1. You are given a parameterized contour, \( C(s) = (x(s), y(s)) \), where \( s \) is the arc length parameter. Let \( n \) be the normal to the contour \( C \), and \( V = (u, v) \) a given smooth vector field in the image. Derive the evolution equation for the following variational principle,

\[
E(C) = \oint_C | < V, n > | ds
\]

Once you derive the evolution equation, you may substitute \( V = \nabla I \). Succinctly give reasons for the purpose of this active contour cost function? Caution: note the absolute value sign on the integrand.

2. If points \( x \) in the left camera and \( x' \) in the right camera correspond to each other in a stereo camera system, what is the equation of the epipolar line in the right camera for any point \( x \) in the left camera in terms of the Fundamental matrix \( F \)? Also, show that \( x \) and \( x' \) are corresponding points if and only if the condition \( x'Fx = 0 \) is satisfied. How many degrees of freedom does the \( F \) matrix (which is a \((3,3)\) matrix) have?

3. The area of a surface \( S(u, v) \) can be written in terms of the coefficients of the induced metric i.e., the First Fundamental form as \( A(S) = \int \int_T \sqrt{(EG - F^2)} \ du \ dv \), where \( E, G \) and \( F \) are the coefficients of the first fundamental form. Now, for the case of a visible surface \( Z(x, y) \), compute the fundamental form coefficients and then minimize the area integral (written in terms of these coefficients) by writing down the Euler Lagrange equations. Simplify your expressions as much as possible by doing the algebra.

4. A Lambertian surface illuminated by a point source has a reflectance map of the form

\[
R(p, q) = \frac{1 + p_s p + q_s q}{\sqrt{1 + p_s^2 + q_s^2} \sqrt{1 + p_s^2 + q_s^2}}
\]

where the light source lies in the direction \((-p_s, -q_s, 1)^t\).

- What value of the gradient \((p, q)\) maximizes this expression? Is your maximum a global one and is it unique?
- For what values of the gradient is \( R(p, q) = 0 \)? What is the corresponding contour in the gradient \((p, q)\) space?

5. In optical flow, if you assume the flow to be a constant over a patch, with the constraint that pixels far from the center of the patch are weighted less than those closer to it, derive a closed form expression for this weighted flow. Explicitly set up the linear system for the weighted flow i.e., show the entries of the matrix in the linear system and the right hand side vector in the linear system. Assume the patch size to be \((k, k)\) and the weight matrix to be \( W \).