### Basic Properties of Logarithms

Log is a one-to-one, monotonic increasing function.

\[ \log_a (b^c) = c \log_a b \]

- \( \log_a 1 = 0 \) for any \( a \)
- \( \log_a (a^x) = x \)
- \( \log_a y = \frac{\log_b y}{\log_b a} \)

### Logarithms

Let \( a \) and \( y \) be positive numbers. If \( a^x = y \), then \( x \) is called the logarithm of \( y \) base \( a \).

- \( \log_a y \) is written as \( \log y \).
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**ANALYSIS OF ALGORITHMS**

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\[(X)^{\infty} = (\sum_{r=0}^{\infty} x^r) \Rightarrow (X)^{\infty} = (X)F \]

\[Q = \frac{Z}{(1-u)u} + a\nu\]

Sum of \(a\), \(b\), \(c\), \(d\) is given by

**Basic Probability**

\[1 = (s)\frac{(1)}{u} \Rightarrow (s) \leq 0\]

A collection of numbers such that \(\{\frac{s}{n} : \frac{s}{n} \in (s) \}\)

Consider an experiment with a collection of possible outcomes.

**Permutations**

\[A, B, C \Rightarrow D\]

A rearrangement of objects.
Rounding Time Functions and O, Θ, and Ω Notations

Definition: A function will be called a rounding time function if

For all \( n \geq m \), where \( n, m \) are positive integers,

\((u)\theta\) for \( (u)f \) for \( (u)g\)

Where there exist positive constants \( c, m \) such that for \( n \geq m \),

\((u)f \leq (u)g \leq (u)f\)

Definition: If there exist real constants \( c, m \) such that for all \( n \geq m \),

\((u)f \leq (u)^{[m]}\)

c

L'Hôpital's Rule: Can use function to obtain new results.

\[\frac{x^{[m]}(1-x)}{1-x} = \frac{x^{[m]}(1-x)}{1} \]

Example:

Let \( x \rightarrow \infty \), 
\[
\frac{x^{[m]}(1-x)}{1-x} = \frac{\lim \left[ x^{[m]}(1-x) \right]}{1} = \frac{\lim \left[ x^{[m]} \right]}{1} = x^{[m]}
\]

Theorem:
Example Application: Insertion Sort

Pseudo Code:

\begin{align*}
\text{key} & \rightarrow [1 + i]V \\
1 - 1 & \rightarrow 1 \\
[i]V & \rightarrow [1 + i]V \quad \text{do} \\
\text{while } \text{key} < [i]V & \text{ and } 0 < i < n \\
1 - f & \rightarrow 1 \\
[1 - f \ldots i]V & \text{ insert into the sorted sequence } [f]V \\
\text{do} & \text{key} \\
[V] & \text{ if } V < f \rightarrow f 
\end{align*}

Example 1: Note that

\[
(\varepsilon u)O + \varepsilon u = \frac{1}{x} 
\]

Example 2: Note that

\[
\frac{1}{x} + \frac{1}{x} + \ldots + \frac{1}{x} = \frac{1}{x} 
\]
Let $f$ be a monotonically increasing function.

$$
(1 + u)f(1 - x) + \cdots + (1 + u)x f + (0 x - 1 x)(0 x) f
$$

**Comparing Sums and Integrals.**

**Step-by-Step Insertion Sort.**

That value of $f$ where number of times the while loop test is true $g$ is executed for

$$
\begin{align*}
(1, u) & g = 8 \\
(1 - f, f) & g = 8 \\
(1 - f, f) & g = 6 \\
I & g = 3 \\
I - u & g = 2 \\
I - u & g = 1 \\
u & \cdot \cdot \cdot \\
\end{align*}
$$

**Analysis of Insertion Sort.**
Example 1. Let $x = (1)(1 + u)f$. Then

\[
(1 + u)f \leq x(1)f + x(1 + u) \leq (1 + u)f.
\]

Hence \( x = f \).