Consider the following schema:

\[
\begin{align*}
SALE & \quad (\text{CUST\_NAME, ITEM\_NAME, WHEN}) \\
CUST & \quad (\text{CUST\_NAME, INDUSTRY}) \\
ITEM & \quad (\text{ITEM\_NAME, CATEGORY, PRICE})
\end{align*}
\]

Write both relational algebra and relational calculus expressions which will answer the following questions:

(1) Which customers bought something on 12-18-04?

RA:

\[
\text{PROJECT (CUST\_NAME) (SELECT (WHEN = 12-18-04) (SALE))}
\]

RC:

\[
\{s.\text{CUST\_NAME} \mid \text{SALE}(s) \text{ and } s.\text{WHEN} = 12-18-04\}
\]

(2) Which customers from the airline industry bought something on 12-18-04?

RA:

\[
\text{PROJECT (CUST\_NAME) (SELECT (WHEN = 12-18-04 \text{ and INDUSTRY = 'airline')}) (SALE * CUST))}
\]

RC:

\[
\{s.\text{CUST\_NAME} \mid \text{SALE}(s) \text{ and } s.\text{WHEN} = 12-18-04 \text{ and EXISTS (c)(CUST(c) and c.CUST\_NAME = s.CUST\_NAME and c.INDUSTRY = 'airline'})}
\]

(3) Which customers have bought something from a category other than “transportation”?

RA:

\[
\text{PROJECT (CUST\_NAME) (SELECT (CATEGORY != 'transportation') (SALE * ITEM))}
\]

RC:

\[
\{s.\text{CUST\_NAME} \mid \text{SALE}(s) \text{ and EXISTS (i)(ITEM(i) and i.ITEM\_NAME = s.ITEM\_NAME and i.CATEGORY != 'transportation'})}
\]
(4) Which customers bought something costing more than $1,000,000 that was not in the category “doohicky”? 

RA: 

PROJECT (CUST_NAME) (SELECT (CATEGORY != 'doohicky' and PRICE > 1000000) (SALE * ITEM)) 

RC: 

{s.CUST_NAME | SALE(s) and EXISTS (i)(ITEM(i) and i.ITEM_NAME = s.ITEM_NAME and i.CATEGORY != 'dohicky' and i.PRICE > 1000000)} 

(5) Which customers from the airline industry did not buy something on 12-18-04? 

RA: 

THOSE_WHO_DID <- PROJECT (CUST_NAME) (SELECT (WHEN = 12-18-04 and INDUSTRY = 'airline') (SALE * CUST)) 

ANSWER <- PROJECT (CUST_NAME) (SELECT (INDUSTRY = 'AIRLINE') (CUST)) - THOSE_WHO_DID 

RC: 

{c.CUST_NAME | CUST(c) and c.INDUSTRY = 'airline' and NOT EXISTS (s)(SALE(s) and s.CUST_NAME = c.CUST_NAME and s.WHEN = 12-18-04)} 

(6) Which customers from the airline industry bought an item from the category “spare part” on 12-18-04? 

RA: 

PROJECT (CUST_NAME) (SELECT (CATEGORY = 'spare part' and INDUSTRY = 'airline' and WHEN = 12-18-04) (SALE * CUST * ITEM)) 

RC: 

{c.CUST_NAME | CUST(c) and c.INDUSTRY = 'airline' and EXISTS (s)(SALE(s) and s.CUST_NAME = c.CUST_NAME and s.WHEN = 12-18-04 and EXISTS (i)(ITEM(i) and i.ITEM_NAME = s.ITEM_NAME and i.CATEGORY = 'spare part'))}
(7) What categories has the customer “Widgets ‘R’ Us” never purchased an item in?

RA:

$\text{WRU\_CATS <- PROJECT (CATEGORY) (SELECT (CUST\_NAME = 'Widgets R Us') (SALE * ITEM))}$

$\text{ANSWER <- PROJECT (CATEGORY) (ITEM) - WRU\_CATS}$

RC:

$\{i.\text{CATEGORY} | \text{ITEM}(i) \text{ AND FORALL (s)(SALE (s) and s.CUST\_NAME = 'Widgets R Us' => NOT EXISTS (ii)(ITEM(ii) and ii.CATEGORY = i.CATEGORY and s.ITEM\_NAME = ii.ITEM\_NAME))}\}$

Note that this is the same as:

$\{i.\text{CATEGORY} | \text{ITEM}(i) \text{ AND FORALL (s)(SALE (s) and s.CUST\_NAME = 'Widgets R Us' => "the item purchased in the sale s was not in the same category as i")}\}$

which is the same as:

$\{i.\text{CATEGORY} | \text{ITEM}(i) \text{ AND FORALL (s)("if s was a sale to the customer 'Widegets R Us' then the item purchased in the sale s was not in the same category as i")}\}$

(8) Which customers are associated with more than one industry?

