Modelling and Foundations of Multi-Sided Patches

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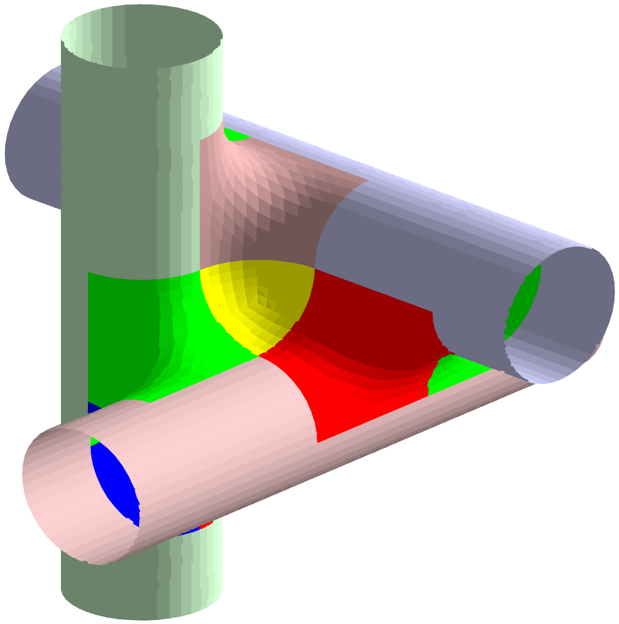
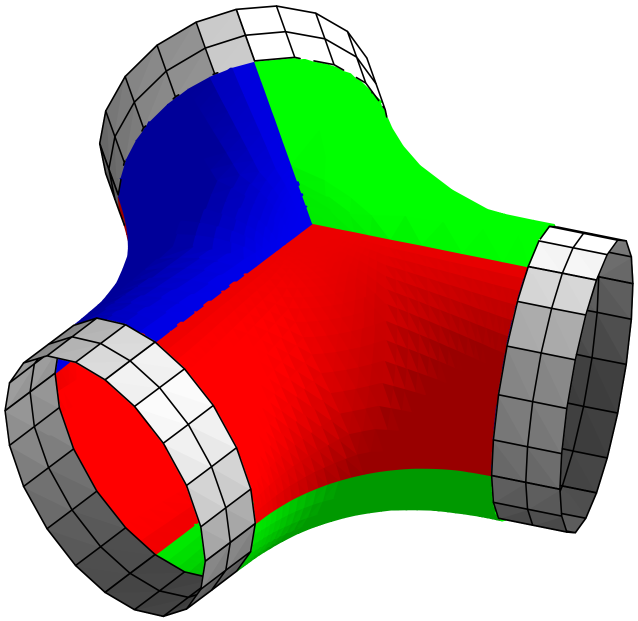
Part I: Modelling with Multi-Sided Patches, Jörg Peters

Outline

1. Multi-sided patches in blending
2. Artifacts of current approaches
3. Combinatorial structure of rational M-patches
4. Tensor-border of multi-sided patches
5. Inclusion of M-patches into B-spline surface
6. Guided and finite subdivision

1 Multi-sided patches in blending





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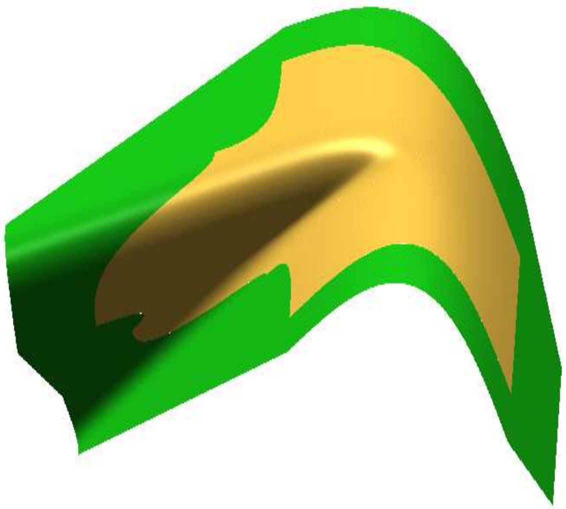
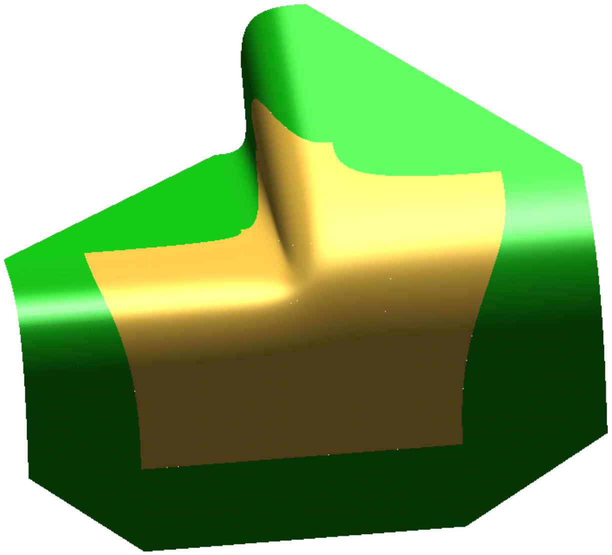
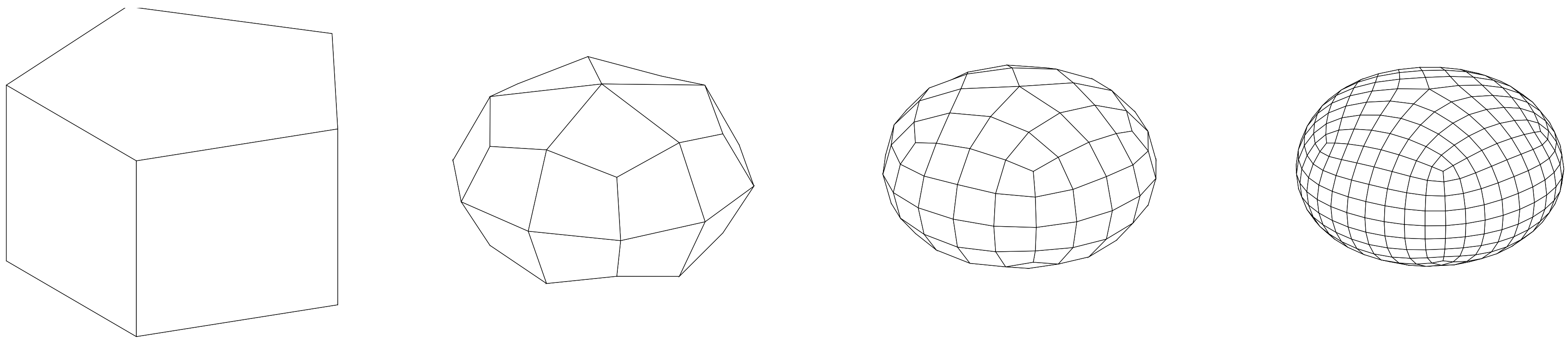
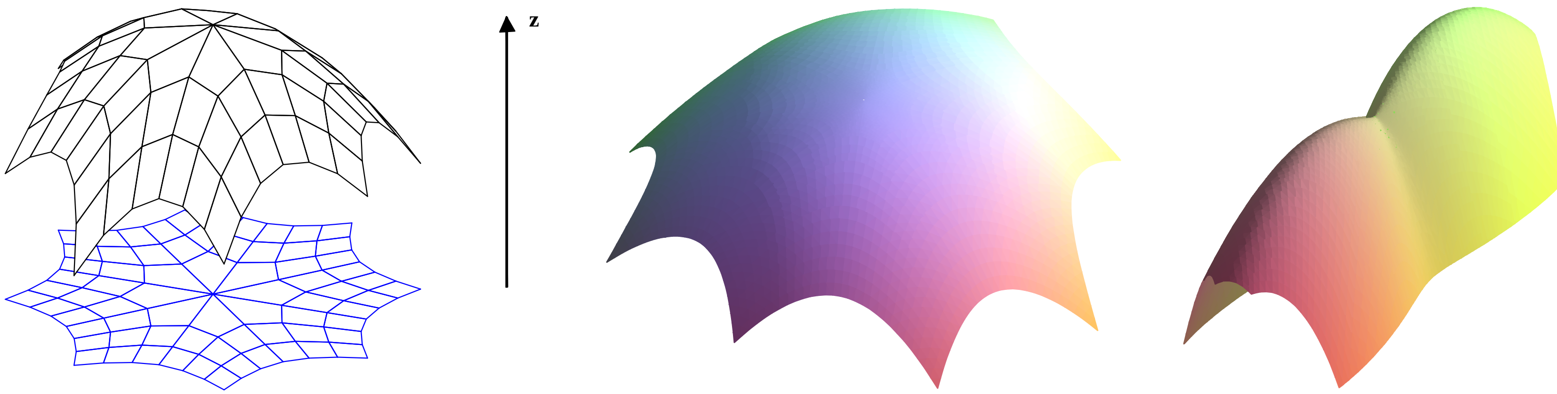




2 Artifacts: Catmull-Clark surfaces



2 Artifacts of Catmull-Clark surfaces

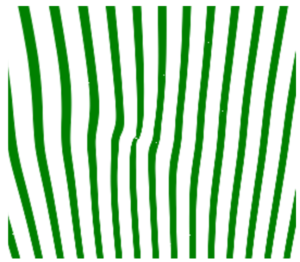


left Control points of characteristic map (blue) are lifted onto paraboloid $z = 1 - x^2 - 0.6y^2$. The corresponding quad mesh (black) looks rather convex – inserting diagonals in direction ”around extraordinary node” we get a convex triangulation.

middle 10 rings of Catmull-Clark subdivision surface.

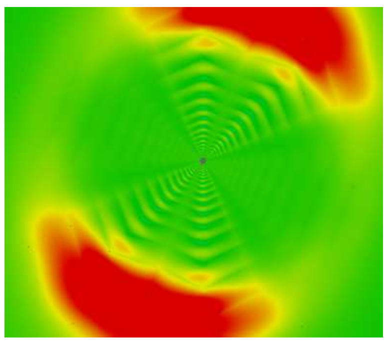
right A look through the microscope – Catmull-Clark surface is scaled by 50 in z-direction; its rings from 10th to 20th are displayed.

2 Shape Artifacts

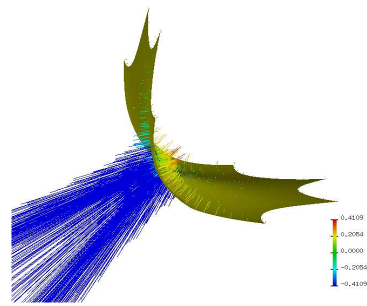


(1)

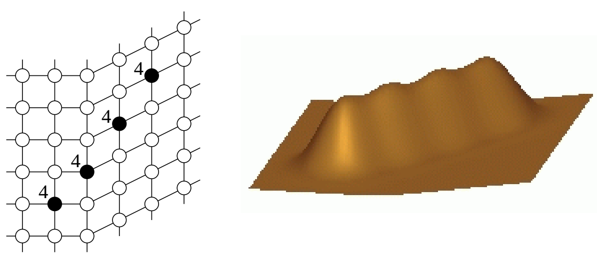
Shape
Defects



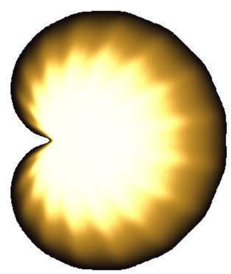
(2)



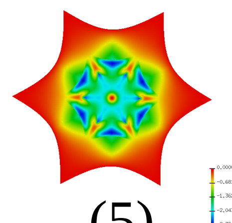
(6)



(3)

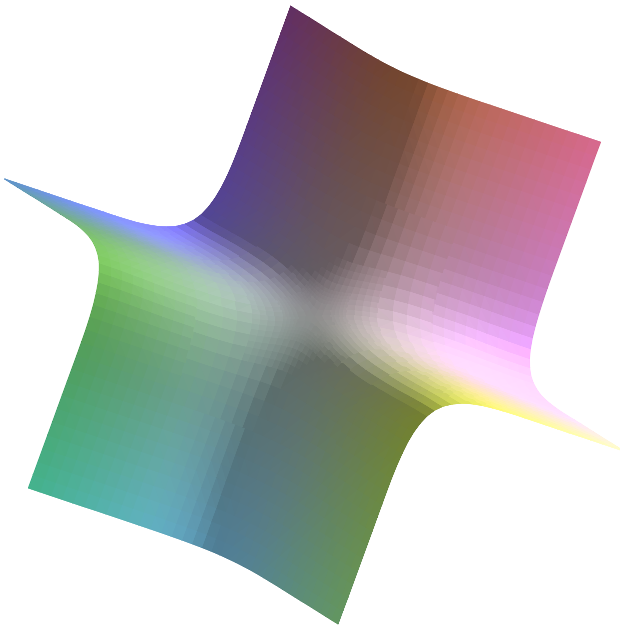
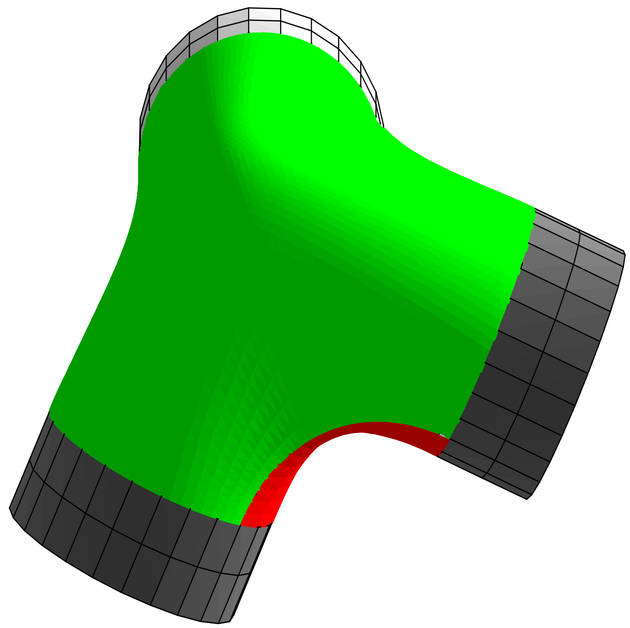


(4)



(5)

2 What blends are difficult to make fair?

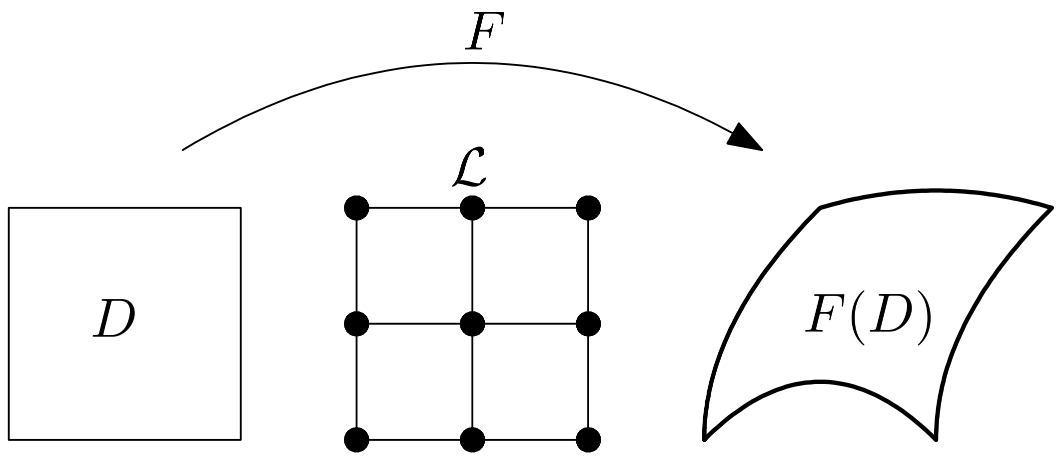
Observation: elliptic shape is a difficult configuration for all surfacing approaches.

2 Artifacts Defined

Consider a blend between multiple primary surfaces (=defining data)

Artifact:= curvature feature that is not present in the defining data.

3 Definition of a rational M-patch



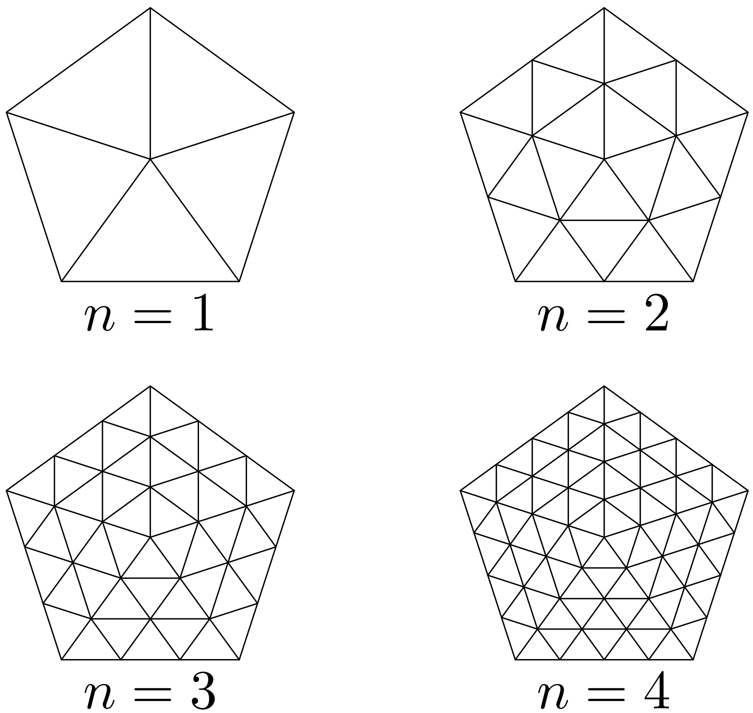
basis functions f_q, $q \in \mathcal{L}$ (index of control net) defined over a *domain* D.

For *control points* $\mathbf{p}_q \in \mathbb{R}^3$ and their *weights* $w_q \in \mathbb{R}$, $q \in \mathcal{L}$,

$$F := \frac{\sum_{q\in\mathcal{L}} w_q \mathbf{p}_q f_q}{\sum_{q\in\mathcal{L}} w_q f_q}.$$

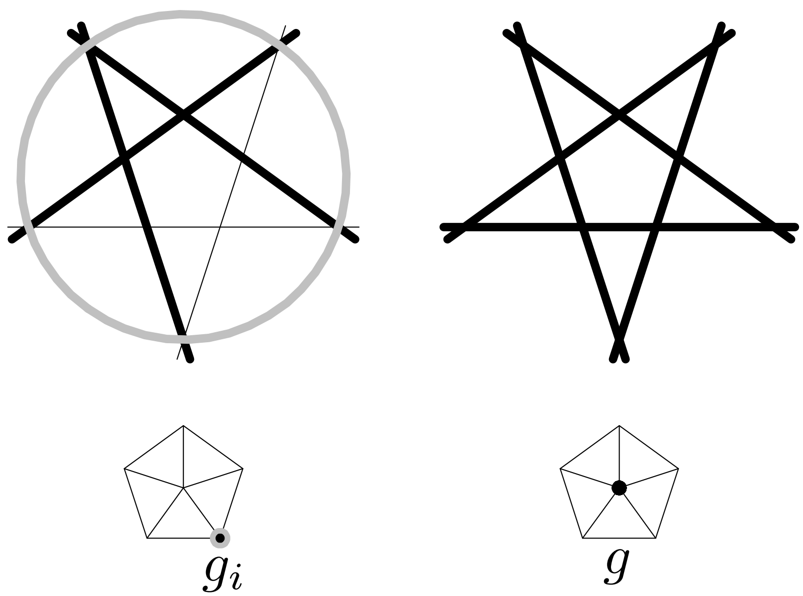
A rational *patch* $:= F(D)$.

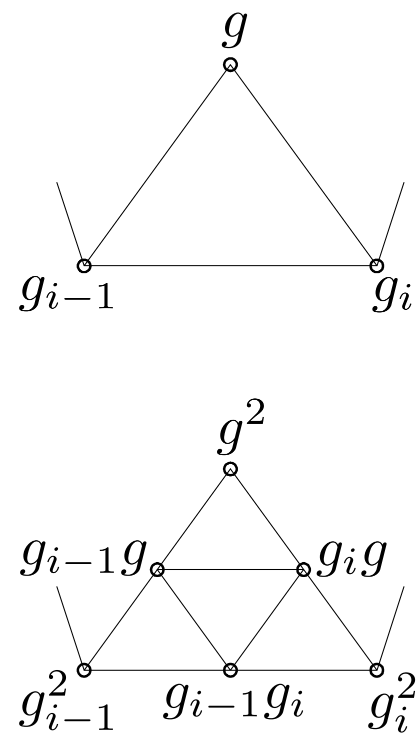
3 Control point net of M-patch



Control nets of M-patches of various depth n.

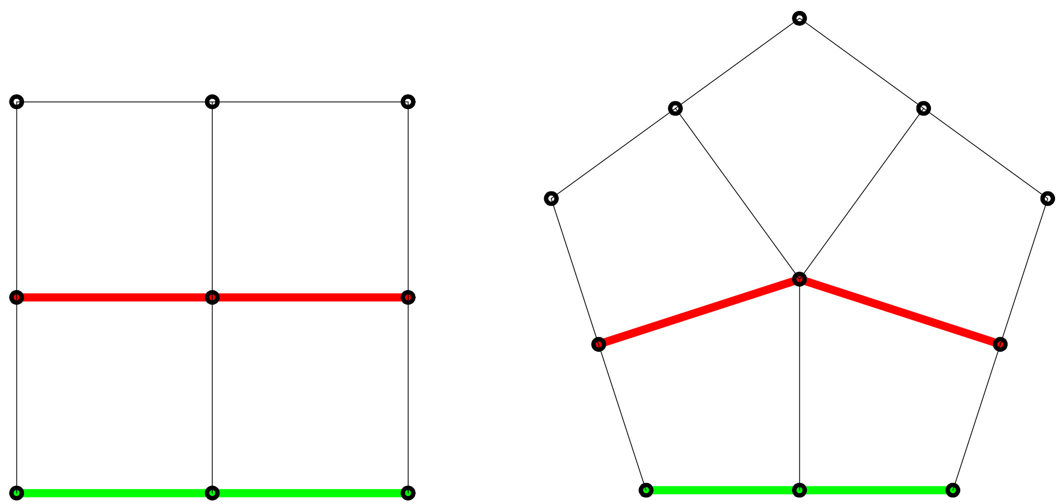
3 Basis functions of M-patch





Monomial structure of basis functions.

4 **Tensor-border nets: degree** 2 **and order** 1



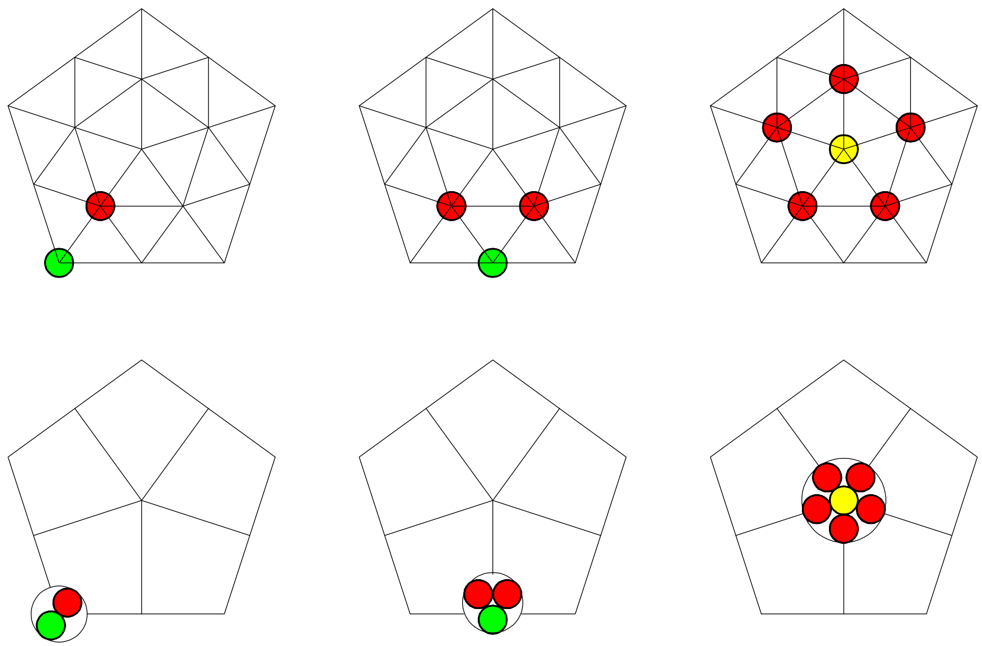
left Biquadratic *tensor-product* net:
$\sum_{i=0}^{2}\sum_{j=0}^{2} c_{ij} b_i^2(u) b_j^2(v)$ where $b_i^2(t) := \binom{2}{i}(1-t)^{2-i}t^i$.

right multi-sided *tensor-border* net of degree 2 and order 1 (Sabin net).

green – define boundary curve;

green $+$ red – first derivative across curve.

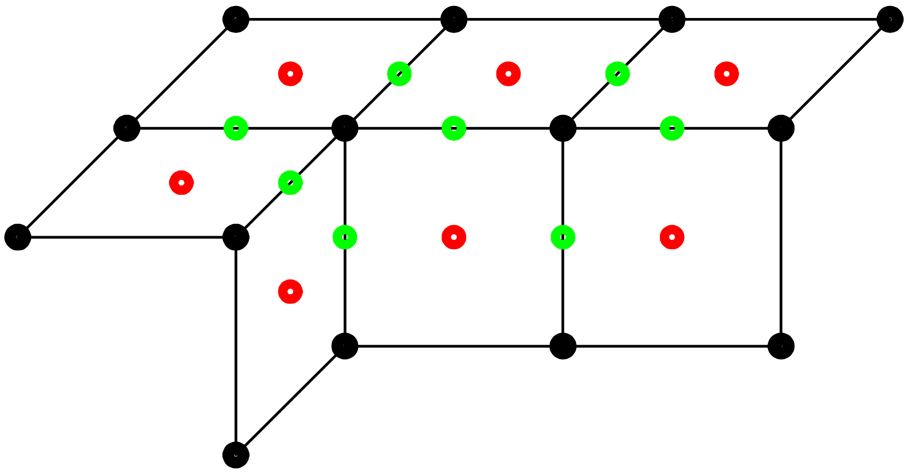
4**A** M**-patch** $\rightarrow$ **order** 1 **tensor-border patch**

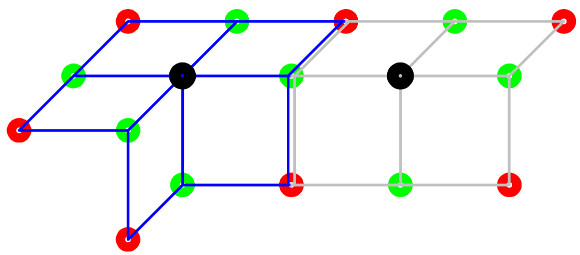


Linearly independent basis functions of order 1 tensor-border patch are derived as the linear combinations of basis functions of M-patch.

M-patch of depth $n \rightarrow$ order 1 tensor-border patch of degree n.

