HomeWork #1

Assigned: Tuesday, January 27, 2004
Updated: Wednesday, January 28, 2004
Due:
   On Campus: Tuesday, February 3, 2004 by 4 PM EST
   FEEDS / NTU: Tuesday, February 10, 2004 by 5 PM EST

Total Points: 100

QUESTION 1 (10 points)

Solve the recurrence relation iteratively (not using the Master’s theorem)
T(N) = 2 T(N/2) + log N

Make any reasonable assumption. Find an exact formula for T(N) and then give a
Theta expression.

QUESTION 2 (15 points)

An electrical engineer is studying a wave signal that repeats the same pattern again and again. In order to study the behavior of the signal, he takes a snapshot of the repeating pattern. The wave starts rising to reach a highest value (peak), immediately drops to a minimum value, and again start rising to the same peak. Three examples of the snapshots of a waveform [15, 18, 22, 0, 2, 6, 8, 10, 15, 18, 22, 0, 2, 6, 8, 10, 15, 18, 22, 0, 2, 6, 8] are [10, 15, 18, 22, 0, 2, 6, 8], [0, 2, 6, 8, 10, 15, 18, 22] and [18, 22, 0, 2, 6, 8, 10, 15]. Provide an efficient algorithm to find the peak value of the wave signal.

QUESTION 3 (10 points)

You are given recursive CombineSort procedure that sorts input sequence A = {a_1, a_2, a_3, …, a_n} . Find its recurrence relation T(n) and use it to calculate worst case complexity of this algorithm. (Note: Combine procedure has cn worst time complexity)

Start = 1;
Finish = n;
BEGIN CombineSort(A, Start, Finish)
   IF (Start < Finish) THEN
      Middle = (Start + Finish)/2;
      CombineSort(A, Start, Middle);
      CombineSort(A, Middle +1, Finish);
      Combine(A, Start, Middle, Finish);
   END IF
END BEGIN
QUESTION 4 (20 points)

a) Let $L$ and $S$ be the largest and smallest in an array $A$ of $n$ integers (you do not know their values). Show that there are two elements $x_1, x_2 \in A$ so that

$$|x_1 - x_2| \leq \frac{(L-S)}{(n-1)}$$

b) Give algorithms to find such a pair of numbers from the array when the array is

- Sorted
- Unsorted

QUESTION 5 (15 points)

In a tribe, marriage of $n$ males and $n$ females happen according to following the strict time order: First the tallest male of the tribe marries the shortest female of the tribe. After the marriage of the first couple is over, the second tallest male marries the second shortest female, and so on till it ends with the tallest female marrying the shortest male (for the ease of calculation, think of $n$ as $2^m + 1$ for some $m$). After the whole marriage ceremony is over, the couples stand left to right in the same time order of their marriage (and the husband and wife stand right by each other). You take a group photograph and this group photograph is the only evidence that you have (it looks like $M_1F_n M_2F_{n-1} \ldots M_{n-1}F_2 M_nF_1$). Using the judgment of your naked eye, find the two persons who represent the individual median height of these $2n$ newlyweds. Are these two persons necessarily a couple, or even from the opposite genders for that matter? Analyze the complexity of your finding method. Assume that with naked eye, you can compare the heights of any two persons if they are separated by at most one person in between them (Hint: the brides are not wearing any high heels!!)

QUESTION 6 (15 points)

Share prices are announced every minute at NASDAQ. Also, the announced share price may take only one of the following 3 courses: 1) rise 1 cent from the previous minute’s price, 2) fall by 1 cent, or 3) remain the same as previous. (no fractional change or more than 1 cent change is allowed). You have a graph of a share value over the day (with time as X axis and value as Y axis). Design an efficient algorithm for finding all the time points when the price of the share was exactly $Z$ cents and analyze its complexity for the following three real-life scenarios:

- The share price was $Z$ cent only once during the whole day
- The share price was $Z$ cent multiple times during the day
- The share price was never exactly $Z$ cents

QUESTION 7 (15 points)

In order to send an important signal back home, the Mars Rover must find out a point of height exactly $X$ meter inside a $n$ unit by $n$ unit square-shaped land. Through satellite images, NASA has the height data of the landscape of that square ($n^2$ points). The land slopes downward both from east to west and from south to north. Give an efficient algorithm to find out if such a point exists in that land.