Effect of Attack on Scale-free Networks Due To Cascading Failures

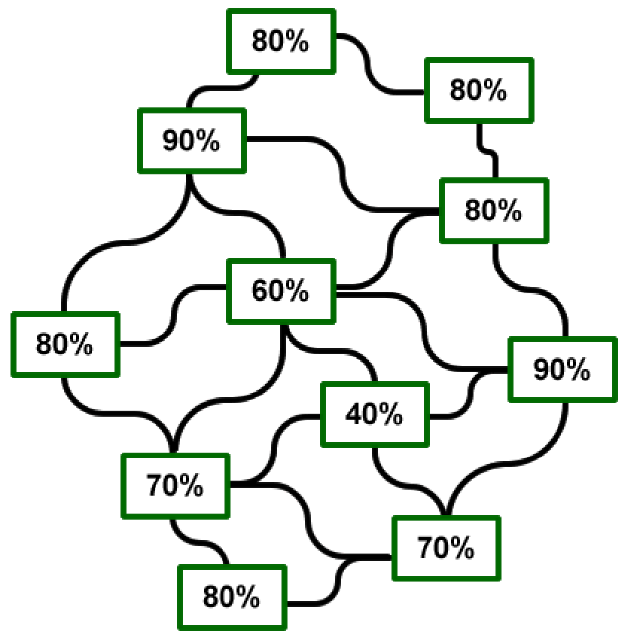
Jian-Wei Wang et al
Presented by Nam Nguyen

Content

- About Cascading Failure
- Models
  - Terminologies
  - A loading-dependent model
    - Finding the Distribution of number of component failures
  - A parameter-dependent model
- Attack Strategies
- Results
- Applications – Open questions

About Cascading Failure

- Def: A failure in a system or network of interconnected parts where a failure of a part can trigger the consequence failures of successive parts.
- Example



Network running normally

About Cascading Failure

- Common in
  - Power grids, causing large backouts of electric power transmission.
  - Traffic, causing traffic jams.
  - Computer networks (such as the Internet)
- Predicting the behavior of cascading failure is important.

