Approximation Schemes for Wireless Networks

Tim Nieberg, Johann Hurink, Walter Kern

Presented by Thang N. Dinh

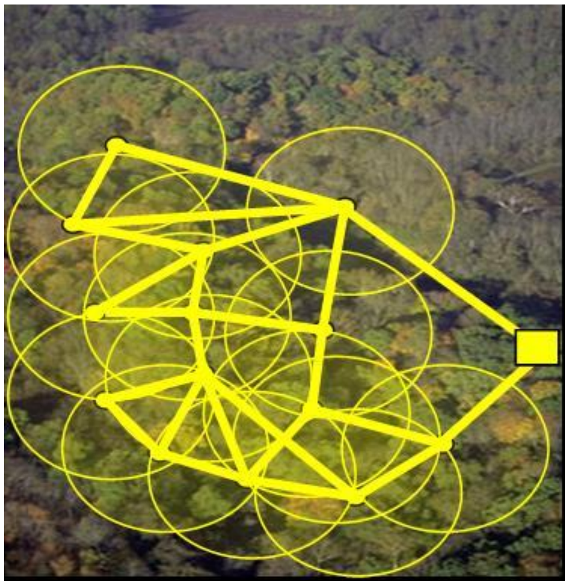
What's interesting?

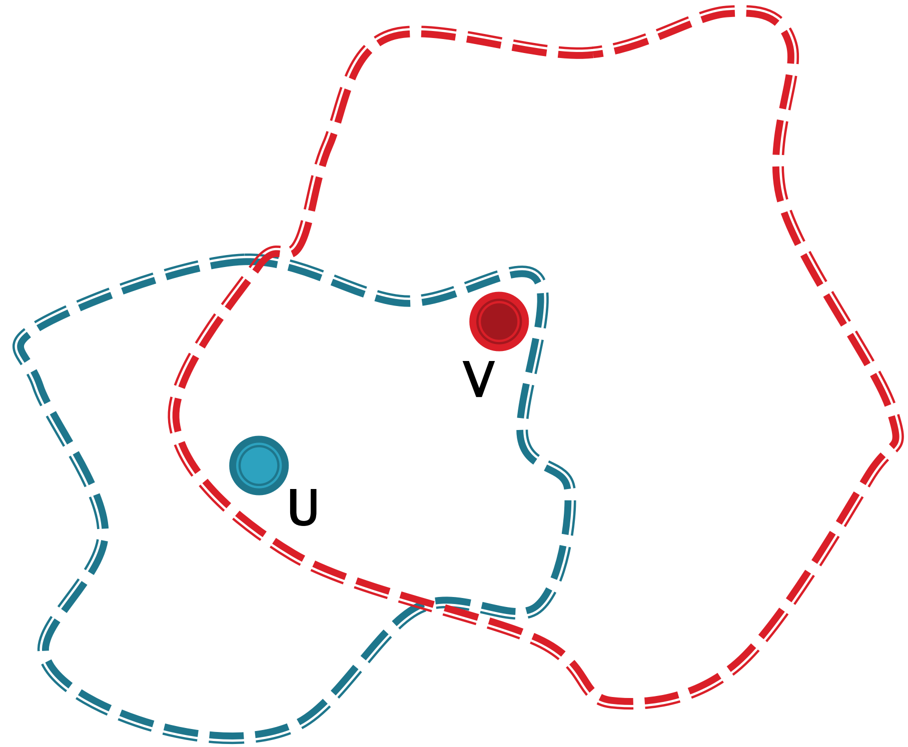
- PTAS for Max (Weighted) IS and Min DS for Wireless Networks
  - WITHOUT geometric information
  - Work for various Wireless Models: Disk Graph , Quasi-Disk Graph, Fading, .etc.
  - Detect if the underlying graph is not UDG, DG, .etc (UDG recognition is NP-hard)
  - Simple algorithms.

Outline

I. Wireless Communication Models
II. Maximum Independent Set and Minimum Dominating Set
III. Polynomial Bounded Growth Graphs
IV. Local Neighborhood-Based Approximation Schemes

Wireless Communication Models





- Node *u*: (p_u : location, A_u : coverage area)
- *Containment model* vs. *Intersection model*

$(u, v) \in E \iff p_u \in A_v.$ $(u, v) \in E \iff A_v \cap A_u \neq \emptyset.$

Wireless Communication Models

- UDG – Idealistic model: Omnidirectional antenna, no obstacles, identical power level,…
- Disk Graph: different transmission ranges
- Quasi-Disk Graphs:

$$(u, v) \in E \qquad \| p_u - p_v \| \leq c^-$$
$$(u, v) \notin E \qquad \| p_u - p_v \| > c^+$$

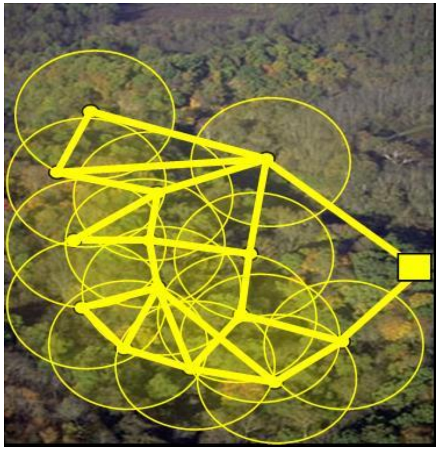


Fig 1. Unit Disk Graph

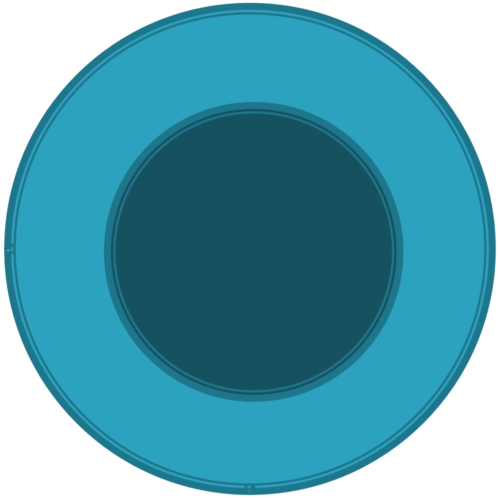
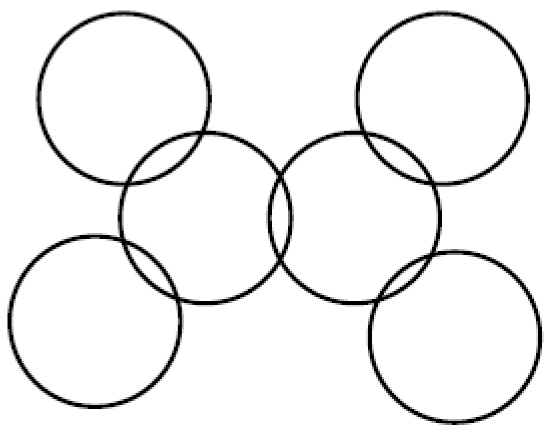
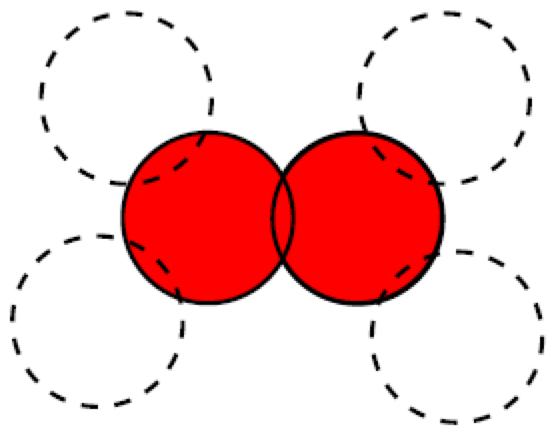


Fig 1. Quasi-Disk Graph

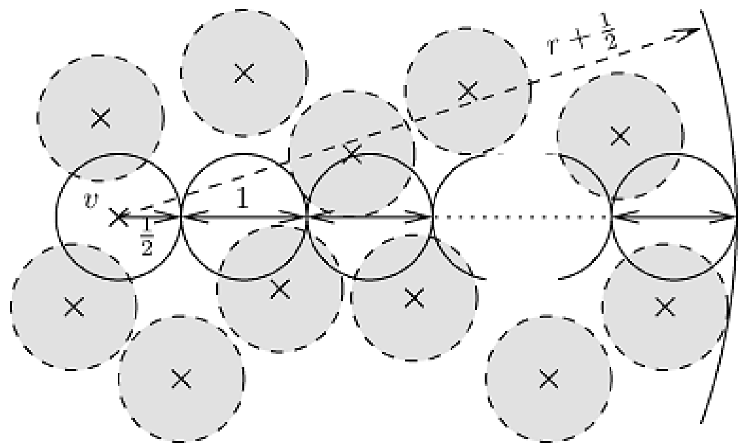
Max IS and Min DS

- Min Weight IS has PTAS in Planar Graphs[B83], UDG[HM85], DG[EJS01]
- Min DS: PTAS in Planar Graphs, UDG, ??? in DG





Polynomial Bounded Growth Graphs



- *f-growth-bounded*: Every *r*-neighborhood in graph contains at most *f(r)* independent vertices
- *Polynomially* bounded: $f = O(r^k)$
- UDG, DG, Quasi-Disk graph are polynomially bounded (Disk fitting).

k–Separated collections

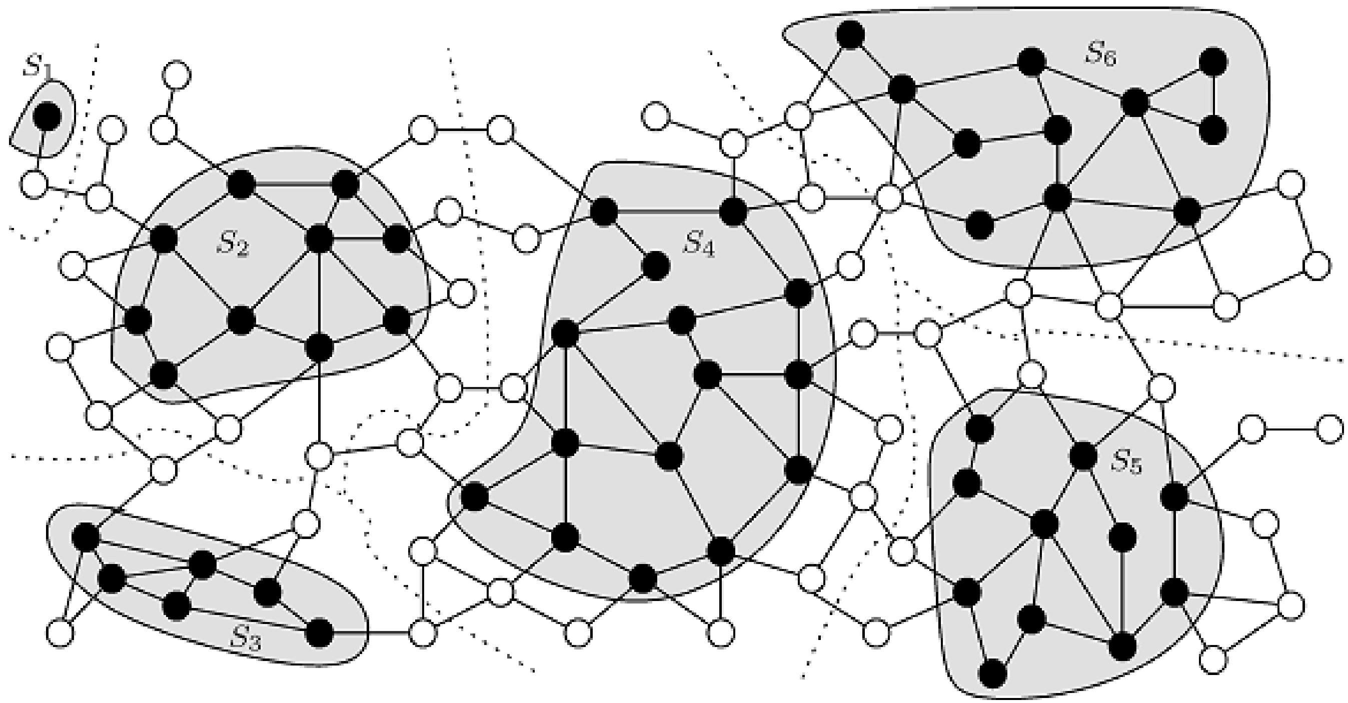


FIG. 2. Example of a 2-separated collection $\mathcal{S} = \{S_1, \ldots, S_6\}$.

