Patching Catmull-Clark Meshes

— completing quadrilateral meshes as smooth Nurbs surfaces —

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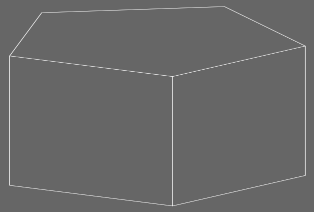

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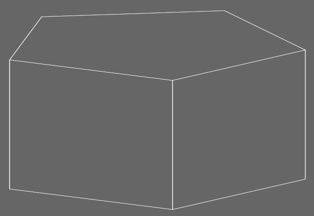

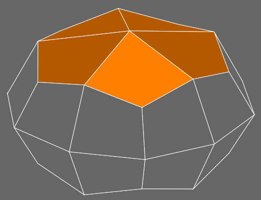

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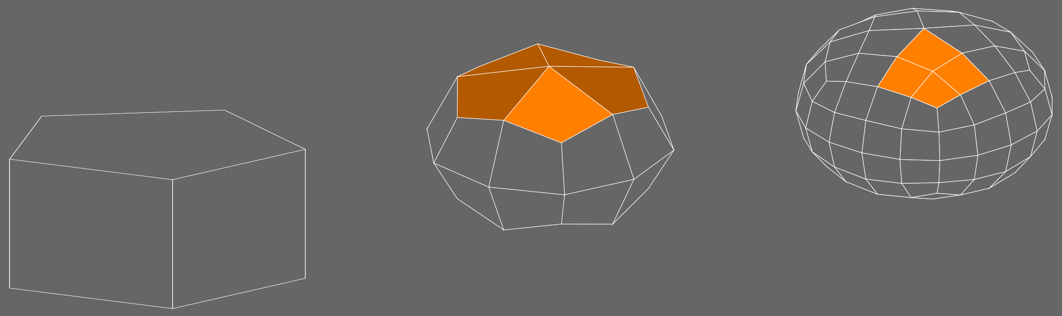

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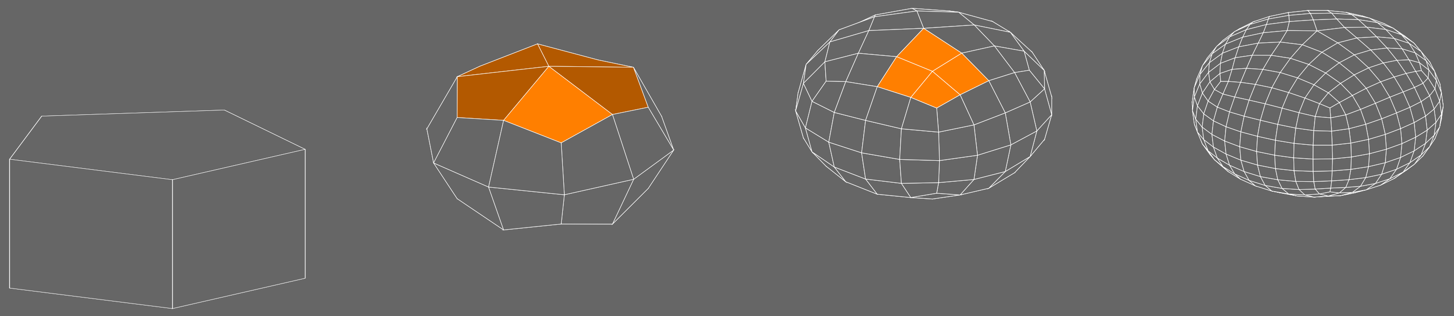

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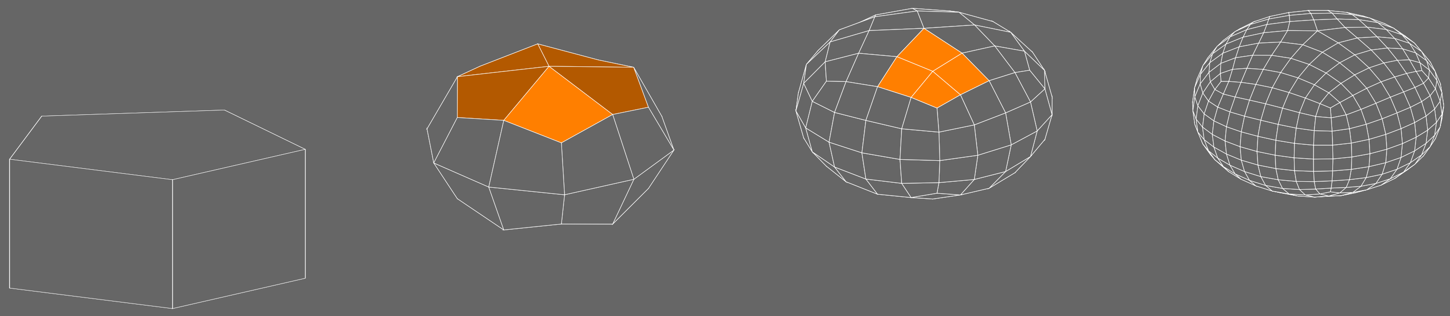

+ separate irregular nodes

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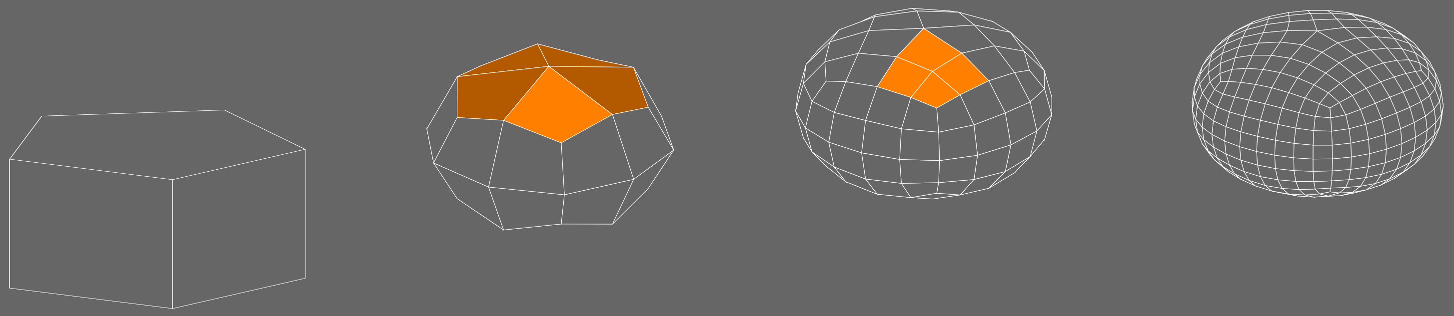

+ separate irregular nodes + distribute curvature or fit data

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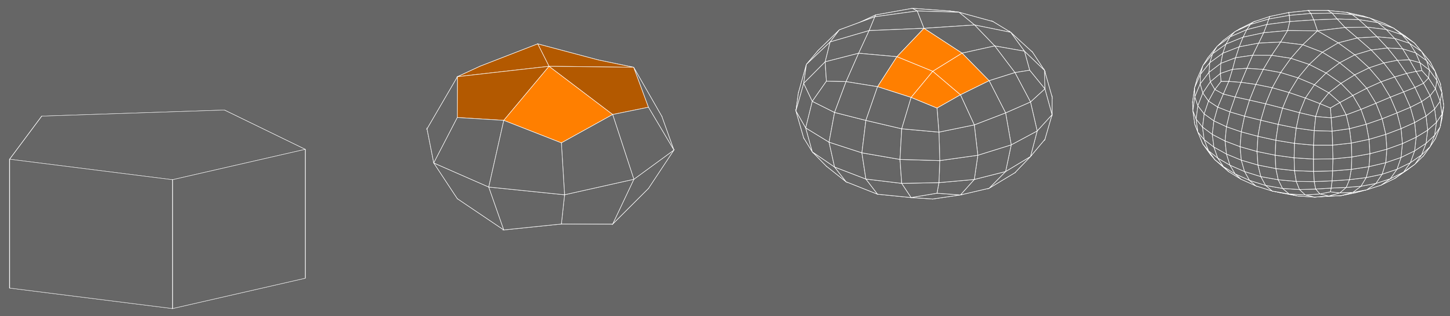

+ separate irregular nodes + distribute curvature or fit data
+ simpler than splines for smooth surfacing with irregular layout

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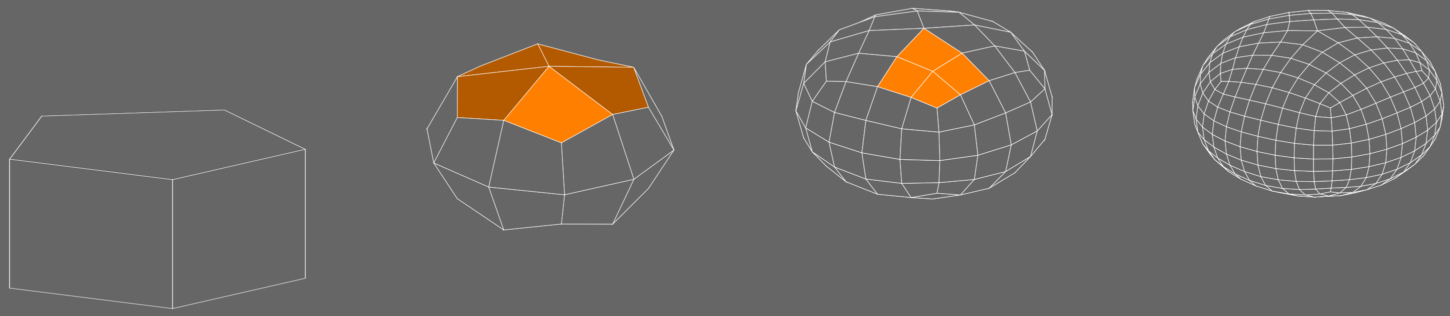

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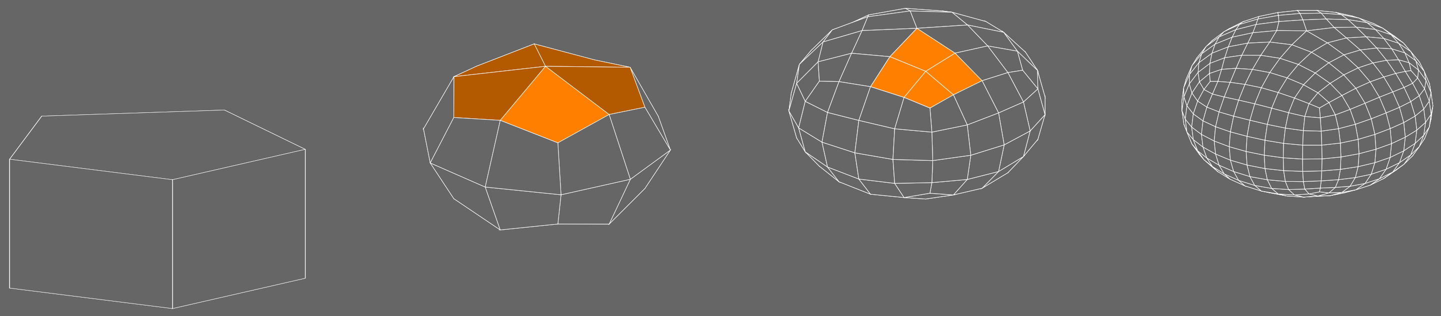

little additional surface generated

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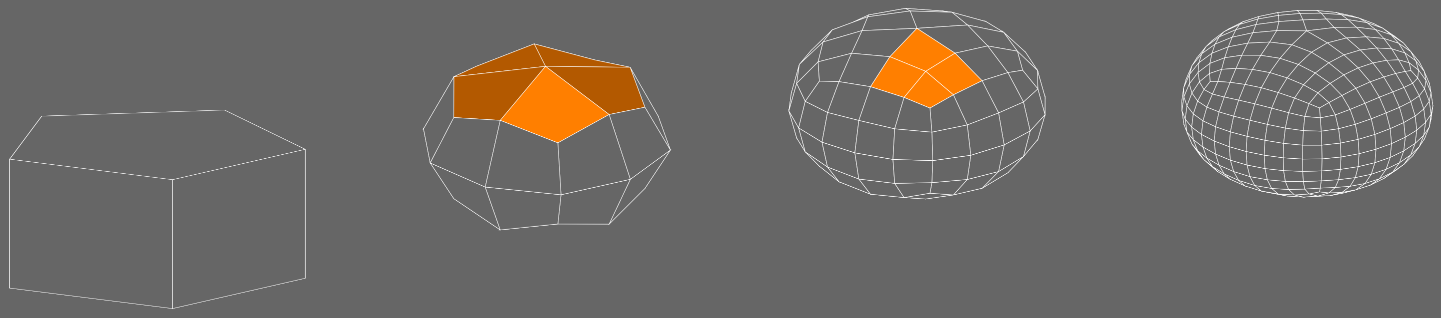

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— completing quadrilateral meshes as smooth Nurbs surfaces —

(parametrized) refinement + finite (standard) representation

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- need both subdivision data structures and surface splines

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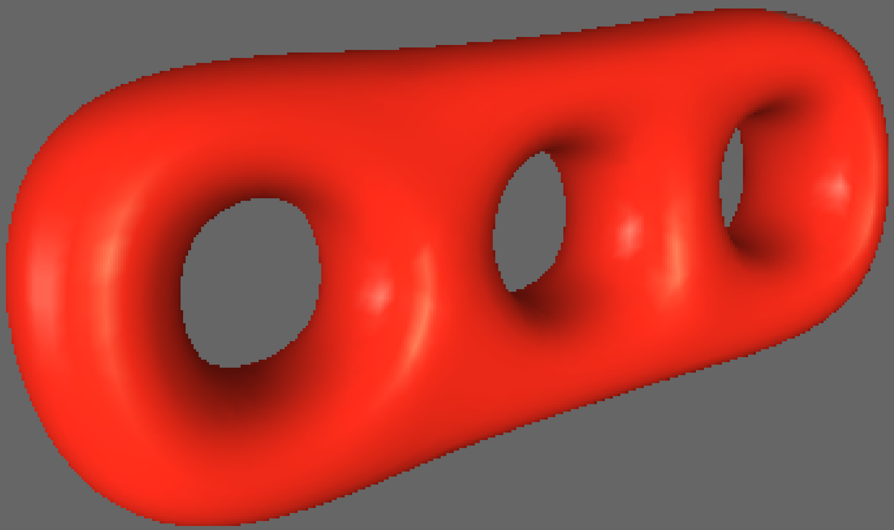
Examples



Overview

- Review of basics and literature
- Algorithm specification
- Discussion of the properties of output
- Array-based, permanent data structures
- Questions: creases, knot spacing

Quick Review: Nurbs

Non-uniform rational basic-spline patch $Q(u,v) \in \mathbf{R}^3$
of order *4* (== *bicubic* tensor-product spline)
is outlined by *control net* formed from k^2 control points $Q_{\mathtt{uv}} \in \mathbf{R}^3$.

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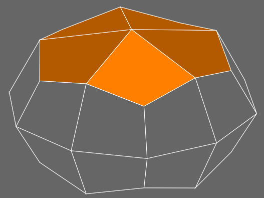
OpenGL

gluNurbsSurface(	obj,	
	uknotcount, *uknot,	vknotcount, *vknot,
	ustride, vstride,	*controlpoints,
	uorder, vorder,	fromto)

gluNurbsSurface(	obj,	
	$k+4$, *uknot,	$k+4$, *vknot,
	3, vstride,	*$Q_{\mathtt{uv}}$,
	4, 4,	GL_MAP2_VERTEX_3)

Knot insertion for splines (increasing k):
subdivide parameter domain
correspondingly refine or [sic] *subdivide* the control net.

$$4F \leftarrow \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \qquad 16E \leftarrow \begin{array}{cc} 1 & 1 \\ 6 & 6 \\ 1 & 1 \end{array}$$

$$64V \leftarrow \begin{array}{ccc} 1 & 6 & 1 \\ 6 & 36 & 6 \\ 1 & 6 & 1 \end{array}$$

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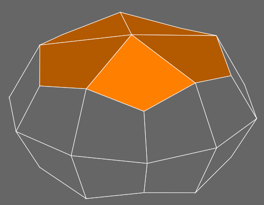

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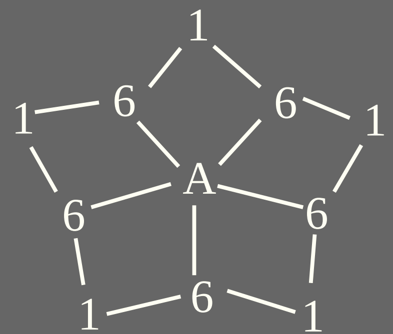
Catmull,Clark 1978:

$4n^2V \leftarrow$



$A = 4n^2 - 7n.$

What we have

Have:

- Catmull-Clark subdivision meshes [Catmull 78]

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- At any subdivision step: apply a simple transformation to get
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Would be nice:

- At any subdivision step: apply a simple transformation to get
- a compact, explicit surface representation:
- maximally large, standard spline (Nurbs) patches
- join as smoothly and largely agree with the Catmull-Clark limit surface.

