1 Section 2.1

Exercise 1: Find the power set of each of these sets, where a and b are distinct elements.
(a) \{a\}, (b) \{a, b\}, (c) \{\emptyset, \{\emptyset\}\}

Solution:
(a) \{\emptyset, \{a\}\}
(b) \{\emptyset, \{a\}, \{b\}, \{a, b\}\}
(c) \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}

Exercise 2: Find the truth set of each of these predicates where the domain is the set of integers.
(a) \(P(x): x^2 < 3\)
(b) \(Q(x): 2x + 1 = 0\)

Solution:
(a) The only integers whose squares are less than 3 are the integers whose absolute values are less than 3. So the truth set is \(\{x \in \mathbb{Z} | x^2 < 3\} = \{-1, 0, 1\}\).
(b) The only real number satisfying this equation is \(x = -1/2\). Because this value is not in our domain, the truth set is empty: \(\{x \in \mathbb{Z} | 2x + 1 = 0\} = \emptyset\).

2 Section 2.2

Exercise 3: Prove the first De Morgan law by showing that if A and B are sets, then \(\overline{A \cup B} = \overline{A} \cap \overline{B}\).
(a) by showing each side is a subset of the other side.
(b) using a membership table.

Solution:
(a) Suppose \(x \in \overline{A \cup B}\). Then \(x \notin A \cup B\), which means that \(x\) is in neither \(A\) nor \(B\). In other words, \(x \notin A\) and \(x \notin B\). This is equivalent to saying that \(x \in \overline{A}\)

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and \( x \in B \). Therefore \( x \in \overline{A} \cap \overline{B} \), as desired. Conversely, if \( x \in \overline{A} \cap \overline{B} \), then \( x \notin A \) and \( x \notin B \). This means \( x \notin A \) and \( x \notin B \), so \( x \) cannot be in the union of \( A \) and \( B \). Since \( x \notin A \cup B \), we conclude that \( x \in A \cup B \) as desired.

(b) The following membership table gives the desired equality, since columns four and seven are identical.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A \cup B</th>
<th>A \cap B</th>
<th>A</th>
<th>B</th>
<th>A \cap B</th>
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</tbody>
</table>

**Exercise 4:** Draw the Venn diagrams for each of these combinations of the sets \( A \), \( B \), and \( C \).

(a) \( A \cap (B - C) \)
(b) \( (A \cap B) \cup (A \cap C) \)
(c) \( (A \cap B) \cup (A \cap \overline{C}) \)

**Solution:** In Figure 1.

### 3 Section 2.3

**Exercise 5:** Determine whether \( f \) is a function from \( \mathbb{R} \) to \( \mathbb{R} \) if

(a) \( f(x) = 1/x \).
(b) \( f(x) = \sqrt{x} \).
(c) \( f(x) = \pm \sqrt{(x^2 + 1)} \).

**Solution:**

(a) This is not a function. The expression \( 1/x \) is meaningless for \( x = 0 \), which is one of the elements in the domain; thus the “rule” is no rule at all. In other words, \( f(0) \) is not defined.
(b) This is not a function. Things like $\sqrt{-3}$ are undefined.

(c) This is not a function. The “rule” for $f$ is ambiguous. We must have $f(x)$ defined uniquely, but here there are two values associated with every $x$, the positive square root and the negative square root of $x^2 + 1$.

**Exercise 6:** Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

(a) $f(x) = 2x + 1$
(b) $f(x) = x^2 + 1$
(c) $f(x) = x^3$

**Solution:**

(a) This function is a bijection. To show that the function is one-to-one, note that if $2x + 1 = 2x' + 1$, then $x = x'$. To show that the function is onto, note that $2((y - 1)/2) + 1 = y$, so every number is in the range.

(b) This function is not a bijection, since its range is the set of real numbers greater than or equal to 1, not all of $\mathbb{R}$.

(c) This function is a bijection, since it has an inverse function, namely the function $f(y) = y^{1/3}$. 