Modeling cascading failure

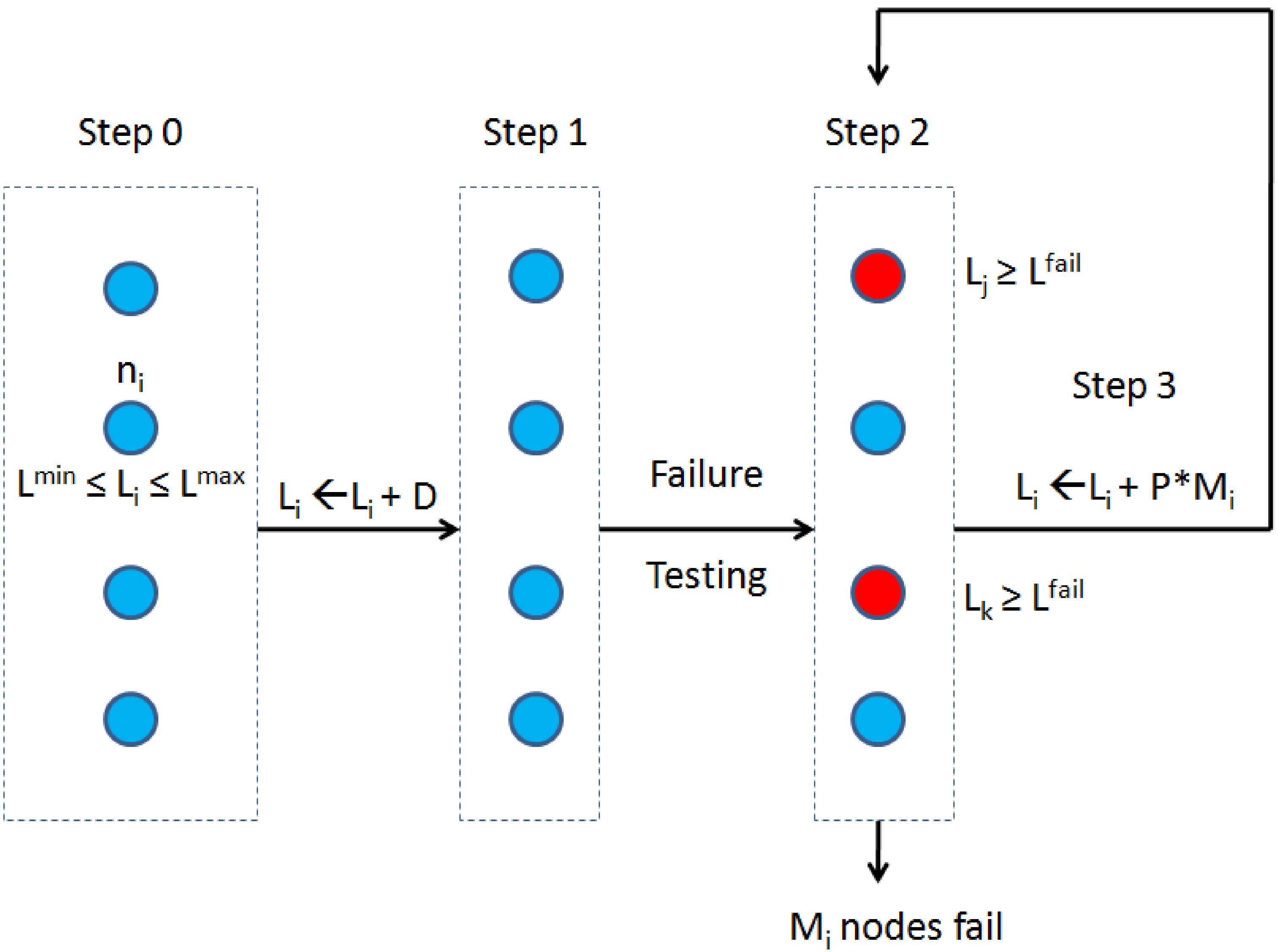
- A loading-dependent model (by Ian Dobson et al)
  - n identical nodes with random initial loads.
  - N_i has initial load L_i and $L_1,..,L_n$ are independent and uniformly distributed in $[L^{min}, L^{max}]$.
  - A component fails when its load exceeds L^{fail}.
  - When a node fails, a fixed amount P is transferred to each of the nodes.
  - Starting with adding an amount D and test of failures. At each stage, add M_i*P to the load of each other nodes.

A loading-dependent model

CASCADE Algorithm

0. All n components are initially unfailed and have initial loads $L_1, L_2, \ldots, L_n$ that are independent random variables uniformly distributed in $[L^{\min}, L^{\max}]$.
1. Add the initial disturbance D to the load of each component. Initialize the stage counter i to zero.
2. Test each unfailed component for failure: For $j = 1, \ldots, n$, if component j is unfailed and its load is greater than L^{fail}, then component j fails. Suppose that M_i components fail in this step.
3. Increment the component loads according to the number of failures M_i: Add $M_i P$ to the load of each component.
4. Increment i and go to step 2.

A loading-dependent model



A loading-dependent model

- If $M_j=0$ then $M_{j+1} = M_{j+2} = \ldots = M_n = 0$;
- We are interested in $S = M_1+M_2+\ldots+M_{n-1}$;
- Normalization

$$\ell_j = (L_j - L^{\min})/(L^{\max} - L^{\min})$$

$$d = \frac{D + L^{\max} - L^{\text{fail}}}{L^{\max} - L^{\min}}, \qquad p = \frac{P}{L^{\max} - L^{\min}}.$$

- Let $N(t)$ = #of n_i where l_i is in $(1-t,1]$;
- $S_j = M_1+M_2+\ldots+M_{j-1}$;

$$S_j = N(d + S_{j-1}p), \qquad j = 0,1,\ldots.$$

A loading-dependent model

$$S_j = N(d + S_{j-1}p), \qquad j = 0, 1, \ldots.$$

$$N(d + Sp) = S,$$

$$N(d + S_j p) > S_j, \qquad -1 \le j < k.$$

- and $N(d + (S_j + r)p) \ge N(d + S_j p) = S_{j+1} = S_j + M_{j+1} > S_j + r.$
- Therefore:

$$S = \min\{s \,|\, N(d + sp) = s, \qquad s \in \{0, 1, 2, \ldots\}\}.$$

- (where S = $M_1 + M_2 + \ldots + M_{n-1}$)

