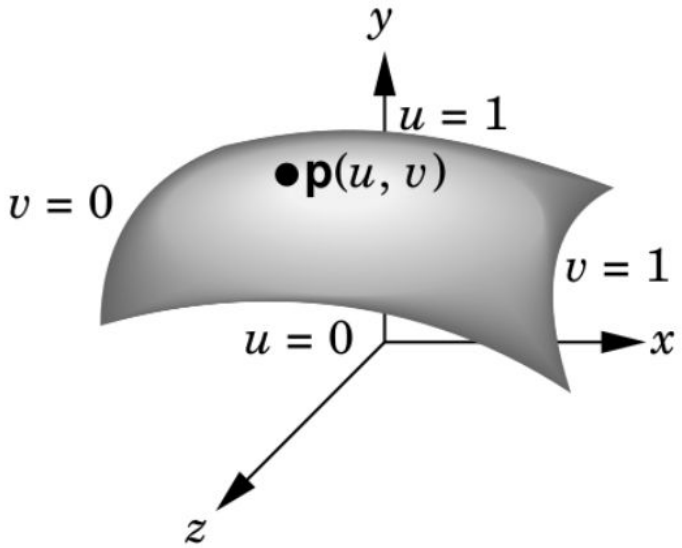
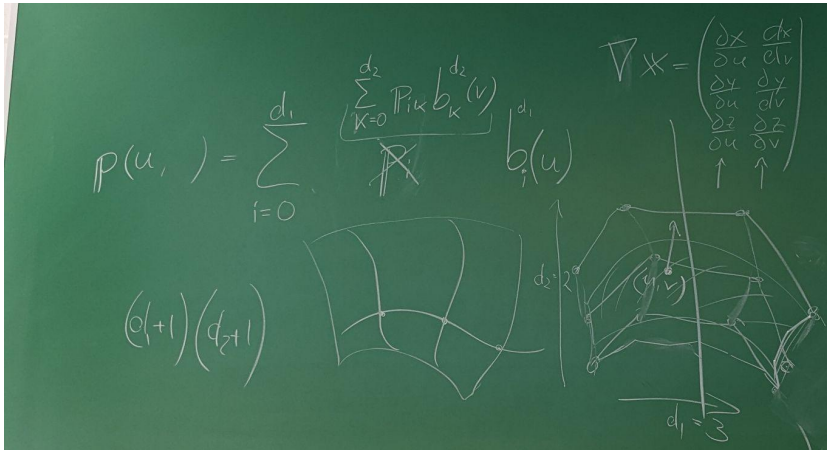
Curved Geometry in 2 variables: tensor-product, 4-sided

Tensor-product BB-form:

$$\mathbf{p}(u,v) = \sum_{i}^{d_1} \sum_{k}^{d_2} \mathbf{p}_{ik} B_i^{d_1}(u) B_k^{d_2}(v)$$