PCCM Patching Catmull-Clark Meshes

Quick review: Nurbs and smooth surfaces

PCCM is new. 1 patch per quad

- *DeRose, Kass and Truong 98* [Catmull-Clark ?78] for rendering. Many small patches (glMap2); not finite.

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- Grimm and Hughes 95 subdivision as a preprocessing.
  High degree HKS rational patches. 9 times as many patches as PCCM.
- *Peters 92* parametrized subdivision as a preprocessing
  for C^1 free-form surfacing. 16 times as many bicubic Bézier patches as PCCM.

Overview

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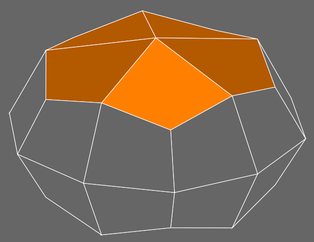
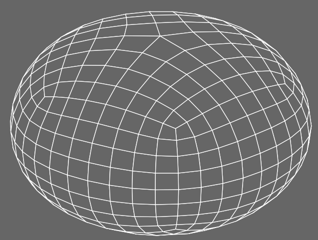
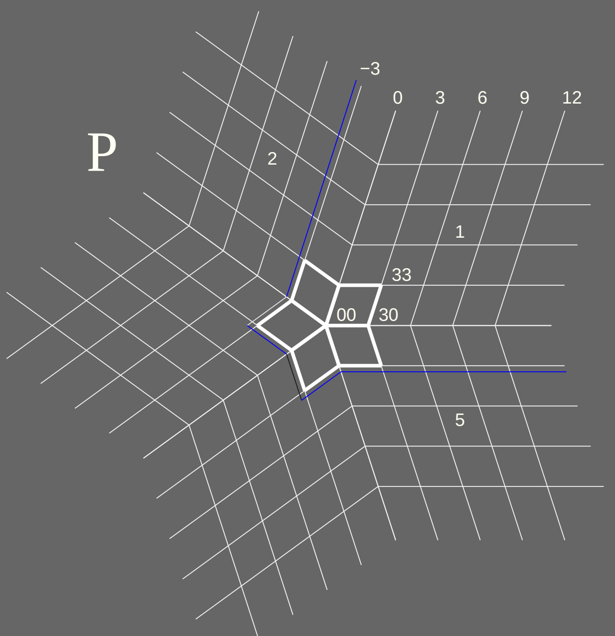
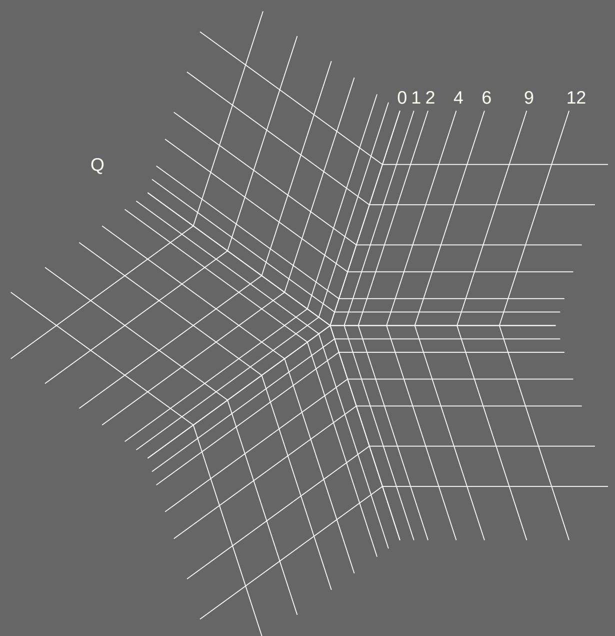
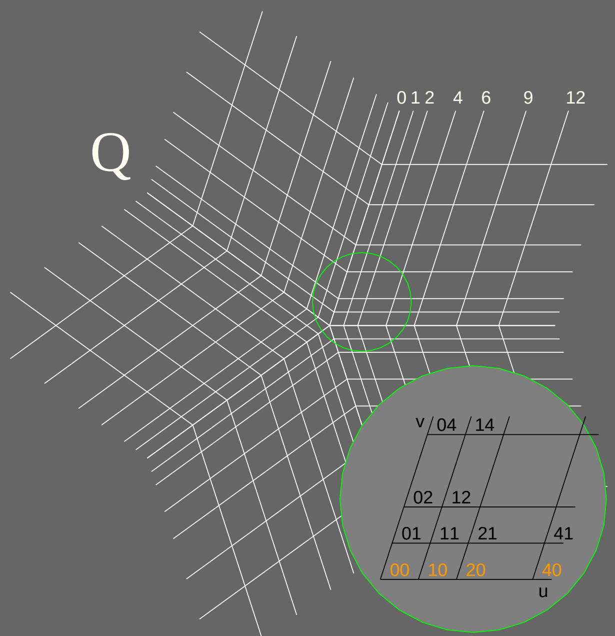
From Mesh to Surface





