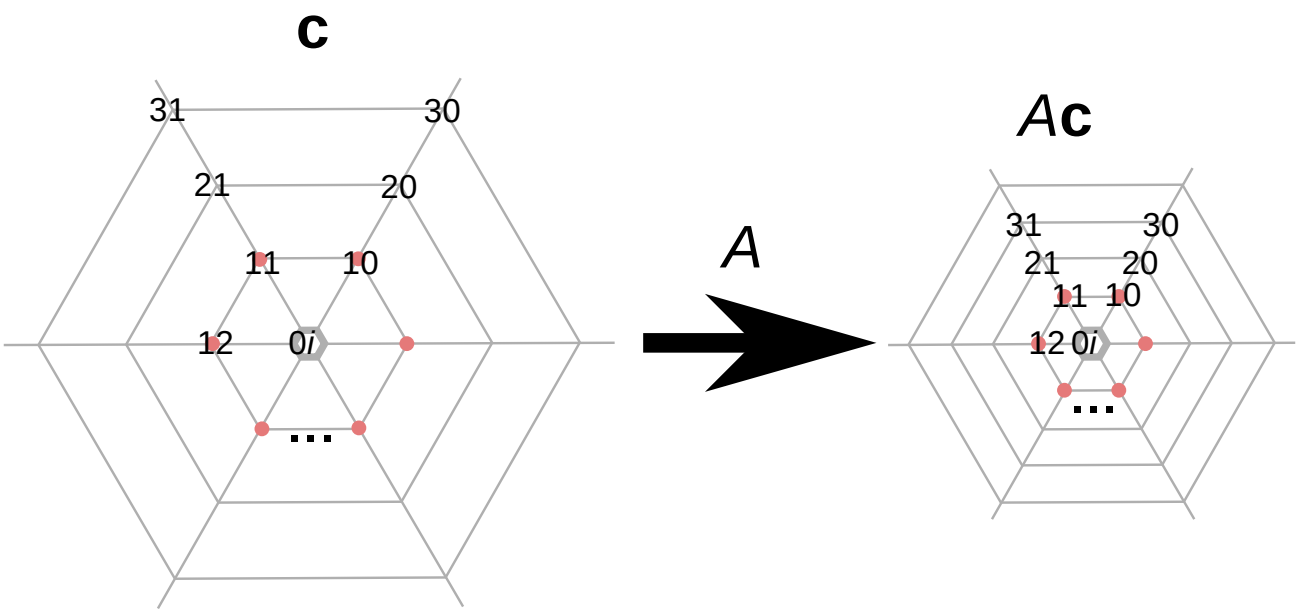
Analysis



$$A = \begin{bmatrix} A_0 & A_1 & \dots & A_{n-1} \\ A_{n-1} & A_0 & \dots & A_{n-2} \\ \vdots & & \ddots & \vdots \\ A_1 & \dots & A_{n-1} & A_0 \end{bmatrix}$$

$$A_0 := \begin{bmatrix} 1/n & 0 & 0 & 0 \\ 1/n & \Gamma_0 & 0 & 0 \\ 0 & 3/4 & 1/4 & 0 \\ 0 & 3/16 & 11/16 & 1/8 \end{bmatrix}, A_i := \begin{bmatrix} 1/n & 0 & 0 & 0 \\ 1/n & \Gamma_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Smooth Parametric Surface Construction

Jörg Peters

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Analysis

Spectral analysis of A (circulant) gives desired properties:

- Eigenvalues: $1, \overbrace{1/2, 1/2}^{\text{Fourier blocks } 1^{st} \text{ \& } n-1^{st}}, \overbrace{1/4, \ldots, 1/4}^{n}.$
- Characteristic map is regular ($\Rightarrow C^1$).
- Geometric multiplicities = algebraic multiplicities for A.

$\Rightarrow$ Bounded curvature.