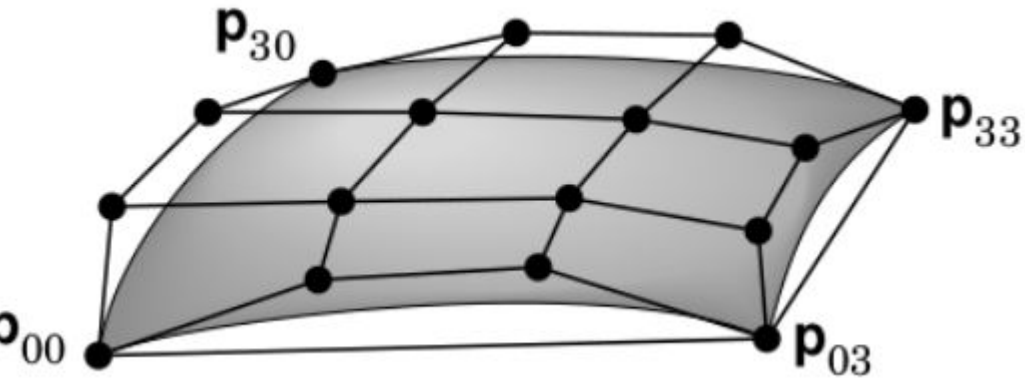


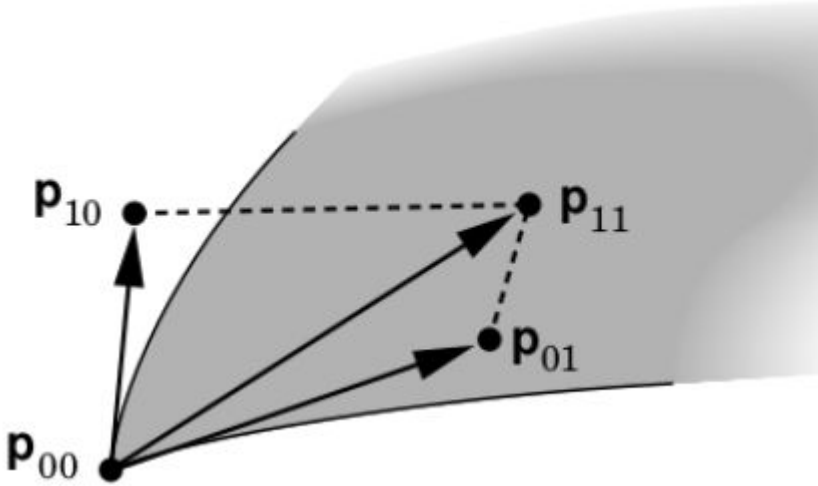


Curved Geometry in 2 variables: Control net, position&derivative at corners

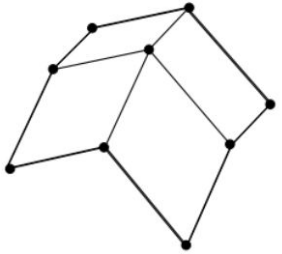
Tensor-product BB-form:

$$\sum_{i}^{d_1} \sum_{k}^{d_2} \mathbf{p}_{ik} B_i^{d_1}(u) B_k^{d_2}(v)$$



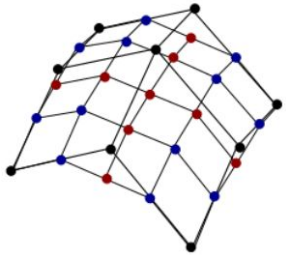


Curved Geometry in 2 variables: deCasteljau, subdivision



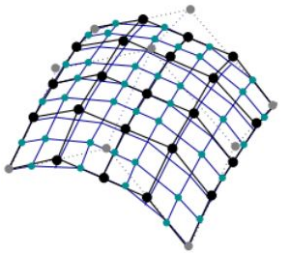
Subdivision begins with a few points connected to form faces

- These are the initial control points used to define the surface



At each step new points are created determined by the surrounding points.

- Original control points
- de Casteljau in front-to-back direction
- Second de Casteljau application