Max Independent Set

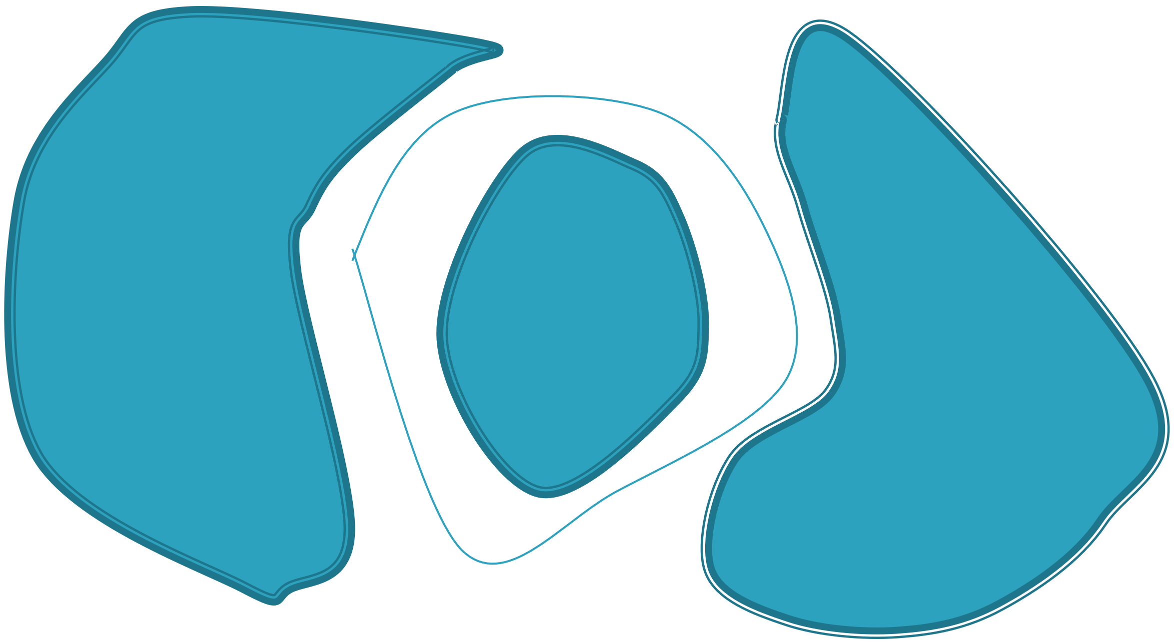
1. Pick up an arbitrarily vertex v
2. Loop until

$$|I_r|(1+\varepsilon) > |I_{r+1}|$$

3. Take I_r and remove all vertices in $(r+1)$ hops from v and repeat.

Where I_r is the optimal IS of nodes at distance at most r from v

Max Independent Set



$$S_r = I_r(v) + I_r(v') + I_r(v'') + \ldots \rightarrow Our\ solution$$
$$S_{r+1} = I_{r+1}(v) + I_{r+1}(v') + I_{r+1}(v'') + \ldots > OPT$$
$$|S_r|(1+\epsilon) > |S_{r+1}|$$

Max Independent Set

- **Correctness**: Union of all sub Independent set is an independent set (all regions are 1–separated)
- **Polynomial runtime**:
  - There exists $c = c(\varepsilon)$ such that $\bar{r} \leq c$.
  - Proof:

$$|I_r| > (1+\varepsilon)|I_{r-1}| > \cdots > (1+\varepsilon)^r|I_0| = (1+\varepsilon)^r.$$ *(Exponentially)*

Number of vertices in I_r at most r^k (*polynomially*)

c = $O(1/\varepsilon \log 1/\varepsilon)$

Weighted MIS

- Using same algorithm

$$W(I_{r+1}) > (1+\varepsilon) \cdot W(I_r)$$

- Pick up the vertex with max weight left at each iteration

$$W(I_r) > (1+\varepsilon) \cdot W(I_{r-1}) > \cdots > (1+\varepsilon)^r \cdot W(I_0) = (1+\varepsilon)^r \cdot w_{\max},$$

Min Dominating Set

- Vertices outside can dominate vertices inside
- Stop condition:

$$|D_{r+2}(v)| > (1+\varepsilon) \cdot |D_r(v)|$$

- Polynomial time:

$$\begin{aligned} p(r+2) \geq |D(\Gamma_{r+2})| &> (1+\varepsilon) \cdot |D(\Gamma_r)| \\ &> \cdots > (1+\varepsilon)^{r/2} \cdot |D(\Gamma_0)| = (\sqrt{1+\varepsilon})^r. \end{aligned}$$

Thank you!

- Q&A:
  - Can we extend the solution for Weighted Min DS?