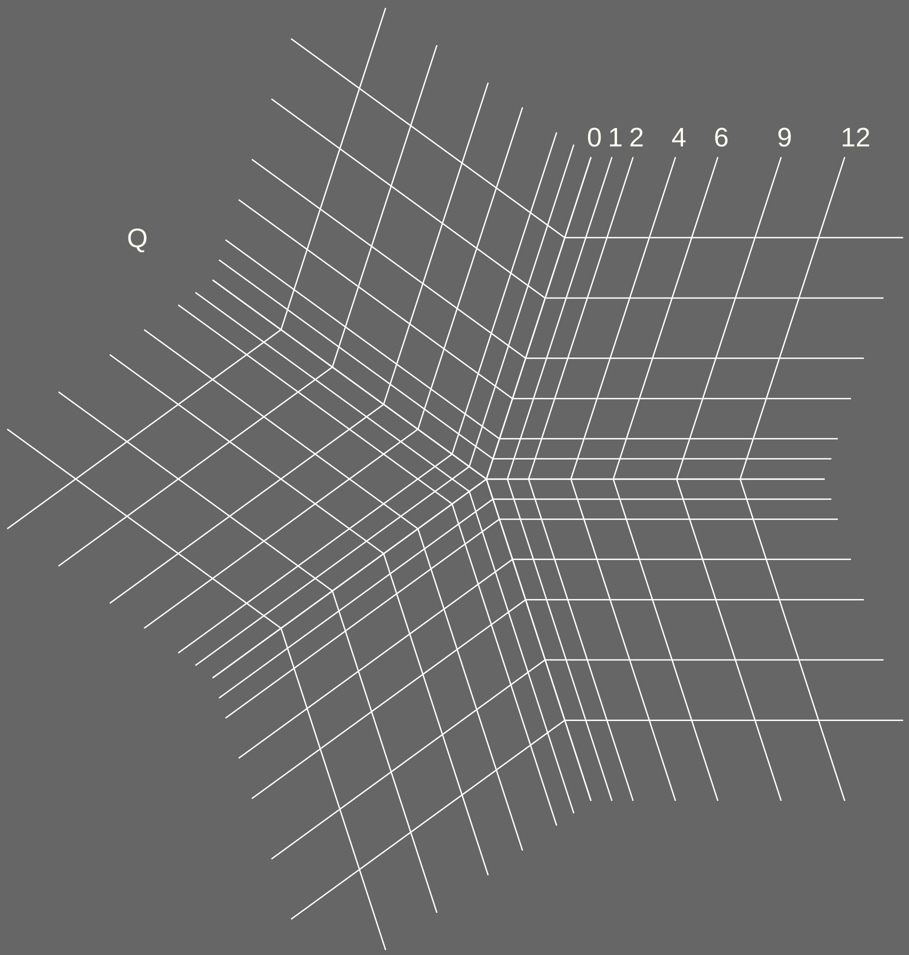






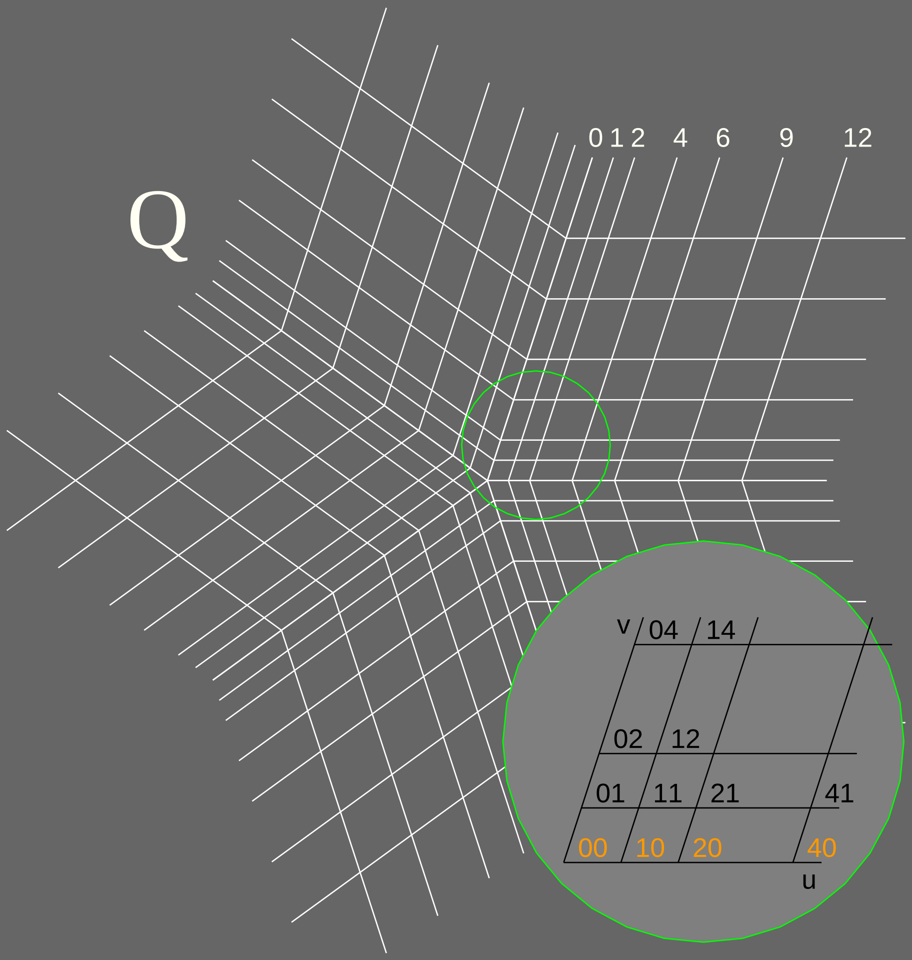
Corner point is placed directly on the Catmull-Clark limit surface:

$$Q_{00}(1) = \ldots = Q_{00}(n) = \frac{\sum nP_{00}(i) + 4P_{30}(i) + P_{33}(i)}{n(n+5)}.$$



Normal is free to choose, eg. as normal of Catmull-Clark limit surface.

PCCM: (2) Corner Smoothing



Nurbs patches C^0 at EON;
C^2 everywhere else.

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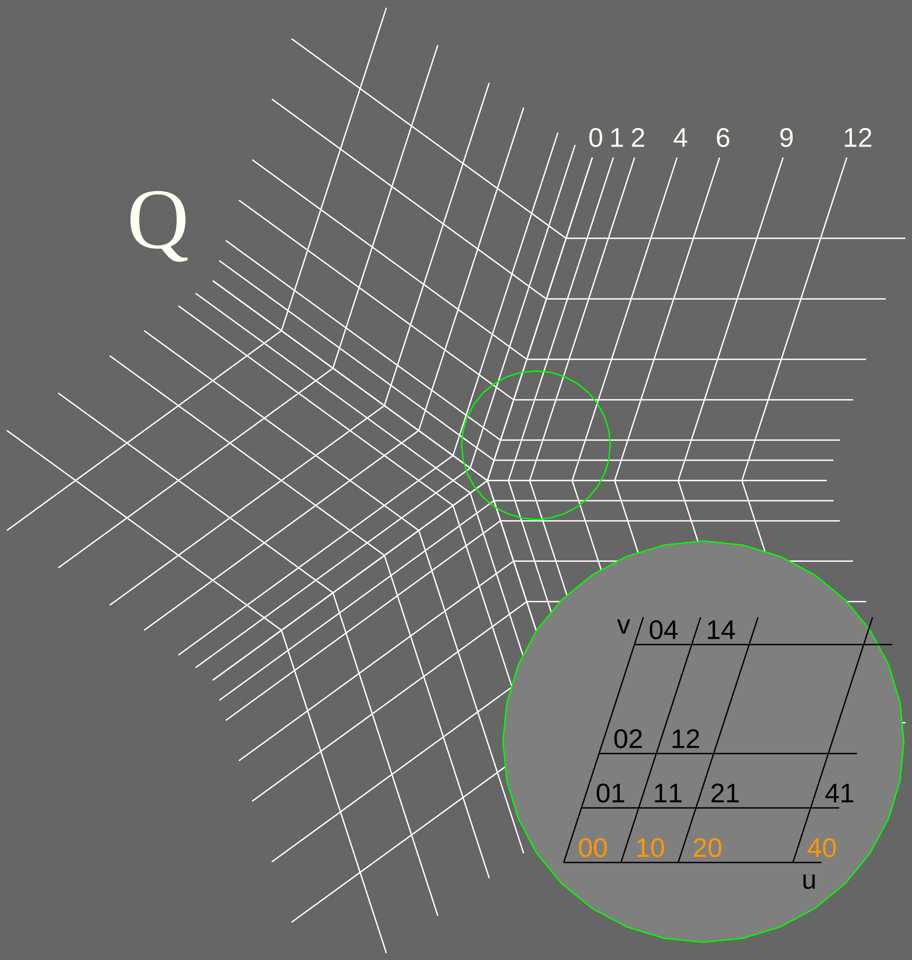

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Collect $\bar{Q}_{\mathtt{uv}}$ for $\mathtt{uv} \in \{00, 10, 20, 40\}$.

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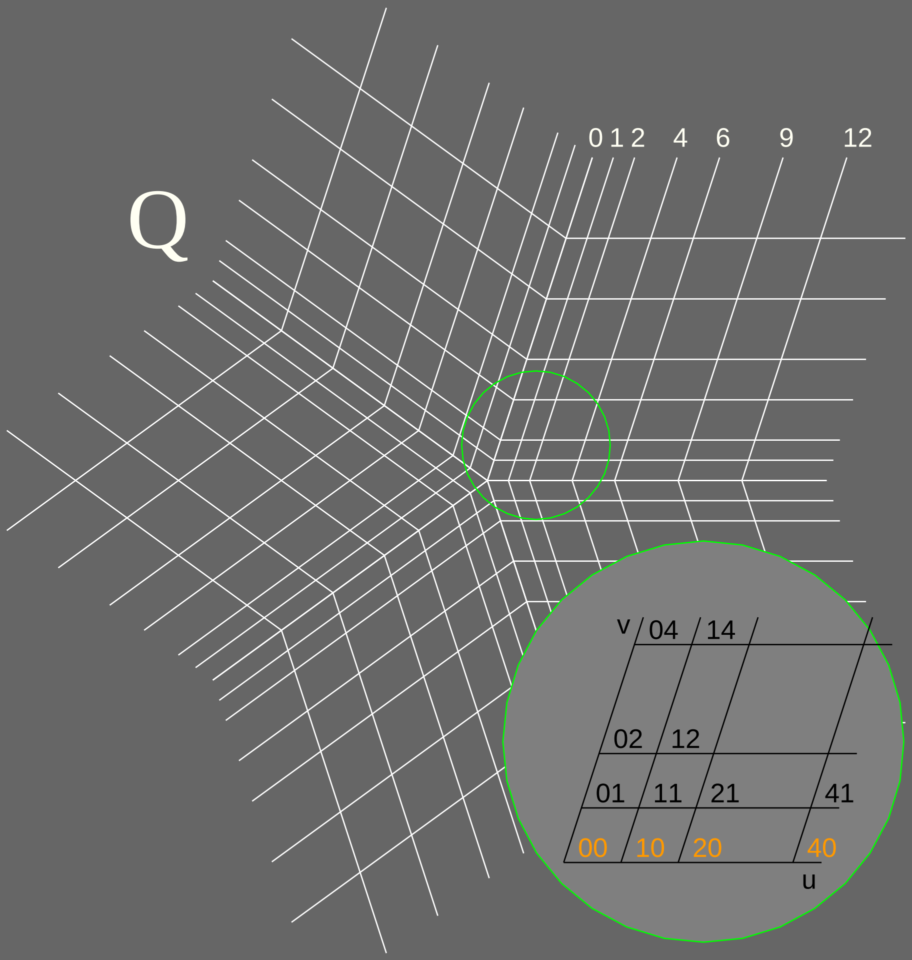

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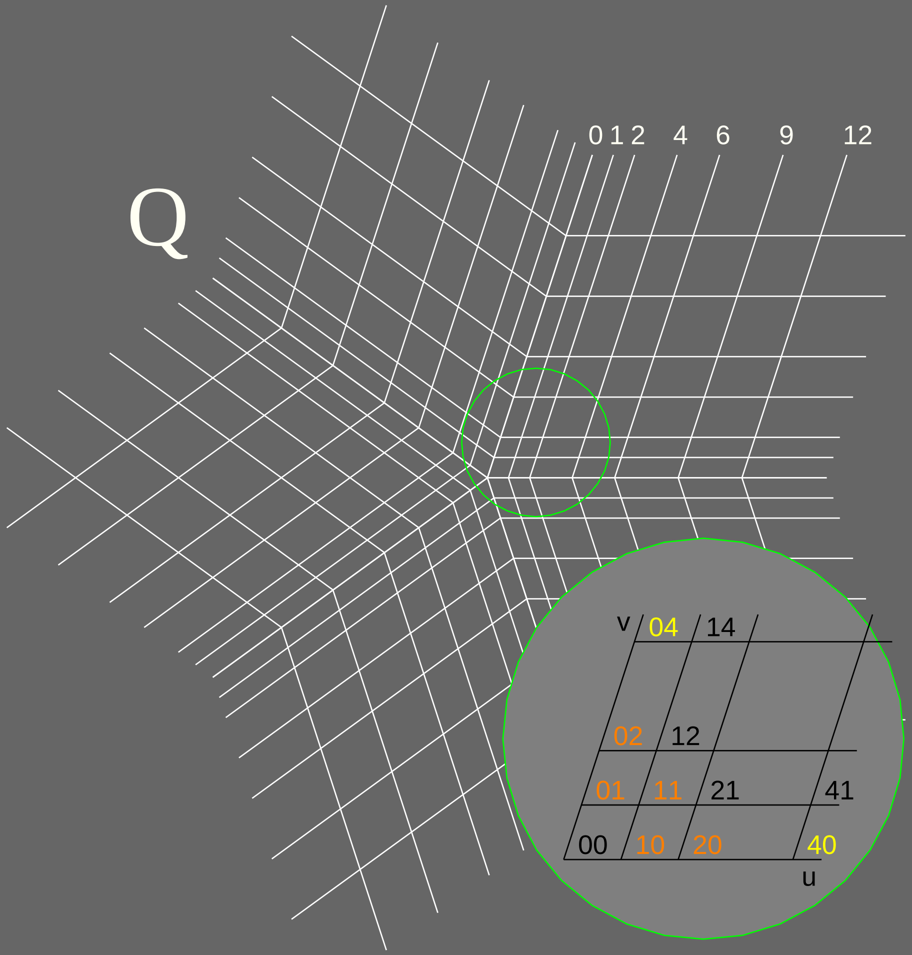
Collect $\bar{Q}_{\mathtt{uv}}$ for $\mathtt{uv} \in \{00, 10, 20, 40\}$

Use $n \times n$ matrices A_n and B_n

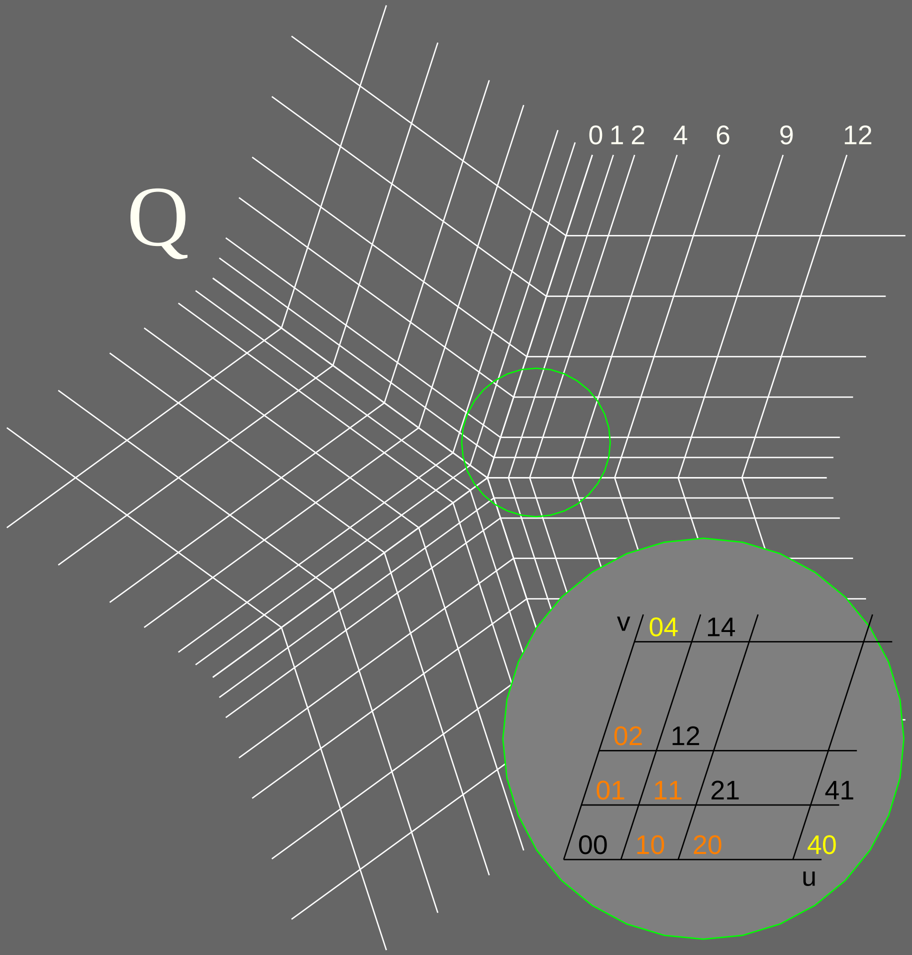


$$Q_{10} = Q_{00} + A_n \bar{Q}_{10}$$

*

$$Q_{20} = (Q_{40} + 6Q_{10} - 2Q_{00})/5$$

$$Q_{11} = B_n\left(Q_{10} + \frac{\cos(2\pi/n)}{6}(Q_{40} - Q_{20})\right)$$



$$Q_{10} = Q_{00} + \alpha A_n P_{30} + \beta(A_n + A_n^+)P_{33}$$
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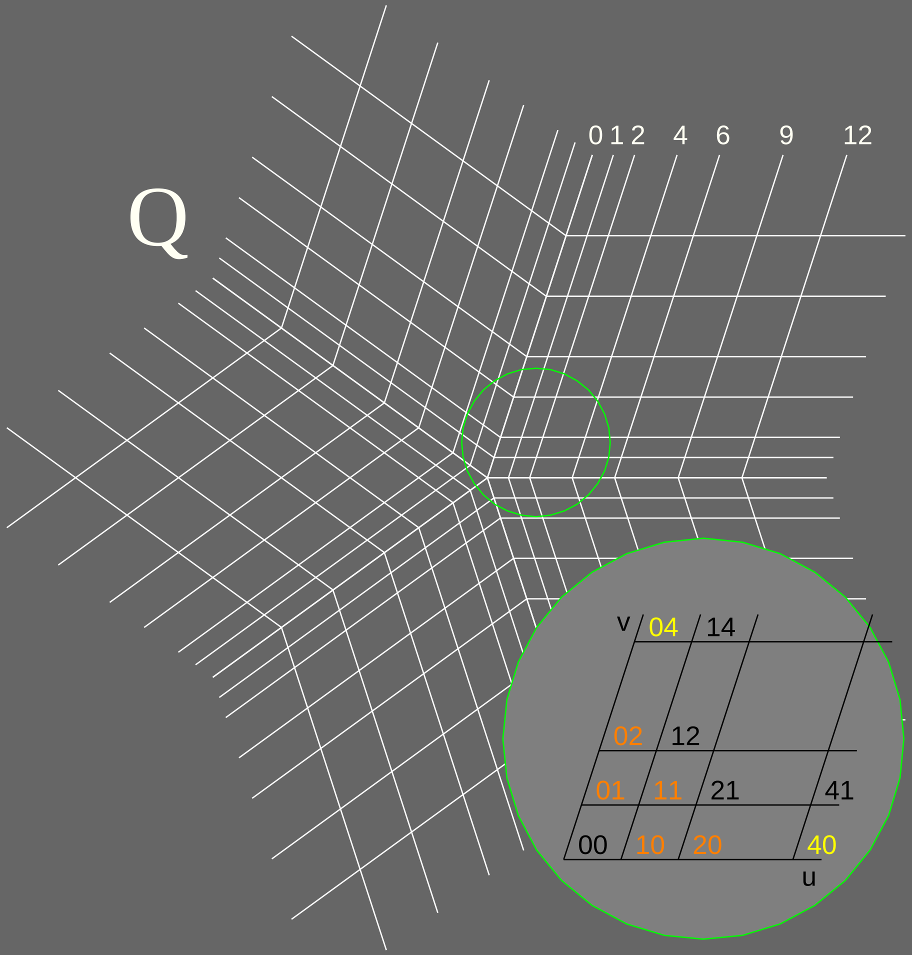

Only if n *is even and greater than 4,*
$r = \sum_{i=1}^{n} (-1)^i \bar{Q}_{40}(i)/n$ and
if $r \neq 0$ add $h_i = -(-1)^i r$ to $Q_{40}(i)$, $Q_{41}(i)$, $Q_{14}(i-1)$.

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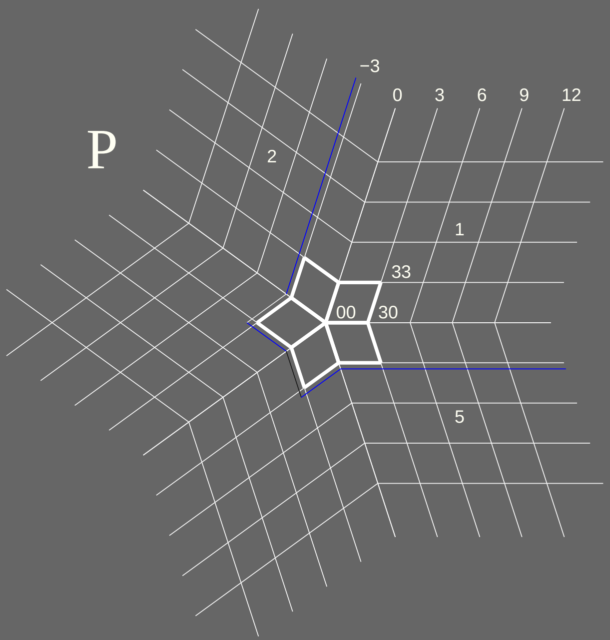
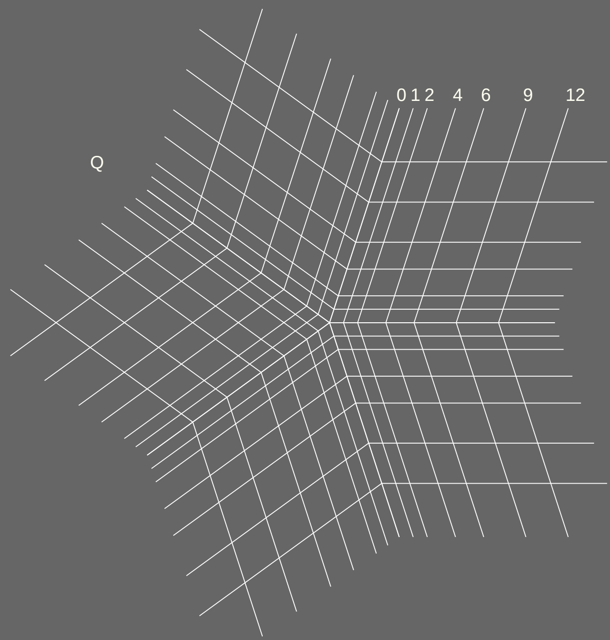
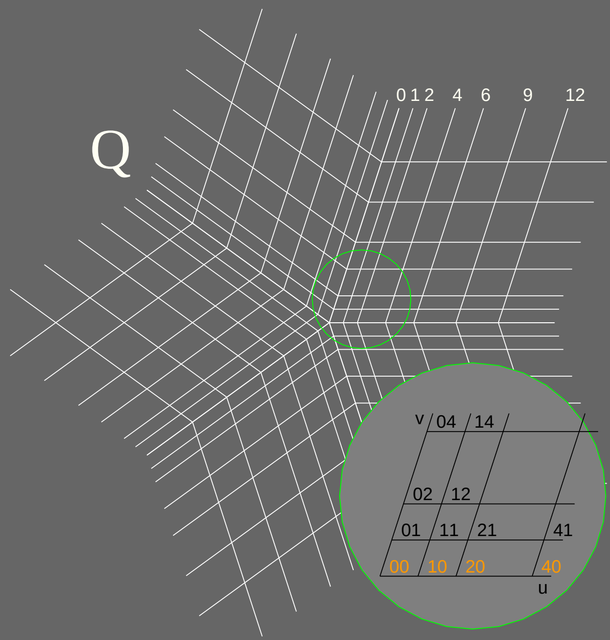
For $i = 1, \ldots, n$,
copy $Q_{\mathrm{v}0}(i+1) = Q_{0\mathrm{v}}(i)$ for $\mathrm{v} \in \{1, 2, 4\}$ and
add $Q_{20}(i) - \bar{Q}_{20}(i)$ to $Q_{21}(i)$ and $Q_{12}(i-1)$.

From Mesh to Surface







Properties of output

- Maximally large Nurbs patches.
  1 patch per quad independent of subdivision level!

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- Nurbs of standard order 4 (*degree 3*),
  in interpolating form with 4-fold knots.

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- The Nurbs patches differ from the limit surface of the Catmull-Clark subdivision only near the EONs. Interpolate position and normal of CC
  (Nurbs have *finite curvature*, Catmull-Clark limit surface infinite.)

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(Nurbs have *finite curvature*, Catmull-Clark limit surface infinite.)

  Difference:
  Generically only in *bi-3 polynomial corner pieces* of NURBS at EON;
  clamped: *positions and tangents agree* with Catmull-Clark
  at boundaries and EON!
  Difference shrinks like $O(h^5)$

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- *Higher-order saddle points* when $n \geq 6$ is even:
  Quadratic boundary segments result in unnecessarily flat sections.
  Fix: adjust second layer, not layer adjacent to EON.

Properties of output

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- C^2 almost everywhere, *tangent continuous* near the EONs.

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- C^2 almost everywhere, tangent continuous near the EONs.
- Nurbs patches, Catmull-Clark subdivision and the PCCM algorithm use the same *array-based data structures*.

