COT 5405 Analysis of Algorithms, Fall 2006.
Homework 3

Due Tuesday, November 21 2006, before class.

Notes

- Page limit: 1 page per question
- Answer each question on a fresh page.
- Write your name and unique ID on the top right hand corner of your homework. Be sure to write your last name as the last word in your name.
- Please write legibly.

1. Let $G = (V, E)$ be a connected undirected graph. Give an $O(V + E)$-time algorithm to compute a path in $G$ that traverses each edge in $E$ exactly once in each direction. Your algorithm should output the path as a sequence of edges.

2. A directed graph $G = (V, E)$ is singly connected if for all $u, v \in V$, if $u \to v$ then there is at most one simple path from $u$ to $v$. (To clarify, note that there can be multiple paths from $u$ to $v$ and yet only one simple path from $u$ to $v$.) Give an $O(V * (V + E))$-time algorithm to determine whether or not a graph is singly connected.

3. Give an $O(V)$-time algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle.

4. A directed graph $G = (V, E)$ is said to be semiconnected if for all pairs of vertices $u, v \in V$, we have $u \to v$ or $v \to u$ or both. Prove that $G$ is semiconnected if and only if the DAG of its strongly connected components have a unique topologically sorted order. Finally, give an efficient algorithm to determine whether or not $G$ is semiconnected.

5. Let $G = (V, E)$ be an undirected, connected graph with weight function $w : E \to \mathbb{R}$, and suppose $|E| \geq |V|$ and all edge weights are distinct. A second best minimum spanning tree is defined as follows. Let $T$ be the set of all spanning trees of $G$, and let $T'$ be a minimum spanning tree of $G$. A second best minimum spanning tree is a spanning tree such that $w(T) = \min_{T'' \in T \setminus \{T\}} \{w(T'')\}$. Show that in this case the minimum spanning tree is unique but the second best minimum spanning tree need not be. Also prove that if $T$ is a minimum spanning tree then there exist edges $(u, v) \in T$ and $(x, y) \notin T$ such that $T - (u, v) + (x, y)$ is a second best minimum spanning tree of $G$.

6. Prove that Dijkstra’s algorithm for single source shortest path in a directed graph reduces to breadth first search if the weight of all edges is identically 1.

7. Suppose we are given a weighted, directed graph $G = (V, E)$ in which edges that leave the source vertex $s$ may have negative weights, all other edge weights are nonnegative, and there are no negative weight cycles. Prove that Dijkstra’s algorithm correctly finds shortest paths from $s$ in this graph.

8. Give an $O(V^3)$-time algorithm for computing the transitive closure of a directed graph.