Due Friday - Sep 7th, 2012. Show your work for all problems.

Problem 1  [Section 1-1 Question 14(a)(c)(e) - 10pts]

Let $p$, $q$ and $r$ be the propositions

$p$: You get an A on the final exam.
$q$: You do every exercise in this book.
$r$: You get an A in this class.

Write these propositions using $p$, $q$ and $r$ and logical connectives (including negations).

a) You get an A in this class, but you do not do every exercise in this book.

b) To get an A in this class, it is necessary for you to get an A on the final.

c) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

Problem 2  [Section 1-1 Question 28 - 10pts]

State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

Problem 3  [Section 1-1 Question 32(a)(c)(d) - 10pts]

Construct a truth table for each of these compound propositions.

a) $p \rightarrow \neg p$

b) $p \oplus (p \lor q)$

c) $(p \land q) \rightarrow (p \lor q)$

Problem 4  [Section 1-3 Question 10(a) - 10pts]

Show that $\neg p \land (p \lor q)] \rightarrow q$ is a tautology by

a) Constructing the truth table.

b) Developing a series of logical equivalences.

Problem 5  [Section 1-3 Questions 22, 26 - 20pts]

Show that the following pairs of propositions are logically equivalent by developing a series of logical equivalences.

a) $(p \rightarrow q) \land (p \rightarrow r)$ and $p \rightarrow (q \land r)$

b) $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \lor r)$

Problem 6  [Section 1-4 Question 6 - 15pts]

Let $N(x)$ be the statement "$x$ has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.

a) $\exists x N(x)$
b) $\forall x N(x)$

c) $\neg \exists x N(x)$

d) $\exists x \neg N(x)$

e) $\neg \forall x N(x)$

**Problem 7** [Section 1-4 Question 10 - 15pts]

Let $C(x)$ be the statement "$x$ has a cat," let $D(x)$ be the statement "$x$ has a dog," and let $F(x)$ be the statement "$x$ has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consists of all students in your class.

a) A student in your class has a cat, a dog, and a ferret.

b) All students in your class have a cat, a dog, or a ferret.

c) Some student in your class has a cat and a ferret, but not a dog.

d) No student in your class has a cat, a dog, and a ferret.

e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

**Problem 8** [Section 1-4 Question 16 - 10pts]

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\exists x (x^2 = 2)$

b) $\exists x (x^2 = -1)$

c) $\forall x (x^2 + 2 \geq 1)$

d) $\forall x (x^2 \neq x)$

**Problem (Bonus Question - 01 extra credit)**

Alice says Bob is lying; Bob says Charlie is lying; and Charlie says Alice and Bob are both lying. Who is/are actually lying? Show your reasons.