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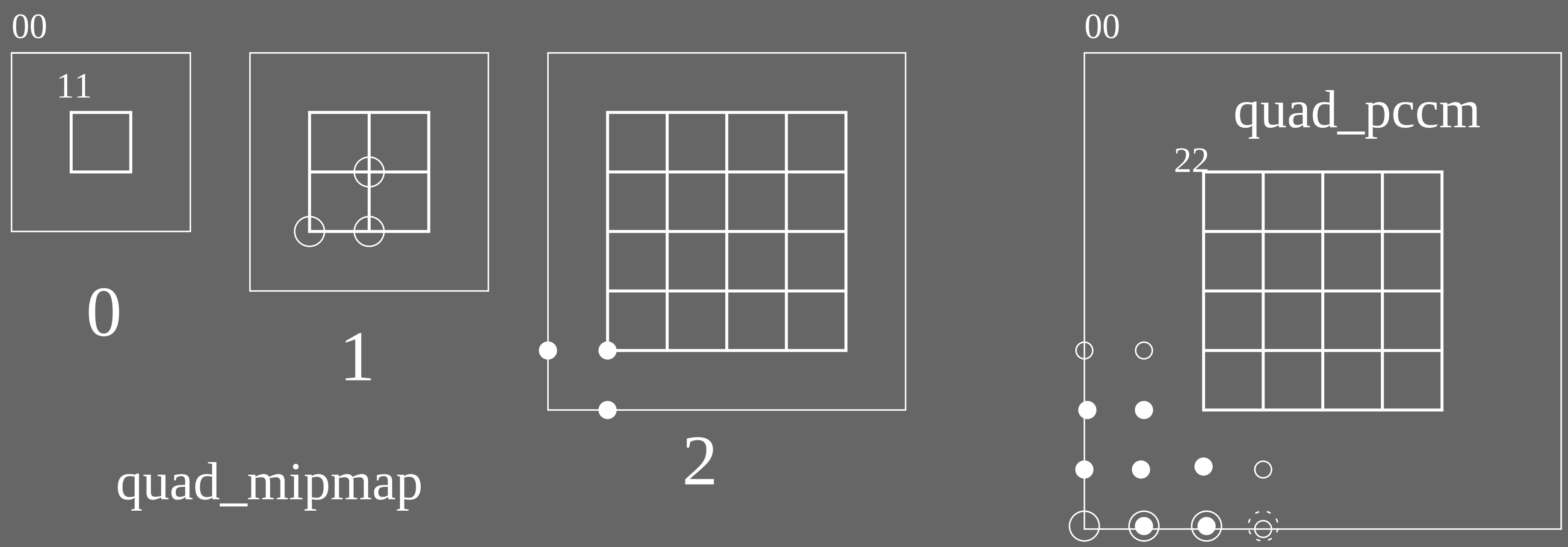
Array-based permanent data structures

Simple algorithm, simple data structures



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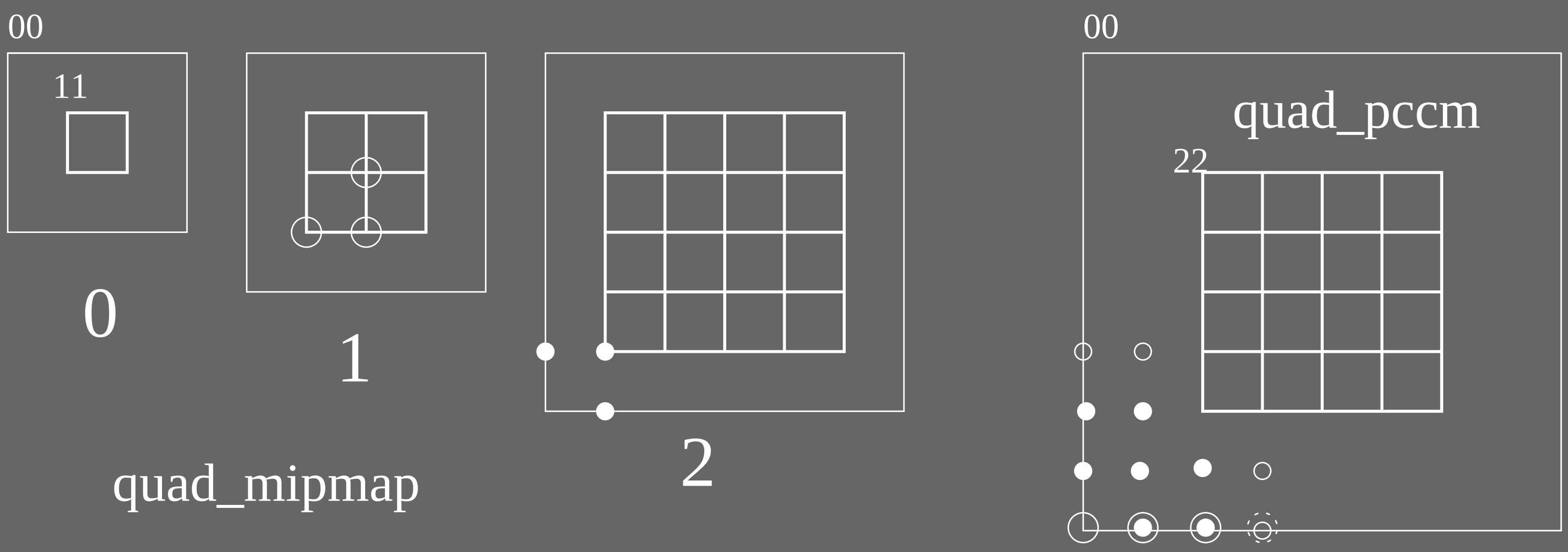
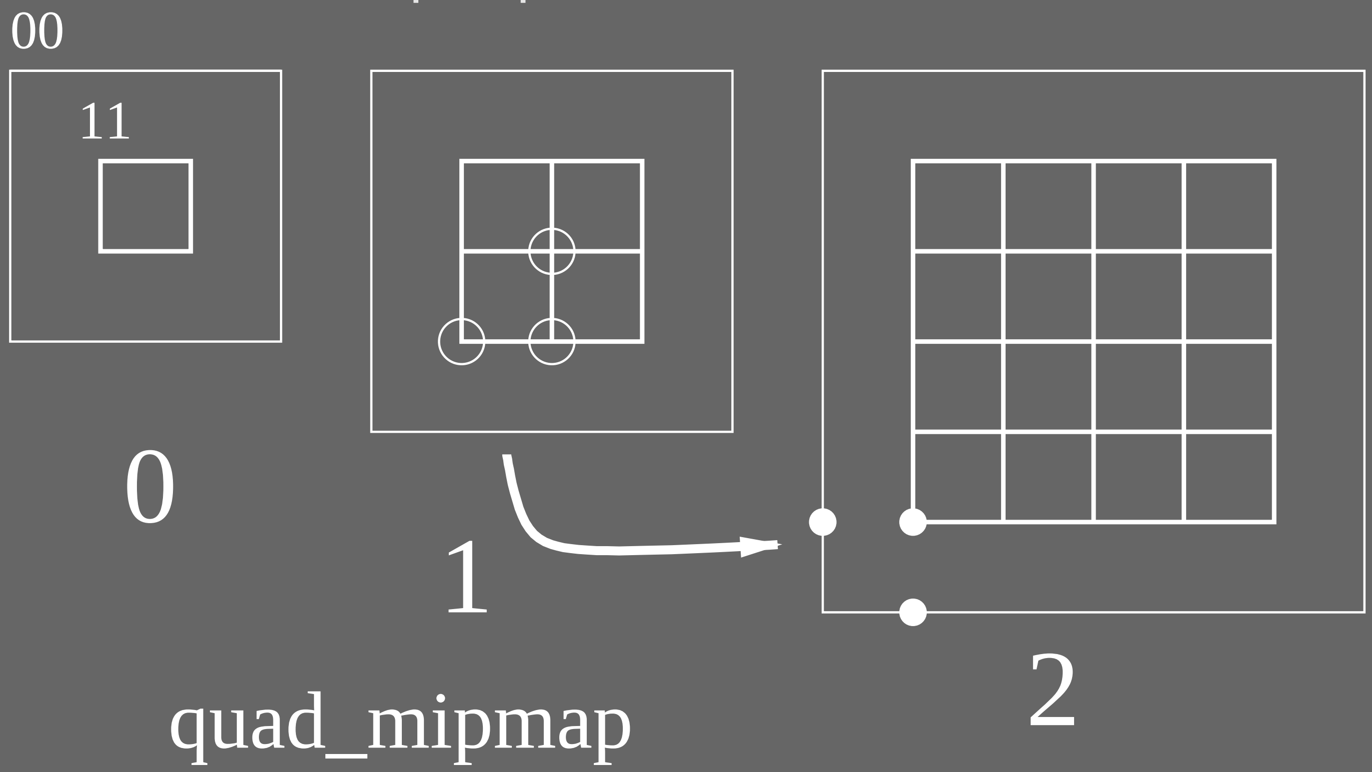


1 For each quad:
Catmull-Clark: *mipmap* of $k+2$ by $k+2$ arrays x, y, z node *positions*
PCCM: $k+4$ by $k+4$. Entry 00 = corner coefficient.