4 A Simplest C^1 spline scheme

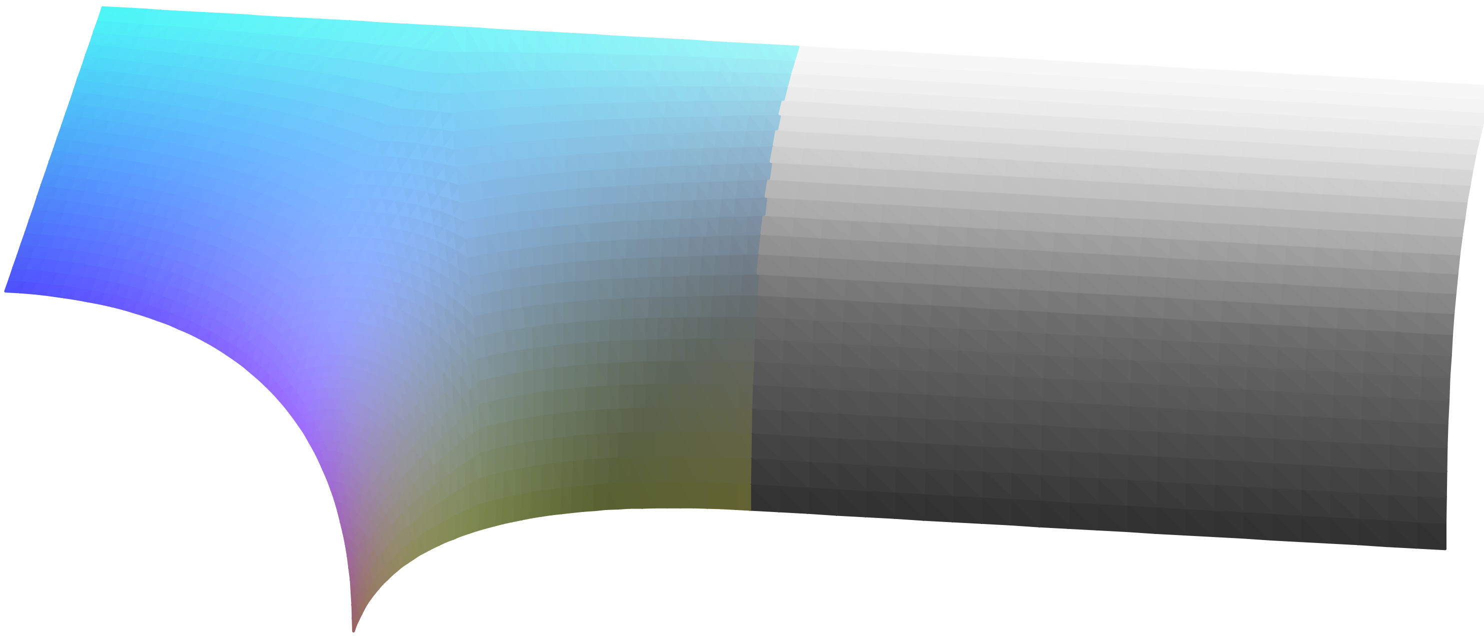




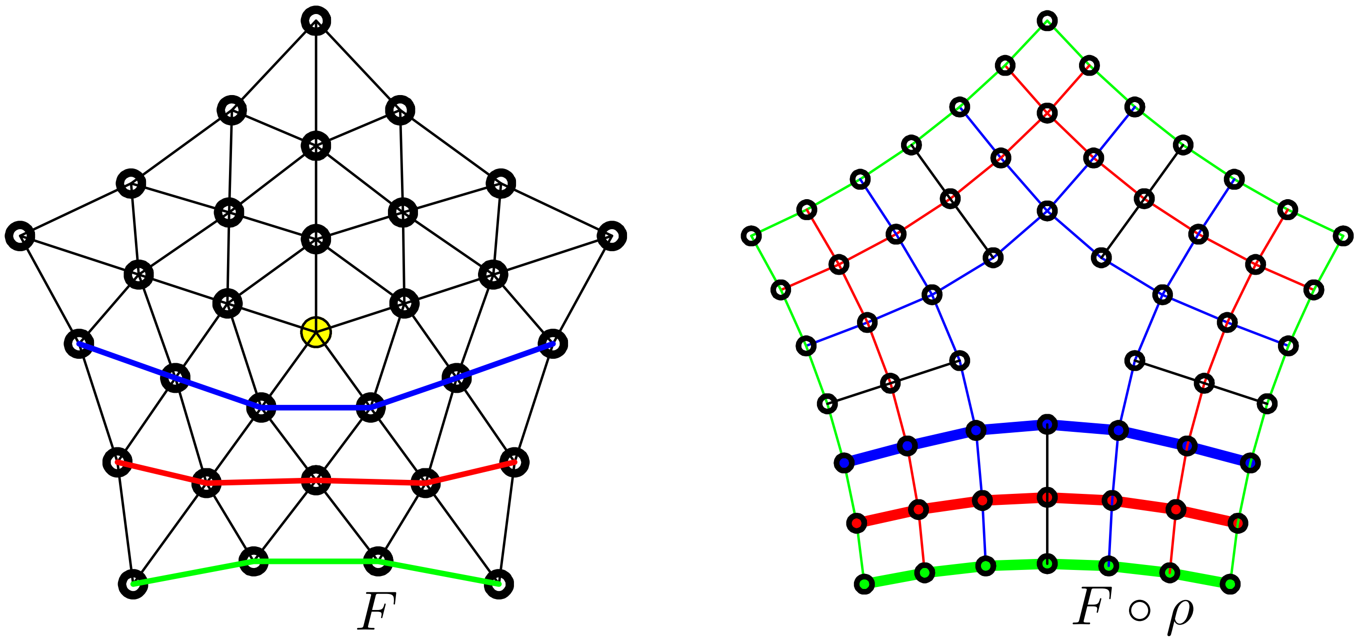
red – centroids of the quads;

green – midedges.

4A Simplest C^1 spline surface



4B Induced order 2 tensor-border of M-patch



Induced order 2 tensor-border of degree 6 of M-patch of depth 3.

New basis functions are dependent.

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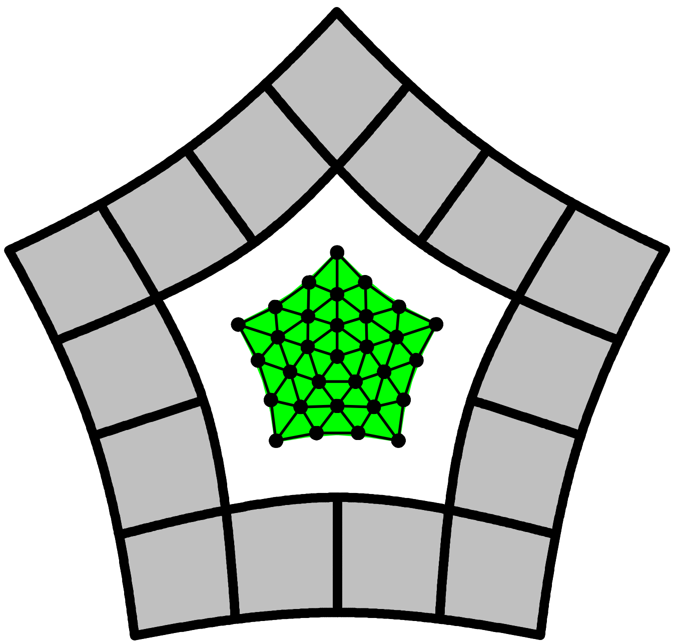
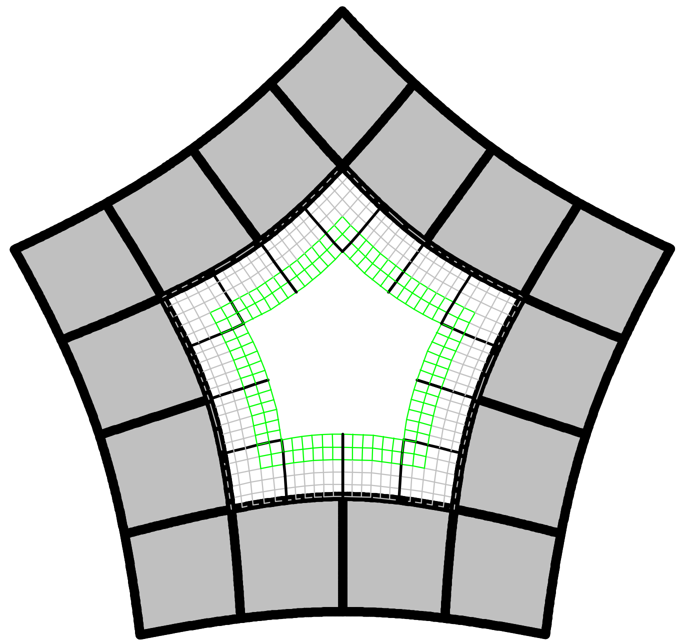
$5\ C^2$ inclusion of M-patch into B-spline surface

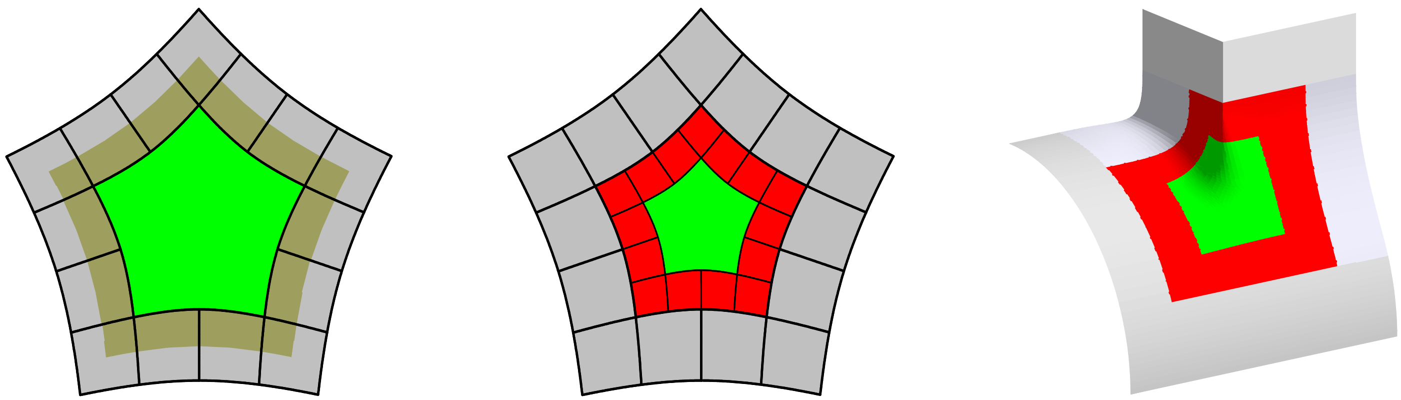


$b_{3,3}$ and F



$b_{3,3} \circ \tau = F \circ \rho$

5 Transition from input data



left M-patch is included into B-spline surface directly.

middle M-patch is included into B-spline surface via intermediate fairing ring (red).