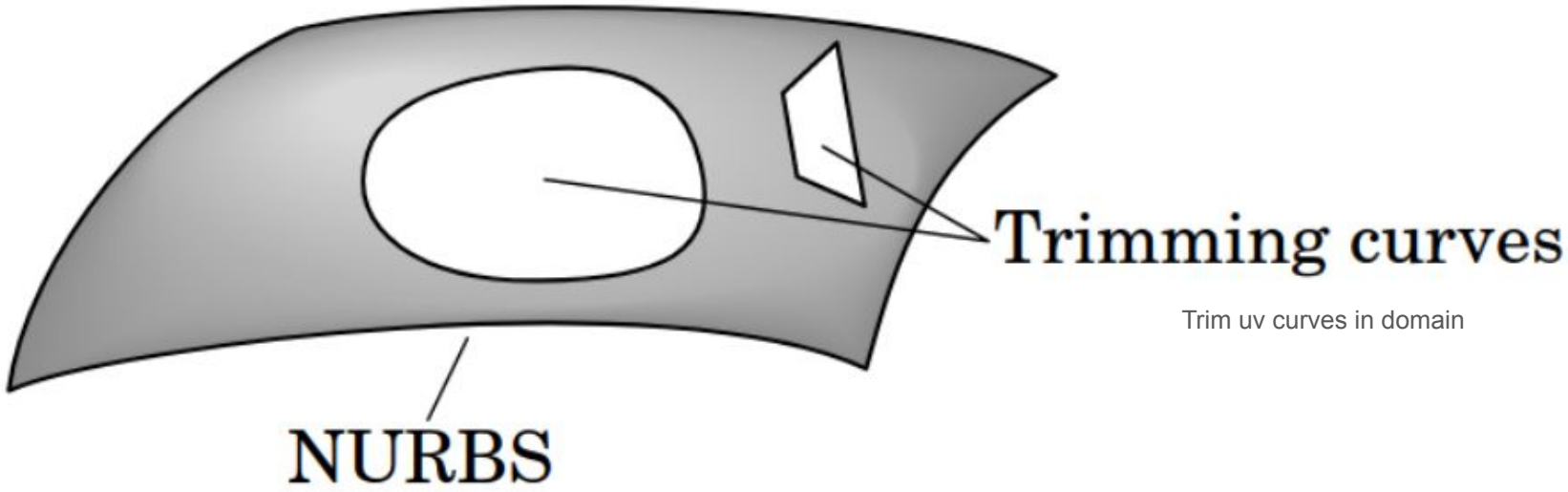


Iteration of de Casteljau=Subdivision

- Input control points
- First Iteration
- Second Iteration

Curved Geometry in 2 variables: restriction of the domain=trimming

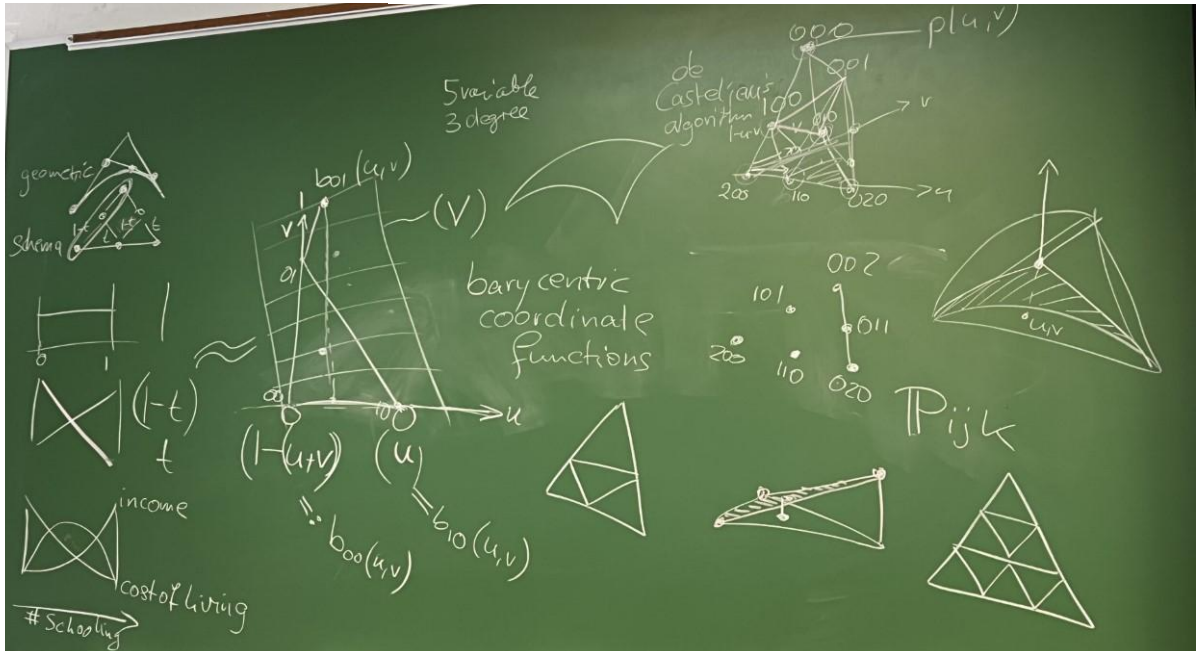
NURBS



Trim uv curves in domain

Curved Geometry in 2 variables: total degree, 3-sided

$$p = \sum_{i+j+k=d} c(i,j,k) B_{i,j,k}; \qquad B_{i,j,k}(u,v,w) = \frac{d!}{i!j!k!} u^i v^j w^k, \qquad u+v+w=1$$



Curved Geometry in 2 variables: de Casteljau

$$p = \sum_{i+j+k=d} c(i,j,k) B_{i,j,k},$$

$$B_{i,j,k}(u,v,w) = \frac{d!}{i!j!k!} u^i v^j w^k, \qquad u+v+w=1$$

for $l = 1..d$

. for $i+j+k = d-l$

. $c(i,j,k) = u \cdot c(i+1,j,k) + v \cdot c(i,j+1,k) + w \cdot c(i,j,k+1)$

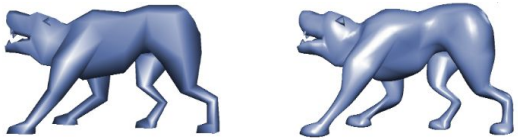
$$n = (c(0,1,0) - c(1,0,0)) \times (c(0,0,1) - c(1,0,0))$$