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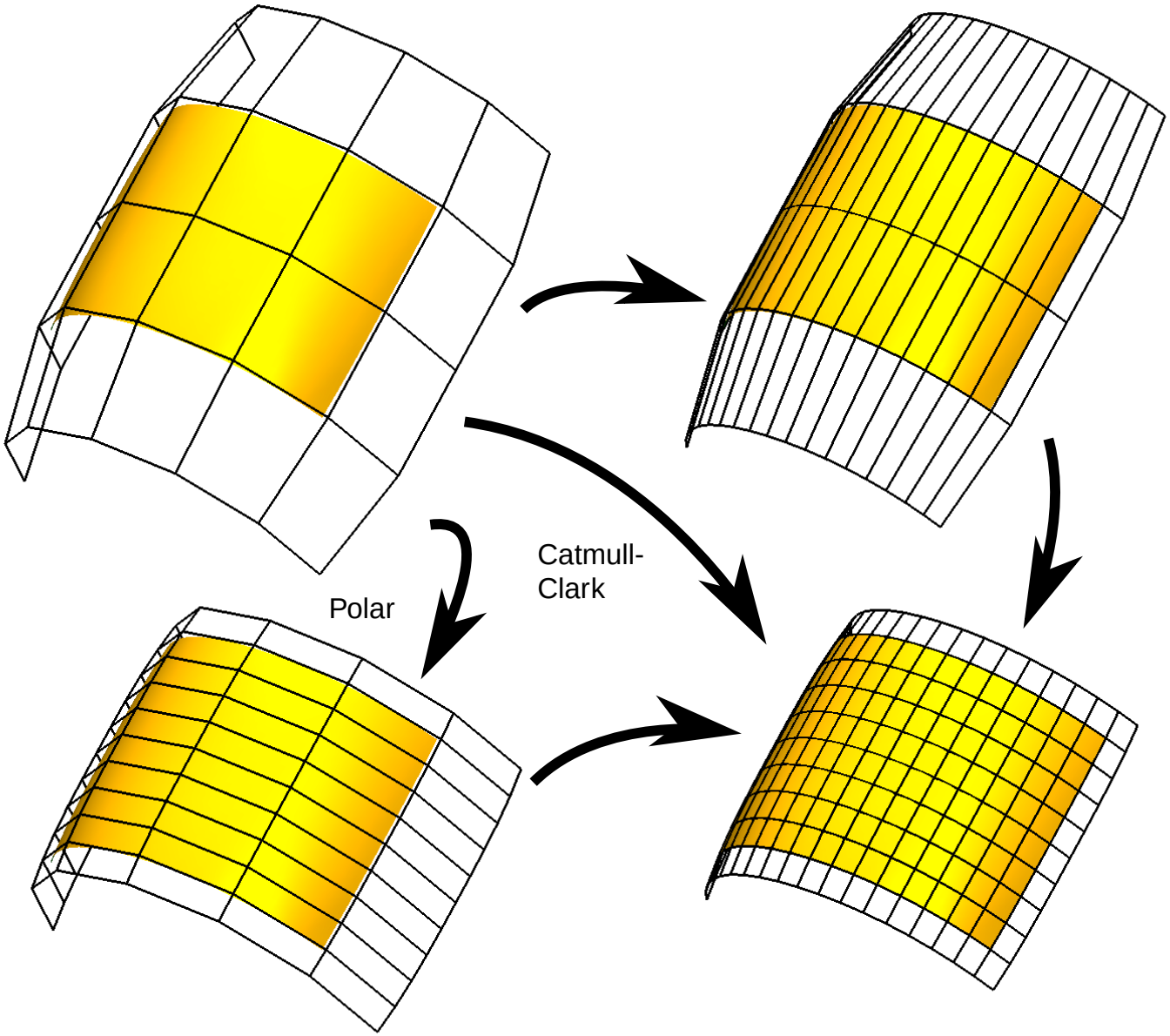
Tensor product B-spline refinement



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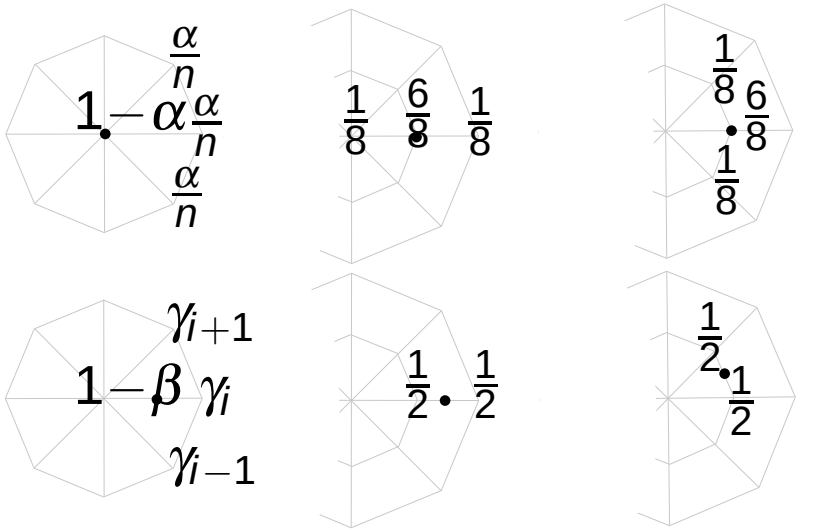
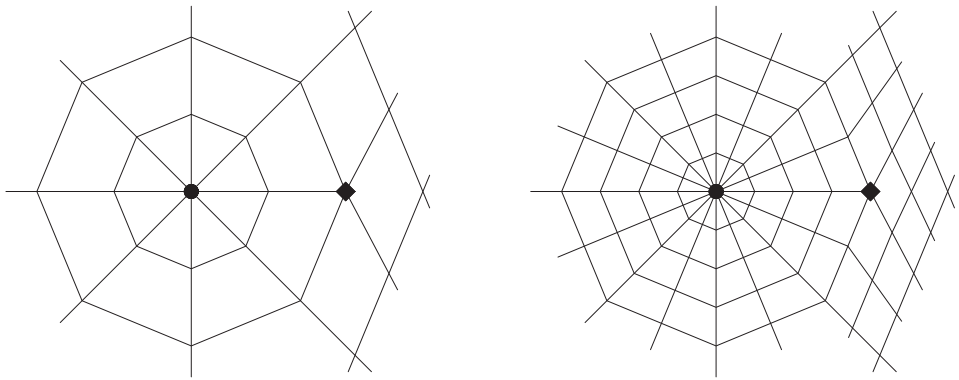
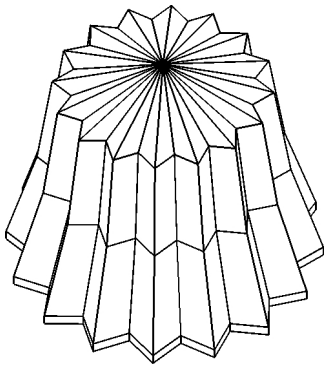
Separating Catmull-Clark and polar extraordinary limit points