2 For each EON, access to adjacent quad corners (== B-rep of quads)

Array-based permanent data structures: Catmull-Clark

To create the mipmap level $\ell + 1$ from level ℓ.



a. For each quad: apply *B-spline subdivision rules*.

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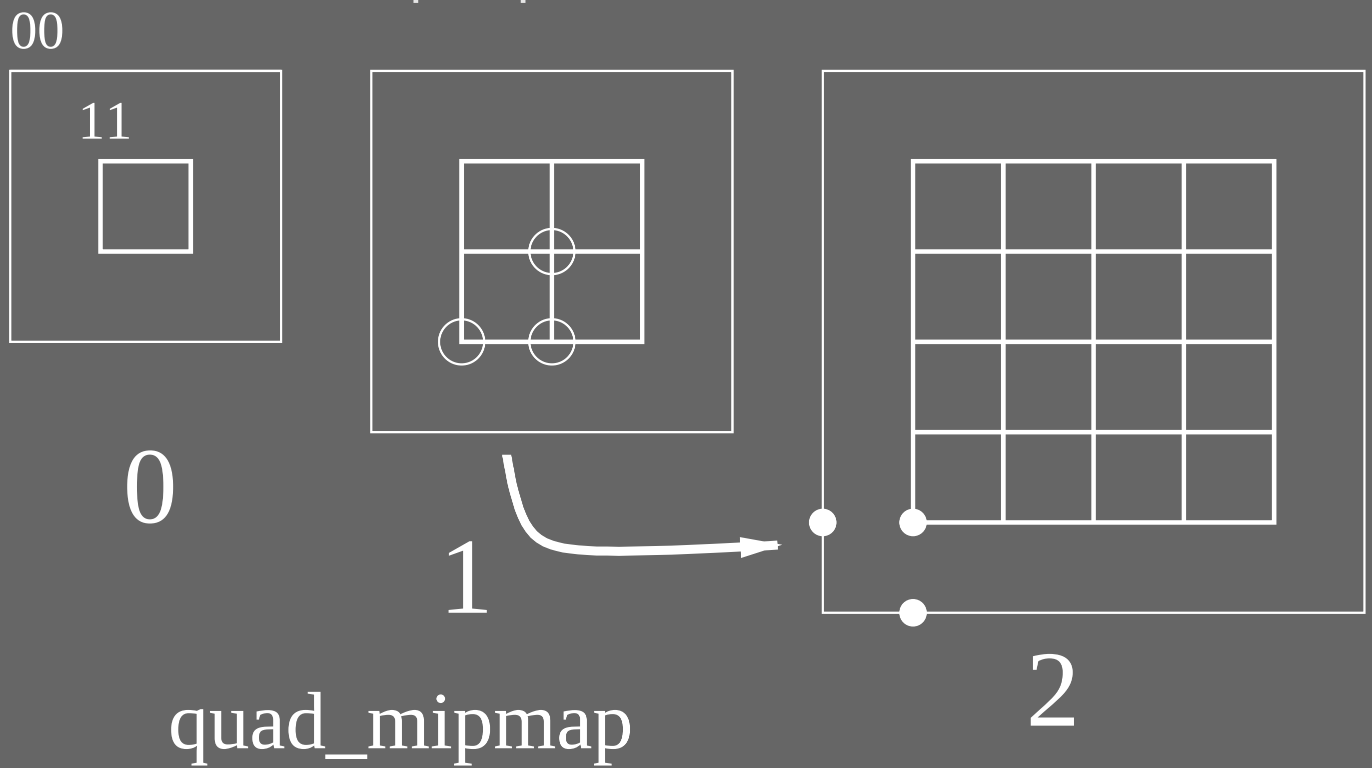
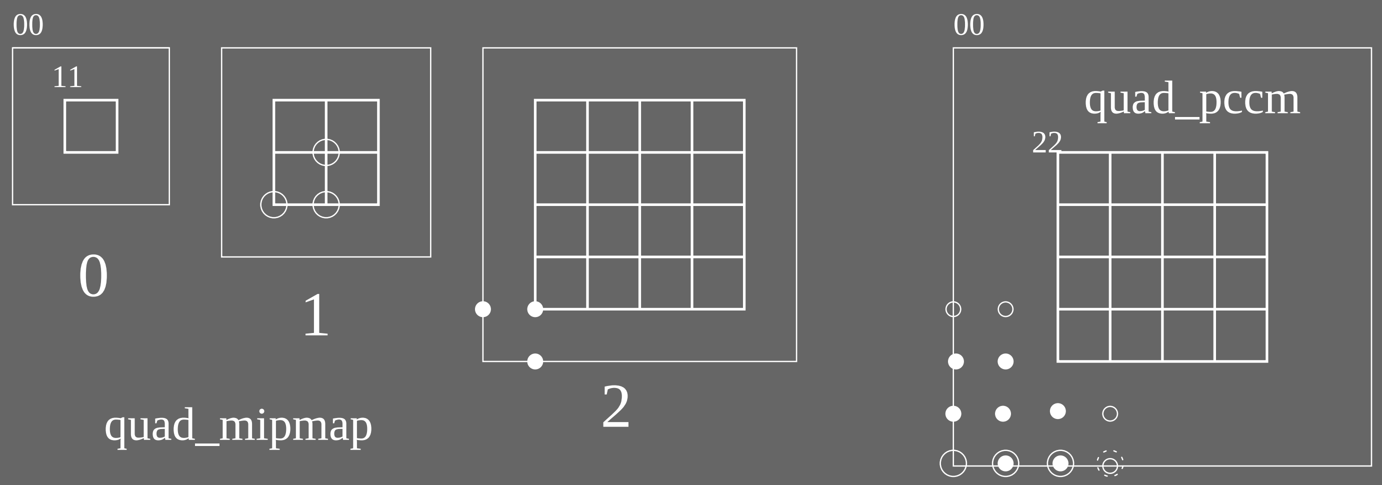


a. For each quad: apply *B-spline subdivision rules*.

b. For each EON:
– *collect* $\circ$ at level ℓ, *compute* CC, *distribute* $\bullet$ to level $\ell+1$.

Array-based permanent data structures: PCCM



a. For each quad: apply *Knot Insertion*.

b. For each EON:
–*collect* $Q_{00}(1)$ and $\bar{Q}_{\mathtt{uv}}(i)$, $\mathtt{uv} \in \{10, 20, 40\}$.
–*Compute* $Q_{\mathtt{uv}}(i)$, $\mathtt{uv} \in \{10, 20, 11\}$, $Q_{20} - \bar{Q}_{20}$ and possibly $Q_{40} - \bar{Q}_{40}$.
–*Distribute* $Q_{\mathtt{uv}}(i)$, $\mathtt{uv} \in \{10, 01, 20, 02, 11\}$ and
add to $\{21, 12\}$ and possibly $\{04, 40, 14, 41\}$.

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+ The quad-arrays can be input directly to `gluNurbsSurface` or displayed as quad-meshes.

Creases in Array-based permanent data structures

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Array of level ℓ captures the blend ratios (or smoothed creases)
of the Catmull-Clark mesh up to level ℓ.
Implementation:
— 8 additional numbers per array and
— one additional pass along the boundary of the array.

Overview

- Review of basics and literature
- Algorithm specification
- Discussion of the properties of output
- Array-based, permanent data structures
- Odds and Ends

Odds and Ends

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- Coming soon: *curvature continuity* at EON.
  (Needs degree ≥ 4)

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For the user: all mesh points are free to move!

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