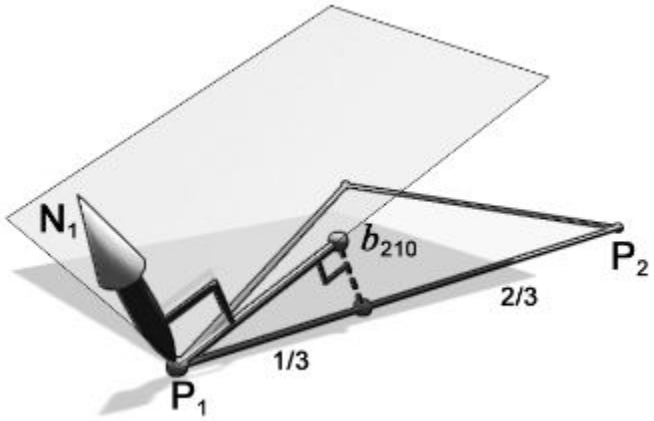
return($puvw = c(0,0,0), normal = n/\|n\|$)

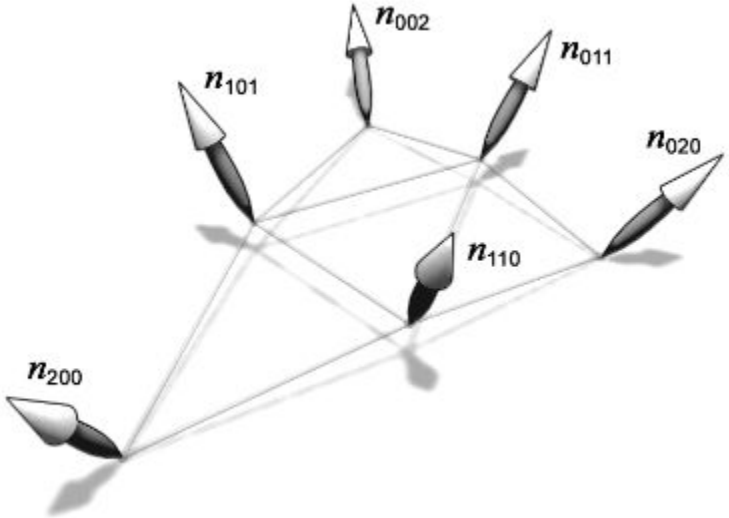
Curved PN triangles

Curved Geometry in 2 variables: construction

Curved PN triangles

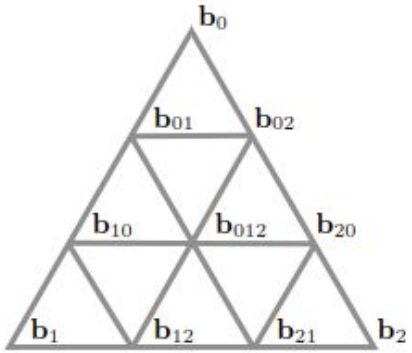


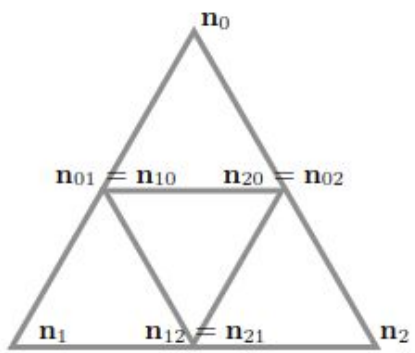


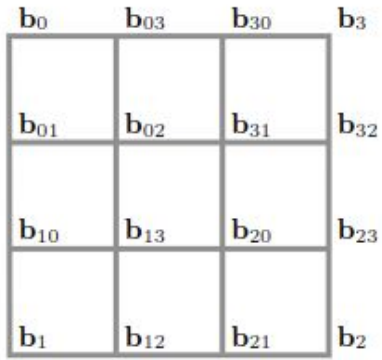


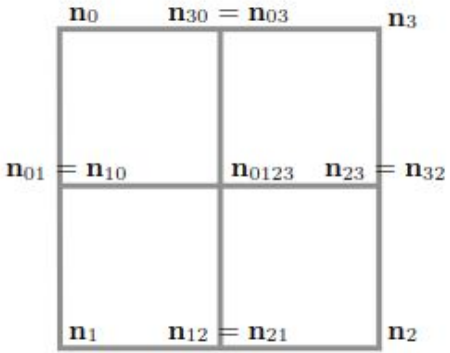
Curved Geometry in 2 variables: PN triangles

Curved PN quads

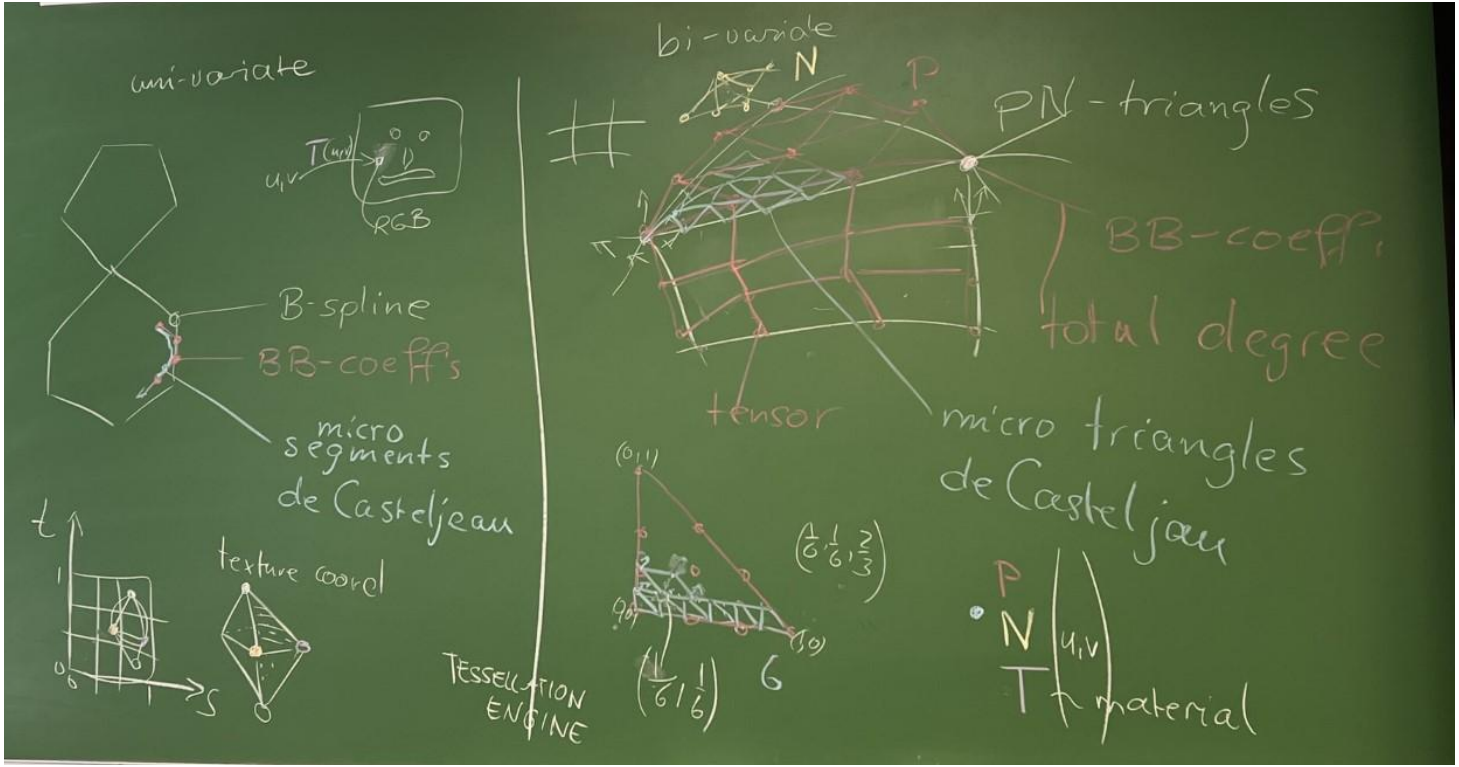








Rendering via the tessellation engine



Curved Geometry in 2 variables

https://en.wikipedia.org/wiki/Covariance_and_contravariance_of_vectors
https://en.wikipedia.org/wiki/Tensor_calculus
https://en.wikipedia.org/wiki/Differential_form