Discussion Problems and Solutions 4

1 Section 2.4

Exercise 19: Show that \( \sum_{j=1}^{n} (a_j - a_{j-1}) = a_n - a_0 \).

Solution: If we just write out what the sum means, we see that parts of successive terms cancel, leaving only two terms:

\[
\sum_{j=1}^{n} (a_j - a_{j-1}) = a_1 - a_0 + a_2 - a_1 + \cdots + a_{n-1} - a_{n-2} + a_n - a_{n-1} = a_n - a_0
\]

Exercise 21: Sum both sides of the identity \( k^2 - (k - 1)^2 = 2k - 1 \) from \( k = 1 \) to \( k = n \) and use Ex19 to find

(a) a formula for \( \sum_{k=1}^{n} (2k - 1) \)
(b) a formula for \( \sum_{k=1}^{n} k \).

Solution:

(a) We use the hint

\[
\sum_{k=1}^{n} (2k - 1) = \sum_{k=1}^{n} (k^2 - (k - 1)^2) = n^2 - 0^2 = n^2.
\]

(b) We can use the distributive law

\[
n^2 = \sum_{k=1}^{n} (2k - 1) = 2 \sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 = 2 \sum_{k=1}^{n} k - n
\]

Thus

\[
\sum_{k=1}^{n} k = (n^2 + n) / 2 = n(n + 1) / 2
\]