1 Section 1.1

Exercise 1: Let $p$, $q$, and $r$ be the propositions

$p$: Grizzly bears have been seen in the area.
$q$: Hiking is safe on the trail.
$r$: Berries are ripe along the trail.

Write these propositions using $p$, $q$, and $r$ and logical connectives.

(a) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

(b) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Express each of these propositions as an English sentence.

(c) $r \land \neg p$

(d) $r \rightarrow (q \leftrightarrow \neg p)$

Solution:

(a) $\neg q \land \neg p \land r$

(b) $(p \land r) \rightarrow \neg q$

(c) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

(d) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

Exercise 2: State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows today, I will ski tomorrow.

(b) I come to class whenever there is going to be a quiz.

Solution:

(a) **Converse**: I will ski tomorrow only if it snows today. **Contrapositive**: If I don’t ski tomorrow, then it will not have snowed today. **Inverse**: If it does not snow today, then I will not ski tomorrow.
(b) The proposition as stated can be rendered “If there is going to be a quiz, then I will come to class.” **Converse:** If I come to class, then there will be a quiz. **Contrapositive:** If I don’t come to class, then there won’t be a quiz. **Inverse:** If there is not going to be a quiz, then I don’t come to class.

### 2 Section 1.2

**Exercise 3:** Show that \((p \rightarrow r) \lor (q \rightarrow r)\) and \((p \land q) \rightarrow r\) are logically equivalent.

**Solution:** We hope to prove

\[
(p \rightarrow r) \lor (q \rightarrow r) \equiv (p \land q) \rightarrow r
\]

**LHS**

\[
\equiv (\neg p \lor r) \lor (\neg q \lor r)
\]

\[
\equiv (\neg p \lor \neg q) \lor (r \lor r)
\]

\[
\equiv (\neg p \lor \neg q) \lor r
\]

**RHS**

\[
\equiv \neg(p \land q) \lor r
\]

\[
\equiv (\neg p \lor \neg q) \lor r
\]

Therefore, **LHS** \(=\) **RHS**.

**Exercise 4:** Show that \((p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r)\) is a tautology.

**Solution:**

\[
(p \lor q) \land (\neg p \lor r) \rightarrow (q \lor r)
\]

\[
\equiv \neg((p \lor q) \land (\neg p \lor r)) \lor (q \lor r)
\]

\[
\equiv (\neg p \lor q) \lor ((p \land \neg r) \lor (q \lor r))
\]

\[
\equiv (\neg p \lor \neg q) \lor ((p \land r) \lor (q \lor r))
\]

\[
\equiv ((\neg p \lor q) \land (\neg q \lor r)) \lor ((p \lor r) \land (\neg r \lor r))
\]

\[
\equiv ((\neg p \land \neg q) \land \mathbf{T}) \lor ((p \lor r) \land \mathbf{T})
\]

\[
\equiv (\neg p \lor q) \lor (p \lor r)
\]

\[
\equiv \neg p \lor q \lor r
\]

\[
\equiv \mathbf{T}
\]

**Exercise 5:** Show that \((p \rightarrow q) \rightarrow r\) and \(p \rightarrow (q \rightarrow r)\) are not equivalent.

**Solution:** To show that these are not logically equivalent, we need only find one
assignment of truth values to p, q and r for which the truth values of \((p \rightarrow q) \rightarrow r\)
and \(p \rightarrow (q \rightarrow r)\) differ. One such assignment is \(F\) for all three. Then \((p \rightarrow q) \rightarrow r\)
is false and \(p \rightarrow (q \rightarrow r)\) is true.

3 Section 1.3

Exercise 6: Translate each of these statements into logical expressions in three
different ways by varying the domain and by using predicates with one and with
two variables.
(a) There is a student in your school who cannot speak Hindi.
(b) A student in your school knows Java, Prolog, and C++.

Solution: Let \(Y(x)\) be the propositional function that \(x\) is in your school or class.
(a) If we let \(H(x)\) be “\(x\) can speak Hindi.” then we have \(\exists x \neg H(x)\) if the
universe is just your schoolmates, or \(\exists x(Y(x) \wedge \neg H(x))\) if the universe is all
people. If we let \(S(x, y)\) mean that person \(x\) can speak language \(y\), then we can
rewrite this last one as \(\exists x(Y(x) \wedge \neg S(x, \text{Hindi}))\).
(b) If we let \(J(x), P(x),\) and \(C(x)\) be the propositional functions asserting
\(x\)’s knowledge of Java, Prolog, and C++, respectively, then we have \(\exists x(J(x) \wedge
P(x) \wedge C(x))\) if the universe is just your schoolmates, or \(\exists x(Y(x) \wedge J(x) \wedge
P(x) \wedge C(x))\) if the universe is all people. If we let \(K(x, y)\) mean that person \(x\)
knows programming language \(y\), then we can rewrite this last one as \(\exists x(Y(x) \wedge
K(x, \text{Java}) \wedge K(x, \text{Prolog}) \wedge K(x, C++)))\).

Exercise 7: Translate these specifications into English where \(F(p)\) is “Printer \(p\) is
out of service,” \(B(p)\) is “Printer \(p\) is busy.” \(L(j)\) is “Print job \(j\) is lost” and \(Q(j)\)
is “Print job \(j\) is queued”.
(a) \(\exists p(F(p) \wedge B(p)) \rightarrow \exists jL(j)\)
(b) \(\forall pB(p) \rightarrow \exists jQ(j)\)
(c) \(\exists j(Q(j) \wedge L(j)) \rightarrow \exists pF(p)\)
(d) \((\forall pB(p) \wedge \forall jQ(j)) \rightarrow \exists jL(j)\)

Solution:
(a) If there is a printer that is both out of service and busy, then some job has
been lost.
(b) If every printer is busy, then there is a job in the queue.
(c) If there is a job that is both queued and lost, then some printer is out of
service.
(d) If every printer is busy and every job is queued, then some job is lost.