1. (15 points) Consider the problem of making change for \( n \) cents using the fewest number of coins. Assume that each coin’s value is an integer.

(a) Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Prove that your algorithm yields an optimal solution.

(b) Give a set of coin denominations for which the greedy algorithm does not yield an optimal solution. Also give a value of \( n \) for which greedy is not the optimal solution. Your set should include a penny so that there is a solution for every value of \( n \).

2. (10 points) Give an efficient algorithm for the following problem: we are given \( n \) intervals on a circle. We want to select a maximal number of disjoint intervals.

3. (15 points) You are the manager in a firm where the length of working time and the start time of work is different for different employees. For example, person X works everyday from 8 to 11 AM, person Y from 9 AM to 1 PM, person Z from 2 to 10 PM etc. Your task is to create a workforce consisting of maximum number of people possible with non-overlapping work hours (if a person leaves at 7 AM and another starts at 7 AM, that is not an overlap). You use the following (Greedy) strategy:

(a) Choose the person (say X) with least number of working hour overlaps with other workers.

(b) Eliminate all people having working hour overlaps with X.

(c) Choose the next person with the least number of overlaps with the remaining people, and so on.

Does this strategy yield the set of maximum people? If yes, prove it. Otherwise, just give a counterexample.

4. (15 points) Assume one of your friends is driving a motorcycle from Gainesville to Los Vegas along I-12. The bike’s gas tank, when full, has enough gas to travel \( k \) miles, and a map gives distances between different gas stations on the route. Your friend wishes to make as few gas stops as possible along the way. Give an efficient method by which you can determine at which gas stations your friend should stop. Prove that your strategy yields an optimal solution.

5. (15 points) Show that Prim’s and Kruskal’s algorithms always construct the exact same minimum-cost spanning tree on a connected and undirected graph in which all edge costs are distinct.

6. (15 Points) Given is an undirected graph of distinct edge costs, \( G = (V, E) \). You have to design an algorithm which takes \( G \) as input, and then
removes a set of edges of minimum total cost, so that the output is a new
graph $G = (V, E)$ containing exactly one path between any two vertices
in it (Note that $G$ should have the exact same set of vertices as $G$).

7. (15 points) Pearls have been prized for their beauty and rarity for more
than four thousand years. Natural pearls are collected in the coastal
regions of Asia, Africa and America. Since this collection is dangerous,
researchers are asked to build an autonomous robot that can do the task. A
task for this robot consists of $n$ potential (pearl) sites, all of them located
along a single coastal line. The robot starts at first site, but can finish at
any site. To make things simpler we assume that all given sites are on a
single line, one after another. The time needed to travel from site $i$ to site
$i+1$ is $t_{i,i+1}$ hours, $1 \leq i \leq n-1$. Assume that the robot’s battery
lasts for $m$ hours. The expected number of pearls to be found per hour is
$p_i$ at a site $i$, $1 \leq i \leq n$.

(a) Design a greedy algorithm to find how much time should be spent
at each site inorder to maximize the number of pearls found. Give the
pseudo-code and the run time complexity.

Assume that each hour of pearl collection at a site decreases the expected
number of pearls to be found at that site in the next one hour by a constant
amount. This amount varies from site to site and is given as $d_i$, for site $i$,
$1 \leq i \leq n$.

(b) Change your algorithm so that it uses the above information to maxi-
mize the total pearls found. Give the pseudo-code and run time complex-
ity.