Distribution of S

- $S = M_1 + M_2 + \ldots + M_{n-1}$

$$S = \min\{s \mid N(d + sp) = s, \qquad s \in \{0,1,2,\ldots\}\}.$$

- Distribution of S

$$P[S = r] = \begin{cases} \binom{n}{r} \phi(d)(d + rp)^{r-1}(\phi(1 - d - rp))^{n-r}, & r = 0,1,\ldots,n-1 \\ 1 - \sum_{s=0}^{n-1} P(S = s), & r = n, \end{cases}$$

$$\phi(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x \le 1 \\ 1, & x > 1. \end{cases}$$

Distribution of S

- Let $P[S = r] = f(r, d, p, n)$
- n = 0: $f(0, d, p, 0) = 1$
- d≤0: no components fail; d≥n: all components fail.

$$f(r, d, p, n) = \begin{Bmatrix} 1 - \phi(d), & r = 0 \\ 0, & 0 < r < n \\ \phi(d), & r = n \end{Bmatrix} \qquad (d \leq 0 \text{ or } d \geq 1) \text{ and } n > 0$$

- Prob. for M_0

$$P[M_0 = k] = \binom{n}{k} d^k (1 - d)^{n-k}$$

Distribution of S

$$P[M_0 = k] = \binom{n}{k} d^k (1-d)^{n-k}$$

$$P[S = r | M_0 = k] = f\left(r - k, \frac{kp}{1-d}, \frac{p}{1-d}, n - k\right)$$

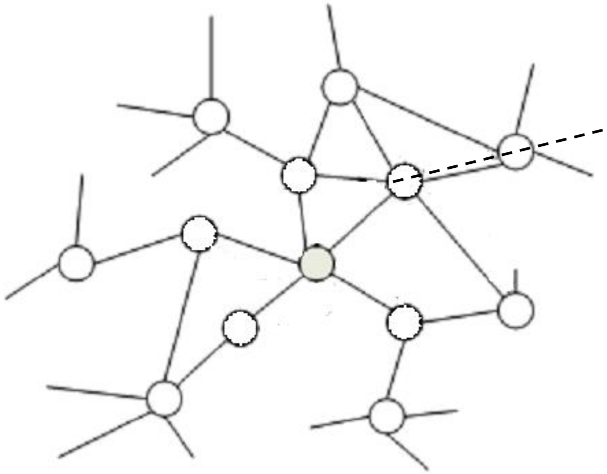
- Thus

$$P[S = r] = f(r, d, p, n) = \sum_{k=0}^{r} P[S = r | M_0 = k] P[M_0 = k]$$

$$= \sum_{k=0}^{r} \binom{n}{k} d^k (1-d)^{n-k} f\left(r - k, \frac{kp}{1-d}, \frac{p}{1-d}, n - k\right),$$

$$0 \le r \le n, \quad 0 < d < 1, \quad n > 0.$$

A Parameter-dependent Model

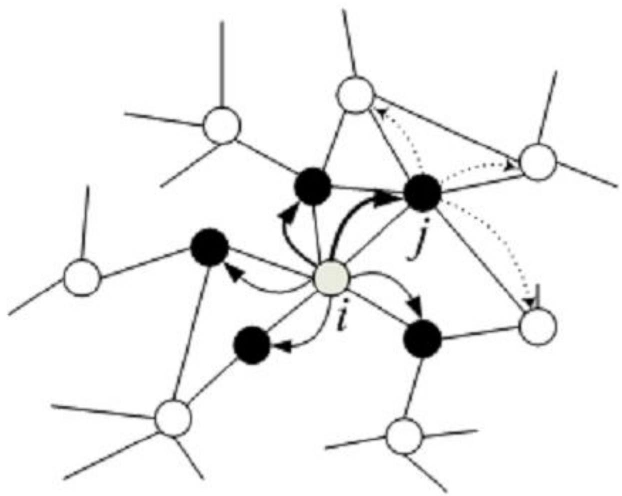


$L_j = k_j^{\alpha}$, (k_j is degree of node j)

$C_j = T * L_j, j = 1, 2, 3, \ldots, N$

is the capacity of node j

Node j fails if $L_j + \Delta L_{ji} > C_j$



When node i fails

$$\Delta L_{ji} = L_i \frac{k_j^{\alpha}}{\sum_{m \in \Gamma_i} k_m^{\alpha}}$$

$$L_j = L_j + \Delta L_{ji}$$

Will node j fail afterward ?

