Homework 11

The homework is due in class (during the first 10 minutes) on Tuesday, 4th December (or before). The problems are mostly from the book by Rosen, 6th edition (Chapter 9). As mentioned in the course syllabus, there are no electronic submissions and no late homeworks will be accepted unless you have an illness spanning the full period from the time the homework was assigned until it was due (and I shall need to see a medical practitioner’s certificate to that effect). Standard academic honesty rules apply. You can discuss problems with one another, but the solutions turned in should be entirely your own. Cases of plagiarism will be dealt with strictly. Also, make sure you write your name and section number on the first homework sheet and also staple all the sheets together. UNSTAPLED OR UNNAMED HOMEWORKS WILL NOT BE GRADED.

1 Matrix Multiplication

Read the wikipedia article on matrix multiplication, i.e. http://en.wikipedia.org/wiki/Matrix_multiplication. Work out the multiplication of two 3 x 3 matrices by hand. Read Theorem 2 on page 628 (section 9.4). Make sure you understand the following property of matrix multiplication: if \( C = AB \) where \( C, A \) and \( B \) are \( n \times n \) matrices, then \( C_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj} \). Now answer the following questions (and submit the answers in this homework :-)): (1) How will you use this theorem to find the length of the shortest paths between two vertices in a graph whose adjacency matrix is given to you? (2) How will you use this theorem to verify whether a graph whose adjacency matrix is given to you, is connected?

2 Graph Isomorphism

1. Section (9.3): 34, 38, 46 (Just use the definition of isomorphism), 50 (Show the one-to-one and onto function clearly).

2. Give an example of a pair of non-isomorphic graphs that have the same number of vertices, edges and the same degree sequence, and with the constraint that both the graphs are connected. We did an example in class in which one of the graphs was disconnected.

3 Connectivity

Section (9.4): 26, 56.
4 Euler and Hamiltonian Paths/Circuits

1. Construct a connected graph that has (1) an Eulerian circuit, but no Hamiltonian circuit (2) a Hamiltonian circuit but no Eulerian circuit (3) both an Eulerian circuit as well as a Hamiltonian circuit (4) neither of the two.

2. Section (9.5): 10, 28.

5 Planarity

Section 9.7: 4, 12, 15, 16 (one more problem: added on 29th Nov), 18 (Hint: the actual relation is \( v - e + r = k + 1 \). Now prove this relation starting from Euler’s formula for a connected planar graph.).

6 Graph Coloring (Excluded from the homework. Not on Final Exam.)

Section 9.8: 4, 15, 27.

7 Optional Problems

1. Section (9.4): 53.

2. Section (9.3): 54(c) [A simple graph is one in which there is at most one edge between any pair of vertices. If there is more than one edge between some pair of vertices in a graph, it is called a multigraph].