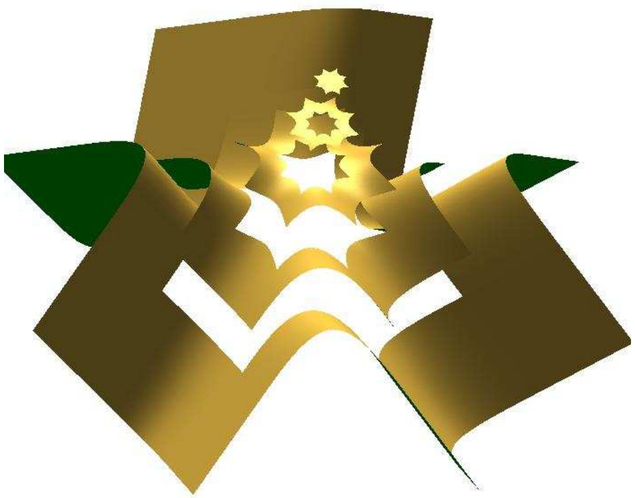
right The features of surrounding surface like creases, incompatible data are pushed into fairing ring leaving M-patch in peace. Example: two intersecting planes are blended with cylinder.

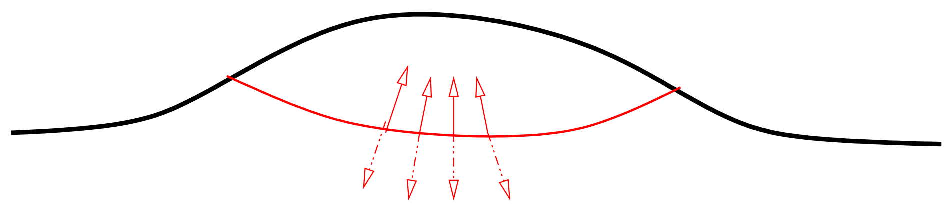
6 Representation of M-patches via rational Bézier patches

A general m-sided M-patch of depth n can be represented as a collection of
m rational Bézier patches of bidegree $((m-2)n, (m-2)n)$.

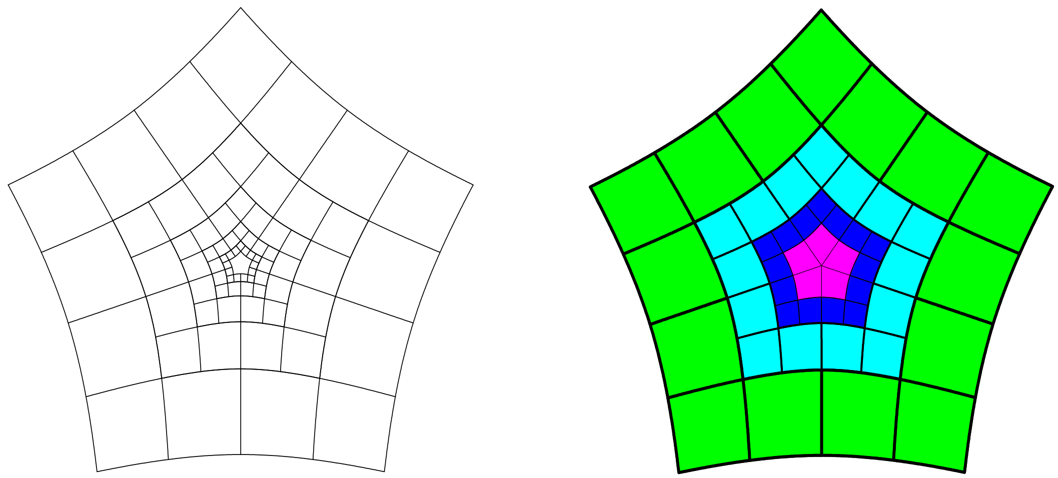
Here $n \geq 4$.

6 Reapproximation: Hermite + Smoothness





6 Guided subdivision and Finite subdivision



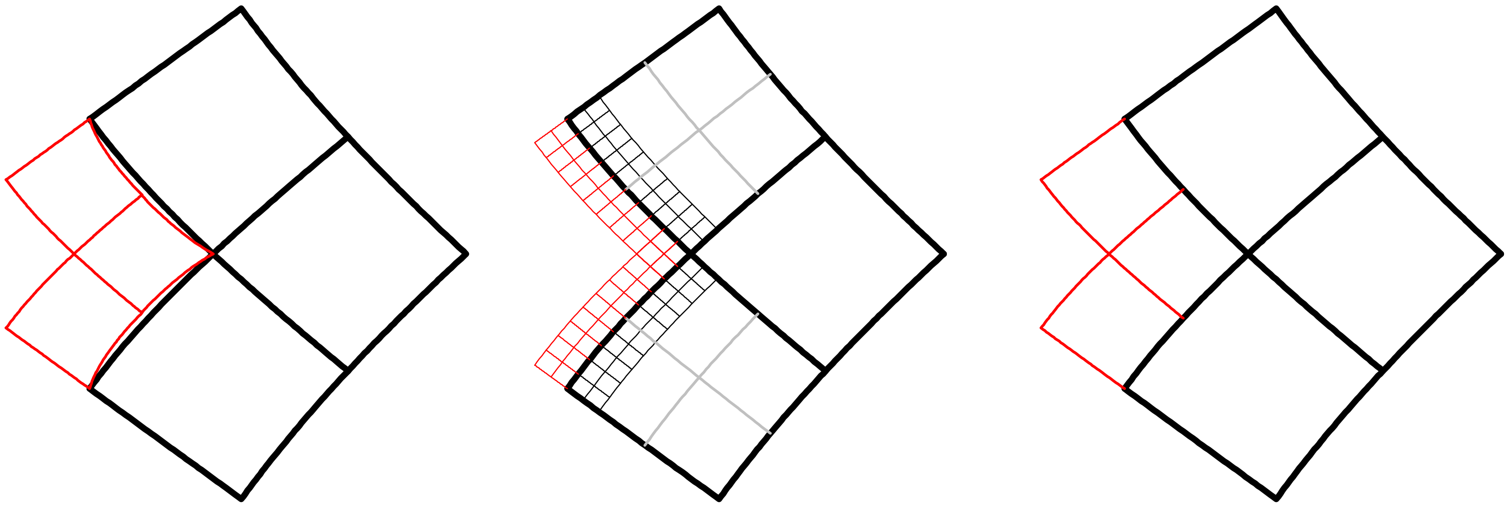
left Infinite sequence of patches:

bidegree $(6, 6)$ patches $\rightarrow$ curvature continuous surface.