Attack the network

- Let CF_i be the number of broken nodes induced by REMOVING node i ($0 \leq CF_i \leq N-1$).
- Let B be the set of attacked nodes and $N_B = |B|$.

$$CF_{\text{attack}} = \frac{\sum_{i \in B} CF_i}{N_B(N-1)}$$

- Critical threshold T_c.
  - $T > T_c$: No cascading failure occurs
  - $T < T_c$: Cascading failure may occur
  - The lower the value of T_c, the stronger the robustness of the network again CF.

Attack Stratergies

- Random Breakdown (RB)
  - Randomly remove nodes in the network.
- Nodes with highest degrees (HD)
  - Attack on nodes with highest potential to cause CF
- Nodes with lowest degrees (LD)
  - Attack on lowest degree nodes
- Node with the lowest average degree of its neighboring nodes (LADN)

Results

- Experiment set up
  - Use Barabási–Albert network generator.
  - Starting at m_0 nodes, each node with m links is attached at each timestep with prob. $\prod_i = k_i / \sum_j k_j$
  - N = 5000 nodes.
  - $m_0 = m = 2$;
  - Average connectivity is 2m = 4;

Results

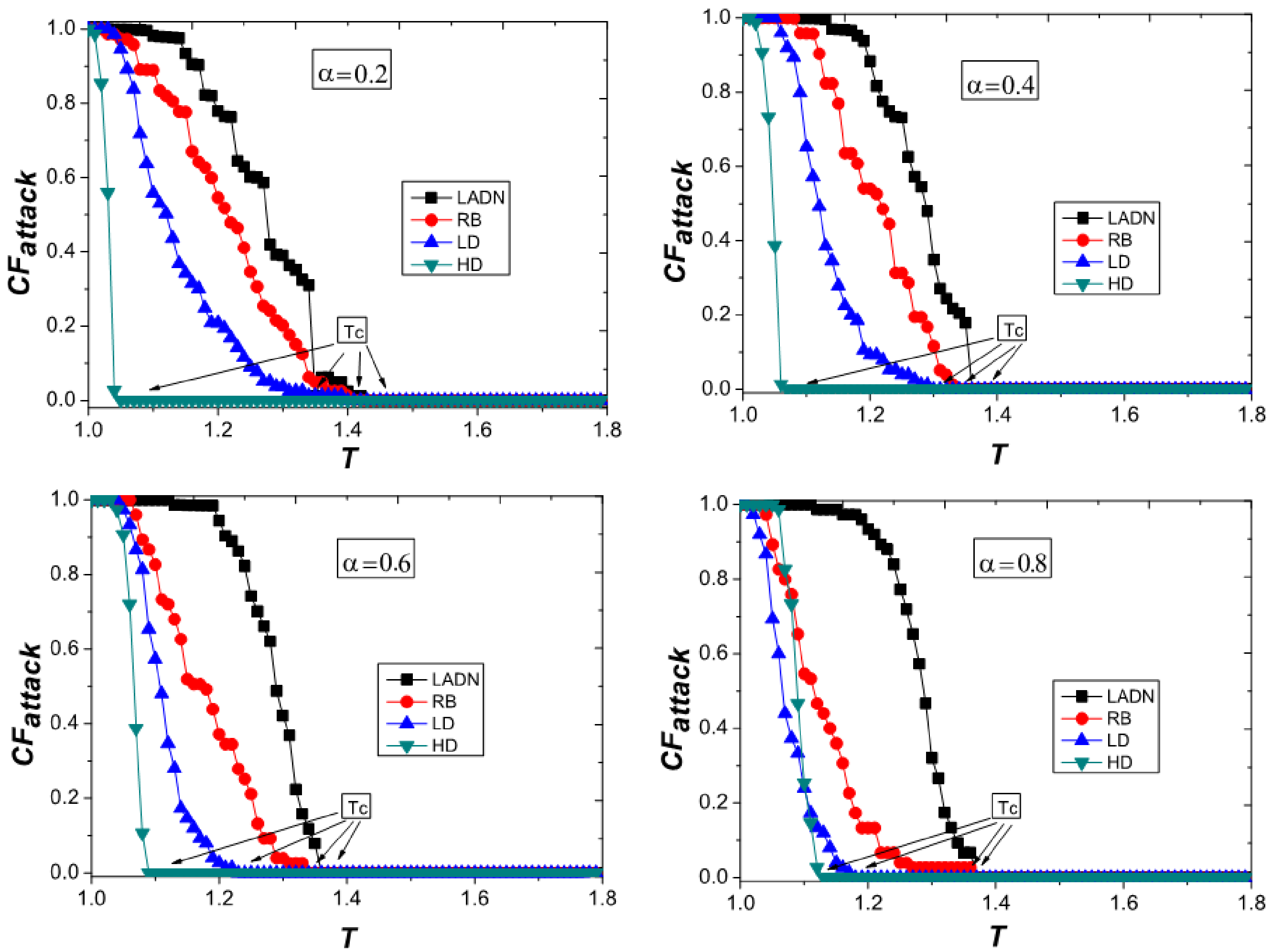


Fig. 2. Comparisons among four different attack strategies in the case of $\alpha < 1$.

Results

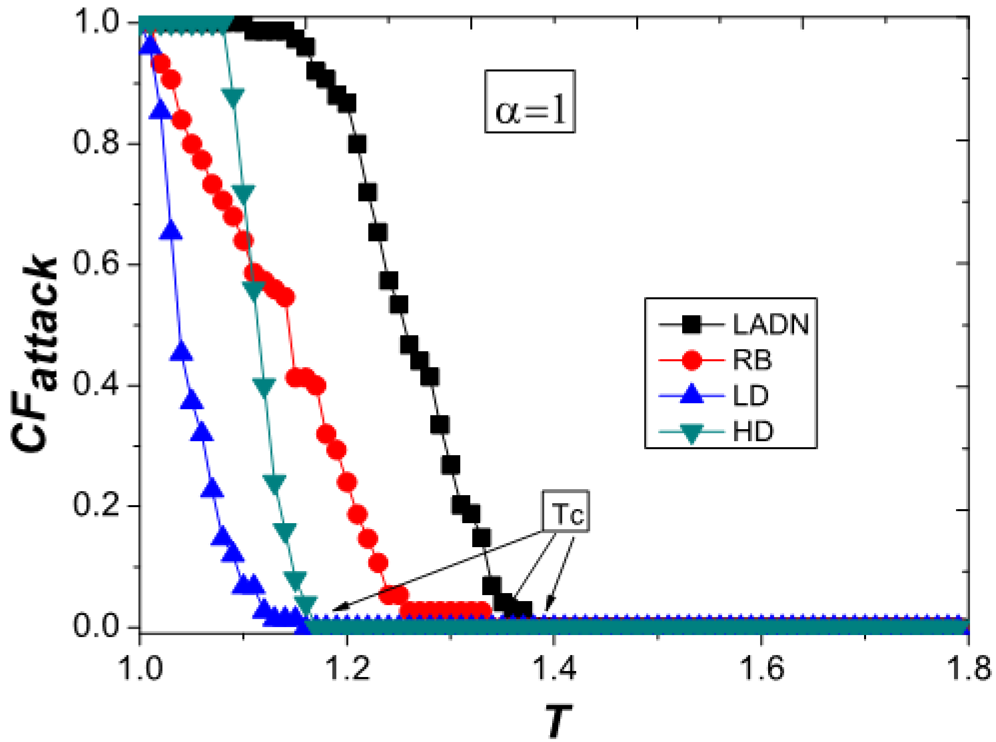


Fig. 3. Comparisons among four different attack strategies in the case of $\alpha = 1$.

