COT5405 Analysis of Algorithms
Homework 6
Due in TA office hour (5:05 - 5:55pm, E309, CSE Building) on Friday, April 15th, 2011

Your solutions should be complete and concise, MUST has your name (last name followed by first
name), section number, 8-digit UFID, clearly typed or handwritten on it. Show your work for all
problems to get full credit. You will be graded exactly by what you write in your homework paper(s).

Please do not copy the answers from the solution (if any). Also, do not copy the solution from your
friends/classmates. Doing so, if detected, will be considered CHEATING.

NO late submissions will be accepted.

On-Campus Students: Turn a hardcopy (stapled) in class, before class begins. NO electronic
submissions for On-Campus students will be accepted.

EDGE Students: Submit through email (cot5405sp11[AT]gmail[DOT]com) by 3:00pm on Tues-
day Apr. 19th, 2011.

Any questions regarding the assignment problems should be directed to TA office hours or email
cot5405sp11[AT]gmail[DOT]com. Inquires to the TA cise emails will be ignored. All the following
problems are from the textbook CLRS (3rd edition). Solving wrong problems by any mean will get
NO credits.

Whenever you are asked to give a NP-complete proof, I expect you to do all of the following:
1. Show clearly the decision version of the problem.
2. Show clearly the problem and instance that you want to reduce from.
3. Show clearly both the forward/reverse directions of the equivalence of your two instances.

Whenever you are asked to give an approximation algorithm for a problem, I expect you to do
all of the following:
1. Provide a concise description of your algorithm followed by the pseudocode.
2. Prove the approximation ratio of your algorithm.
Do all the following problems:

**Problem 1** (35pts) Given an undirected connected graph $G = (V, E)$ and a positive integer $k \leq |V|$, we say that two vertices $u$ and $v$ are connected if and only if there exists at least one path from $u$ to $v$. For all the $\binom{n}{2}$ possible vertex pairs, we want to remove $k$ vertices from $G$, so that the number of connected vertex pairs in the resulting graph is minimized.

- (10pts) Write an integer program formulation for finding these $k$ vertices. Show the correctness of your formulation.
- (10pts) Show the dual of the LP relaxation of your formulation.
- (15pts) Show that this problem is NP-Complete.

**Problem 2** (15pts) Given an undirected graph $G = (V, E)$. Find a spanning tree of $G$ with the maximum number of vertices that have degree 1. Show that this problem is NP-complete.

**Problem 3** (15pts) Given a set $U$ and a collection of subsets $T_1, \ldots, T_k$ of $U$, find a subset $A \subseteq U$ such that $A$ has a minimum size and $A$ intersects every $T_i$, namely $A \cap T_i \neq \emptyset$, $\forall i = 1 \ldots k$. Show that this problem is NP-complete.

**Problem 4** (20pts) In Problem 3, for each element $e \in U$, define $d_e$ as the number of subsets $T_i$ that contain $e$. Let $\delta = \max_{e \in U} d_e$ and $H(\delta) = \sum_{i=1}^{\delta} 1/i$, which is called a harmonic function of $\delta$. Devise an $H(\delta)$-approximation algorithm to Problem 3.

**Problem 5** (15pts) Given an undirected graph $G = (V, E)$ with nonnegative edge costs, and an integer $k$, find a partition of $V$ into sets $S_1, \ldots, S_k$ so that the total cost of edges running between these sets is maximized. Give an algorithm for this problem that achieves a factor of $(1 - \frac{1}{k})$. Is the analysis of your algorithm tight?