bidegree $(5, 5)$ patches $\rightarrow$ curvature bounded surface.

right Ordinary patches are capped by degree (11,11) patches.

6 **Gluing adjacent rings**



left Inner (red) and outer (black) annulus are juxtaposed.

middle The outer patches are subdivided;
the subdivided data is C^2 extended to the inner ring.

left Outer ring and corrected inner ring are C^2 connected.

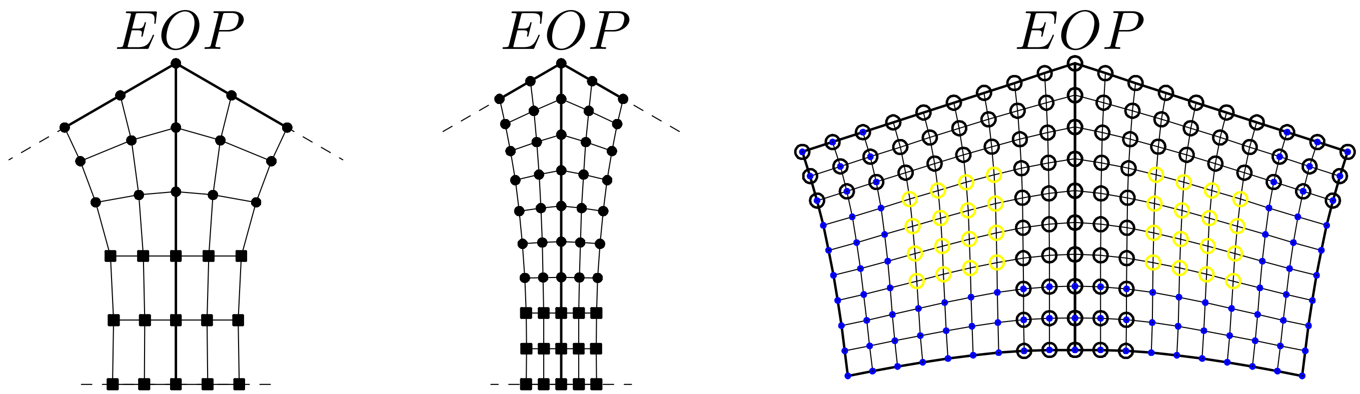
Summary

1. A finite spline scheme that empirically produces fair surfaces.

2. Shape for filling a multisided hole is from the M-patch.

3. The shape is captured by polynomial patches of moderate degree (guided finite subdivision).

Challenges: Problems of piecewise approaches

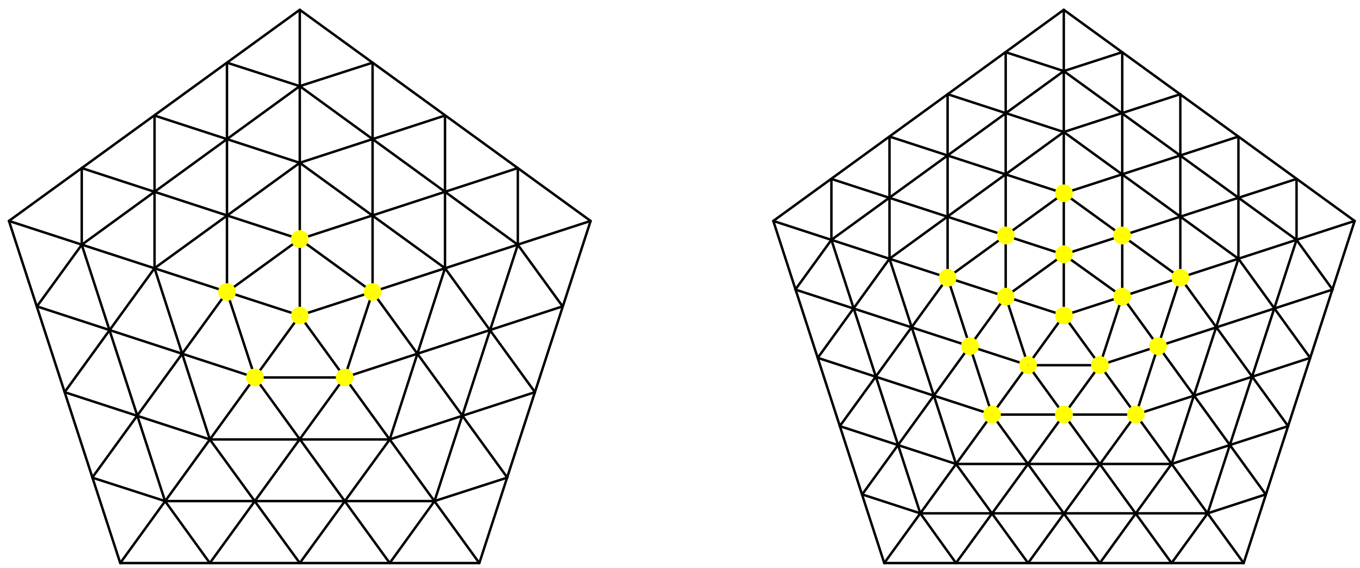


1) For low degree patches, the smoothness constraints result in very stiff connection of control points. This restricts the geometry..

2) For high degree patches, the control structures are underconstrained and difficult to control.

3) In particular, unconstrained control points (yellow) are difficult to set to match an elliptic shape intent.

Unconstrained control points of M-patch



left M-patch of depth 4 – current version in C^2 construction; unconstrained (yellow) points are under control.

right M-patch of depth 5; unconstrained (yellow) points – current problem.