Results

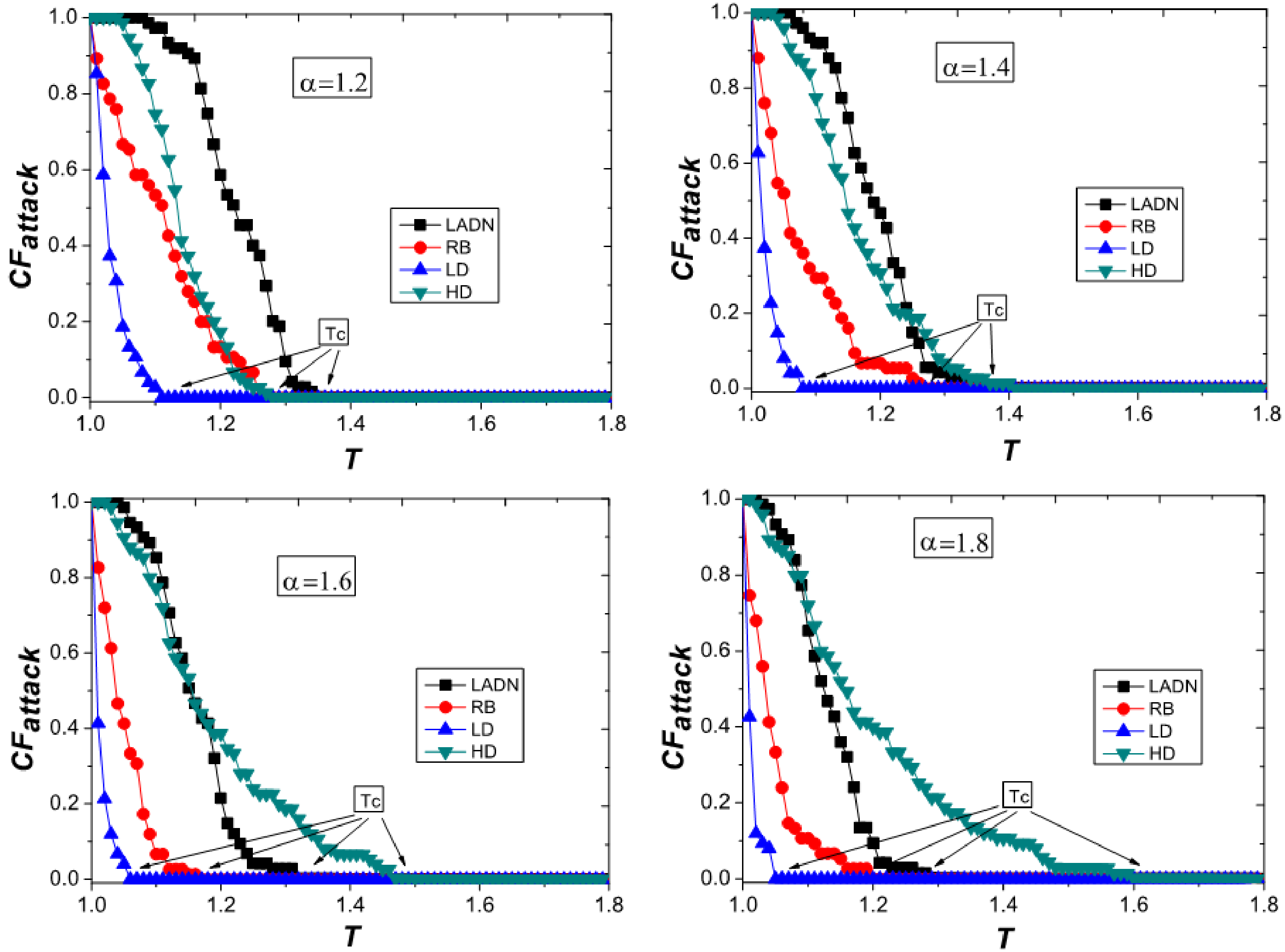


Fig. 4. Comparisons among four different attack strategies in the case of $\alpha > 1$.

Conclusions

- The parameter-dependent model is more realistic in networks.
- However, there is neither distribution for the number of broken nodes nor the probability of a specific node will be broken given one or more attacked nodes.
  - Problem: Find them.
- Problem: Assume there are some important nodes that we don’t want them to be broken. Given that some other nodes are being attacked. What is the least number of nodes to be removed so that the important nodes are safe after the cascading failure ?

References

- [1] Effect of Attack on Scale-Free Networks Due To Cascading Failure, Jian-Wei Wang, Li-Li Rong and Liang Zhang, Modern Physics Letter B, Vol. 23, No. 12 (2009).
- [2] A Loading-Dependent Model of Probabilistic Cascading Failure, Ian Dobson, Benjamin A. Carreras and David. E. Newman, Probability in the Engineering and Informational Sciences, 19, 2005.
- Wikipedia, *http://en.wikipedia.org/wiki/Cascading_failure*.