RA:

$\text{PAIRS (C1, I1, C2, I2) <- CUST X CUST}$

$\text{ANSWER <- PROJECT (C1) (SELECT (C1 = C2 and I1 != I2) (PAIRS))}$

RC:

$\{c.\text{CUST\_NAME} | \text{CUST (c) and EXISTS (cc) (CUST(cc) and c.CUST\_NAME = cc.CUST\_NAME and c.INDUSTRY != cc.INDUSTRY)}\}$

(9) What category has the most expensive item?

RA:

$\text{PAIRS (I1, C1, P1, I2, C2, P2) <- ITEM X ITEM}$
This next step gives us all items with someone who costs more:

$$\text{ONES\_WITH\_LARGER (ITEM\_NAME, CATEGORY) <- PROJECT (I1, C1)}$$
$$(SELECT (P1 < P2) (PAIRS))$$

And all items minus all items with someone who costs more gives our answer:

$$\text{ANSWER <- PROJECT (CATEGORY) (PROJECT (ITEM\_NAME, CATEGORY)}$$
$$(\text{ITEM) - ONES\_WITH\_LARGER})$$

RC:

$$\{i.CATEGORY \mid \text{ITEM}(i) \text{ and NOT EXISTS (ii)(ITEM (ii) and i.PRICE < ii.PRICE)}\}$$

(10) Which customers have bought every item in the “spare part” category?

10 and 11 are NASTY!!!!

RA:

First, we get all person/part combos for every type of spare part:

$$\text{ALL\_PAIRS <- PROJECT (CUST\_NAME) (CUST) X PROJECT (ITEM\_NAME)}$$
$$(\text{SELECT \ (CATEGORY = 'Spare Part') (ITEM))}$$

Now, we use that to get everyone who has NOT bought every spare part

$$\text{NOT\_ALL <- PROJECT (CUST\_NAME) (ALL\_PAIRS - PROJECT (CUST\_NAME, ITEM\_NAME) (SALE))}$$

And the answer is everyone else!

$$\text{ANSWER <- PROJECT (CUST\_NAME) (CUST) - NOT\_ALL}$$

RC:

It is easier in RC:

$$\{c.CUST\_NAME \mid \text{CUST(c) and forall (i)(ITEM(i) and i.CATEGORY = 'spare part' => EXISTS (s)(SALE(s) and s.CUST\_NAME = c.CUST\_NAME and i.ITEM\_NAME = s.ITEM\_NAME))}\}$$
This is the same as:

\{c.CUST\_NAME | CUST(c) and forall (i)(ITEM(i) and i.CATEGORY = 'spare part' => "the customer c.CUST\_NAME has bought i.ITEM\_NAME")}

Or:

\{c.CUST\_NAME | CUST(c) and forall (i)("if i is a spare part from the ITEM table, then the customer c.CUST\_NAME has bought i.ITEM\_NAME")\}

(11) Which customers have bought exactly the same set of items as “Chris’ Lobster Shack”?

RA:

This is not too different from #10, but it's even harder!

First we need to get all person/item combos, where the item is one from Chris' Lobster Shack:

\texttt{ALL\_COMBOS} <- \texttt{PROJECT (CUST\_NAME) (CUST) X PROJECT (ITEM\_NAME) (SELECT (CUST\_NAME = 'Chris' Lobster Shack') (SALE))}

We use that to find the people who are missing one of the items from Chris' Lobster Shack:

\texttt{NOT\_ALL} <- \texttt{PROJECT (CUST\_NAME) (ALL\_PAIRS - PROJECT (CUST\_NAME, ITEM\_NAME) (SALE))}

Now we can get everyone who has bought all of those items:

\texttt{GOT\_EM\_ALL} <- \texttt{PROJECT (CUST\_NAME) (CUST) - NOT\_ALL}

But, we're not done. We may have people who bought some extra items. So we need everyone who bought something that Chris did not.

\texttt{CHRIS\_MISSED} <- \texttt{PROJECT (ITEM\_NAME) (ITEM) - PROJECT (ITEM\_NAME) (SELECT (CUST\_NAME = 'Chris' Lobster Shack') (SALE))}

\texttt{EXTRA} <- \texttt{PROJECT (CUST\_NAME) (SALE * CHRIS\_MISSED)}

And subtract those people out to get the answer:

\texttt{ANSWER} <- \texttt{GOT\_EM\_ALL - EXTRA}
WOW!

RC:

We want something like:

\{c.CUST_NAME \mid \text{CUST (c) and } "\text{Chris has bought everything that}
\text{c.CUST_NAME has and c.CUST_NAME has bought everything that Chris has}" \}

The RC for this is:

\{c.CUST_NAME \mid \text{CUST (c) and}

\text{FORALL (s)(\text{SALE(s) and s.CUST_NAME = c.CUST_NAME } \Rightarrow
\text{EXISTS (ss)(\text{SALE(ss) and ss.ITEM_NAME = s.ITEM_NAME and ss.CUST_NAME = 'Chris' Lobster Shack'})}}

\text{and}

\text{FORALL (s)(\text{SALE(s) and s.CUST_NAME = 'Chris' Lobster Shack'} \Rightarrow
\text{EXISTS (ss)(\text{SALE(ss) and ss.ITEM_NAME = s.ITEM_NAME and ss.CUST_NAME = s.CUST_NAME}))))}\}