Note

All questions of this homework come from the textbook. The due date is Monday, Oct. 8, 2012.

Question 1 [Chapter 1 Supplementary Exercise 30, p.113 (10 points)]

If ∀y∃xP(x, y) is true, does it necessarily follow that ∃x∀yP(x, y) is true? Prove or disprove it.

Question 2 [Section 2.1 Exercise 10, p.125 (10 points)]

Determine whether these statements are true or false.

a) ∅ ∈ {∅}
b) ∅ ∈ {∅, {∅}}
c) {∅} ∈ {∅}
d) {∅} ∈ {{∅}}
e) {∅} ⊂ {∅, {∅}}
f) {{∅}} ⊂ {∅, {∅}}
g) {{∅}} ⊂ {{∅}, {∅}}

Question 3 [Section 2.1 Exercise 24, p.126 (10 points)]

Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

a) ∅
b) {∅, {a}}
c) {∅, {a}, {∅, a}}
d) {∅, {a}, {b}, {a, b}}

Question 4 [Chapter 2 Supplementary Exercise 6, p.187 (10 points)]

Let A and B be sets. Show that A ⊆ B if and only if A ∩ B = A.

Question 5 [Chapter 2 Supplementary Exercise 8, p.187 (10 points)]

Suppose that A, B and C are sets. Prove or disprove that (A − B) − C = (A − C) − B.

Question 6 [Section 2.2 Exercise 10, p.136 (10 points)]

Show that

a) A − ∅ = A.
b) ∅ − A = ∅.

c) (A − B) − C ⊆ A − C.
d) (B − A) ∪ (C − A) = (B ∪ C) − A.

Question 7 [Section 2.2 Exercise 18 (c,e), p.136 (10 points)]

Prove the first distributive law of set identities

A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)
Question 9  [Section 2.2 Exercise 30, p.137 (10 points)]

Can you conclude that $A = B$ if $A$, $B$ and $C$ are sets such that

a) $A \cup C = B \cup C$

b) $A \cap C = B \cap C$

c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$

Question 10  [Section 2.2 Exercise 36 (10 points)]

The symmetric difference of $A$ and $B$, denoted by $A \oplus B$, is the set containing those elements in either $A$ or $B$, but not in both $A$ and $B$. Prove that $A \oplus B = (A - B) \cup (B - A)$.

Question Bonus Question  [(1 point for course total)]

The elements of a Cartesian product set are ordered pairs $(a, b)$, which means, the order of the two elements $a$ and $b$ matters: $(a, b) \neq (b, a)$. But the elements of general sets are unordered. How can we define an ordered pair using the set language? One of the definitions is given by Kuratowski:

$\langle a, b \rangle := \{\{a\}, \{a, b\}\}$

a) Please prove that this definition is correct, which means, $\langle a, b \rangle = \langle c, d \rangle$ if and only if $a = c$ and $b = d$.

b) Consider an alternative definition of ordered pair using set language:

$\langle a, b \rangle := \{a, \{b\}\}$

show that this definition is incorrect.

References: