COT 5405  Analysis of Algorithms    Spring 2004
On-Campus Exam #2

Name: __________________________________________

UFID: ____________ - ____________

E-mail: _________________________________________

Instructions:

1. Write neatly and legibly
2. While grading, not only your final answer but also your
   approach to the problem will be evaluated
3. You have to attempt only TWO problems. If you
   attempt more than two problems, we will grade ANY
   two problems of OUR choice. Clearly CROSS OUT the
   problem that you don’t want us to grade
4. Each problem carries 10 points. Total points are 20
5. Time yourself well. Do not spend more than 20 minutes
   on any one problem. Total time for the exam is 50 minutes
6. You are not allowed to use a calculator for this exam

I have read carefully, and have understood the above
instructions. On my honor, I have neither given nor
received unauthorized aid on this examination.

Signature: ________________________________

Date: ____ (MM) / ____ (DD) / ___________ (YYYY)
**Question 1:**
Give an algorithm that, given a set \( \{x_1, x_2, x_3, \ldots, x_n\} \) of sorted points on the real line, determines the smallest set of unit length closed intervals that contains all the given points. Also argue that your algorithm is correct.

E.g., points \(<1.0, 1.2, 1.7, 2.7, 2.9, 3.1, 4.0, 4.5, 5.1>\) may be divided into the following intervals: \([1.0, 2.0], [2.5, 3.5], [4.0, 5.0], [5.0, 6.0]\) as drawn below (though not to the scale)
Question 2:
You are the Project Manager in a company where the length of working hours and the work start time varies for different employees. For example, person A works everyday from 8 AM – 10 AM, person B from 9 AM – 3 PM, person C from 1 PM – 10 PM etc. You are required to assemble a workforce consisting of maximum possible number of people with non-clashing working hours. You use the following Greedy strategy:

While (true) {
    1. Choose the person (say person X) with least number of working hour clashes with other employees (break ties arbitrarily). Add him/her to the solution set S.
    2. Eliminate people having working hour clashes with person X.
    3. If no more employees to choose from, quit and Print contents of set S.
}

Does this strategy necessarily yield the set of maximum number of people? If yes, prove it. Else, just provide a counterexample.
Question 3
You are given a connected undirected graph $G = (V, E)$ of distinct edge weights. Design an algorithm that does the following:

- Takes graph $G$ as input, and outputs graph $G' = (V', E')$
- $V = V'$ (same set of vertices), and $E' = E - R$ (obviously $E' \subseteq E$) for some $R$
- $G'$ contains exactly one path between any two vertices.
- The totals weight of edges in $R$ (the set of removed edges) is minimal.