You should try to solve all these problems yourself and the solutions will be available only after Sunday morning.
There will be five problems on the exam.

1. Show that the function \( f(x) = (x+2) \log(x^2+1) + \log(x^3+1) \) is \( O(x \log x) \).

2. Consider two finite sets \( A = \{A_1, A_2, \ldots, A_m\} \) and \( B = \{B_1, B_2, \ldots, B_n\} \). Given a function \( f : A \rightarrow B \), write a correct algorithm that will determine if \( f \) is one-to-one. Give \( O \)-estimates for the time-complexity of this algorithm. In your algorithm, assume that you can access the element in \( B \) that some \( A_i \) maps to by saying \( f(A_i) \).

3. How many ways are there to seat 6 people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.

4. A test contains 100 true/false questions. How many different ways can a student answer the questions on the test, if answers may be left blank.

5. Use mathematical induction to show that \( 1 \cdot 2^0 + 2 \cdot 2^1 + 3 \cdot 2^2 + \ldots + n \cdot 2^{n-1} = (n - 1) \cdot 2^n + 1 \), whenever \( n \) is a positive integer.

6. How many subsets with more than two elements does a set with 100 elements have? (Problem 17 in 5.3.)

7. A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes (Problem 19 in 5.3.)
   (a) are there in total?
   (b) contain exactly two heads?
   (c) contain at most three tails?
   (d) contain same number of heads and tails?

8. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men? (Problem 34 in 5.3.)
9. How many bit strings contain exactly eight 0s and 10 1s if every 0 must be immediately followed by an 1? (Problem 35 in 5.3.)

10. A computer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the networks that are directly connected to the same number of other computers. (Problem 32 in 5.2.)

11. Show that among any group of 20 people (where any two people are either friends or enemies), there are either four mutual friends or four mutual enemies. (Problem 26 in 5.2.)

12. There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive integers. (Problem 40 in 5.2.)

13. Let $x$ be an irrational number. Show that for some positive integer $j$ not exceeding $n$, the absolute value of the difference between $jx$ and the nearest integer to $jx$ is less than $1/n$. (Problem 41 in 5.2.)

14. Show that if there are 101 people of different heights standing in a line, it is possible to find 11 people in the order they are standing in the line with heights that are either increasing or decreasing. (Problem 22 in 5.2.)

15. Use the product rule to show that there are $2^{2^n}$ different truth tables for propositions in $n$ variables. (Problem 58 in 5.1.)

16. Use tree diagram to determine the number of ways that the World Series can occur, where the first team that wins four games out of seven wins the series. (Problem 54 in 5.1.)

17. How many ways are there to arrange the letters $a$, $b$, $c$ and $d$ such that $a$ is not followed immediately by $b$. (Problem 53 in 5.1.)

18. Examples 9 and 10 in 4.4. (We went over similar examples in class).

19. Give a recursive algorithm for finding the sum of the first $n$ positive integers. (Problem 8 in 4.4.)

20. Give a recursive algorithm for finding the maximum (or minimum) of a finite set of integers, making use of the fact that the maximum (or minimum)
of \( n \) integers is the larger of the last integer in the list and the maximum (or minimum) of the first \( n - 1 \) integers in the list. (Problem 10 in 4.4.)

21. Problem 39 in 4.3.

22. Which amounts of money can be formed using just two dollar bills and five-dollar bills? Prove your answer using strong induction. (Problem 7 in 4.2.)

23. Problem 12 in 4.2.

24. Problem 29 in 4.2.


27. Problem 49 in 4.1.

28. Problem 6 in 3.3.

29. Problem 9 in 3.3.

30. Problem 61 in 3.2.

31. Problem 62 in 3.2.

32. The complexity table on page 196 (Table 1 in 3.3).

33. Problems 5, 7, 8, 10 12 14 in 3.2. (These are all similar problems.)

34. Problems 16, 19, 20, 21 in 3.2. (Make sure that you can do these problem quickly.)

35. Greedy Algorithms and Halting Problem in 3.1. (You need to read the textbook from page 174 to page 176)