radial circular



Make predictable ripples



Catmull-Clark

Our method

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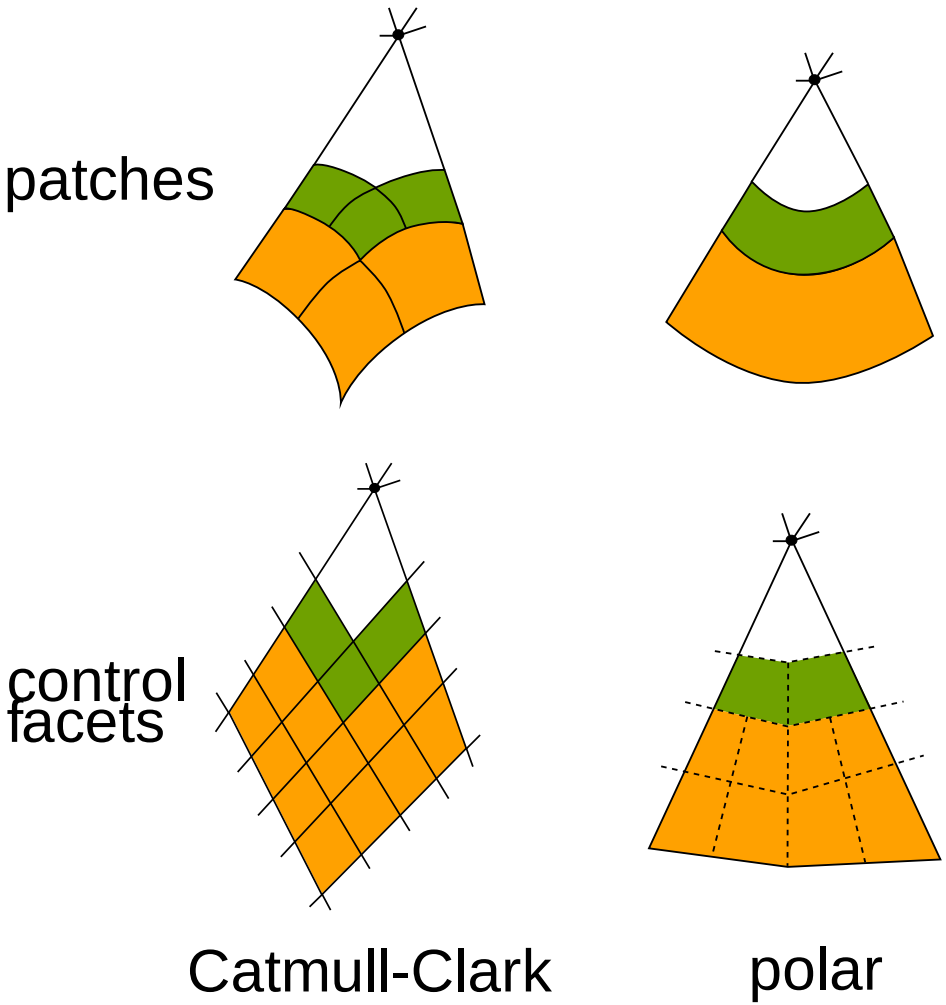
T-corners



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