1. Sol
In the input sequence, the first element corresponds to the root. Using
the root, we will check if we can divide the rest of the sequence into two
consecutive parts – all elements less than root followed by all elements
greater than root. If we find that there is an element less than root, after
we find an element greater than root, then we return this sequence as
invalid. Ex: 3,1,4,5,2 is invalid since we have element 2(<root) after 4
(>root). If we were infact able to divide the sequence into two parts, then
we’ll check the constraint on those two subparts recursively.
The scan can in fact be done in a single pass but requires a stack and an
additional feature/color.

Correctness: The recursive check will identify all the invalid pre-orders.
If we are not able to find such a partition on the sequence, then we can see
that a value less than root will be in the constructed binary tree. Since
this should be a search tree, that sequence is invalid.

If a sequence is validated by this constraint, we can easiy see that a BST
can be constructed in the order of the recursive calls.

2. Soln Cycle property: Let C be any cycle in G, and let e be the max cost
edge belonging to C. Then the MST T does not contain e.

• Suppose e belongs to T, and let’s see what happens.
• Deleting e from T creates a cut S in T. (i.e., change the second T in
the sentence to G)
• Edge e is both in the cycle C and in the cutset D corresponding to
S ⇒ there exists another edge, say f, that is in both C and D. Also
notice that f does not belong to T otherwise T would have the cycle
C.
• T’ = T ∪ {f} − {e} is also a spanning tree.
• Since e ≤ f, cost(T’) < cost(T).
• This is a contradiction.

3. Sol Perform a topological sort on the given graph. Let V_1, V_2, V_3, ..., V_n,
be the order imposed by the sort. Now check if there is an edge from each V_i
to V_{i+1}. Even if one such edge does not exist, then there is no hamiltonian
path.

Correctness: If there are such edges possible, it is easy to see that there
will be a hamiltonian path consisting of V_1, V_2, ..., V_n, in that order.
If atleast one such edge is not there, that means, the corresponding two
nodes can be interchanged in a topological sort. But, by the definition of
a hamiltonian path, only one topological sort is possible. So, we correctly
recognize the 'no' cases.