Sample Test: piecewise polynomial curves

February 12, 2023

– State any simple, reasonable assumption used to arrive at your answer.
– A 'yes' or 'no' answer without reasoning is worth 0 points.
– Zero points if the writing is hard to decipher. Use a black pen if in doubt.
– Indicate with arrow if you use the back of the previous page (last page for page 1).

1   2D Curves

A curve segment on the unit interval has the Bézier coefficients

$$\left[\begin{smallmatrix} -81 \\ -27 \end{smallmatrix}\right], \left[\begin{smallmatrix} -27 \\ 27 \end{smallmatrix}\right], \left[\begin{smallmatrix} 27 \\ -27 \end{smallmatrix}\right], \left[\begin{smallmatrix} 81 \\ 27 \end{smallmatrix}\right].$$

(a) (1 point) Sketch the curve segment

(b) (2 points) Compute the position and normal at $t = 1/3$ using *de Casteljau's algorithm*.

2 2D Curves

A curve segment p_1 in Bézier form has coefficients

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

and another curve segment p_2 has coefficients

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}.$$

(a) (1 point) For what values x_0, y_0 do the two curves meet with equal first derivative?

(b) (1 point) For what values x_2, y_2 do the two curves meet with equal second derivative?

(c) (1 point) Can the curve segments be joined so that the third derivative matches?

(d) (1 point) Estimate the area between the x-axis and the curve segment on $x \in [-2 \ldots 2]$. (Hint: use the convex hull of the two curve segments!)