Multiple-Choice Random Network for Server Load Balancing

Ye Xia, Alin Dobra and Sueng Chul Han
Computer and Information Science and Engineering Department
University of Florida
Gainesville, FL 32611-6120
Email: {yx1, adobra, schan}@cise.ufl.edu

***Abstract*—In many networking applications involving online file/data access, structured peer-to-peer networks are increasingly used in dynamic situations with fluctuating load, which require proper load balancing. The relationship between the network structure and its load-balancing properties has not been fully understood. In this paper, we focus on the Plaxton-type networks, which are broad enough to include Pastry, Tapestry, and hypercube. We first use hypercube as an example and demonstrate that replicating files at nodes in decreasing order of the length of the common prefix with the original server leads to perfectly balanced load, and does so fast and efficiently. Moreover, this replication strategy coincides with a simple on-demand replication/caching strategy based on the observed load. One of our main contributions is to show that such desirable properties also exist for a large class of random networks, which are less restrictive and more practical than the hypercube. More importantly, we have discovered a multiple-choice random network, which drastically reduces the statistical fluctuation of the load: The maximum load over all replication servers is at most three times the average load for systems of practical sizes. The main insight is that this algorithm is related to a variant of the multiple-choice balls-in-bins problem.**

I. Introduction

A. Motivation

Structured networks are attractive platforms for building overlay networks through which network applications and services can be deployed. They have many desirable characteristics, such as allowing fast resource location, decentralizing massive computation or data access, enabling large-scale resource sharing, simplifying routing, and improving service quality and fault-tolerance due to path redundancy. They have been applied to diverse applications such as application-level multicast, persistent data storage, file/data access or sharing, media streaming and other content distribution. Among recently proposed systems, as pointed out in [1], Chord [2], Tapestry [3], and Pastry [4] are related to the hypercube topology, CAN [5] is based on the torus topology, Koorde [6] and ODRI [7] are based on the de Bruijn graph, Viceroy [8] and Ulysses [9] are based on the Butterfly topology, and FISSIONE is based on Kautz graphs [1].

Most structured networks were originally invented in the parallel processing and switching communities for their efficient self-routing property. That is, the network is constructed with a static routing structure that supports very fast lookup, hence, eliminating the need of a dynamic routing protocol or complicated routing-table lookup. Another research effort lies in creating and studying networks with small network diameters. However, structured networks are increasingly applied to large-scale file/data access, distribution, storage, and sharing, which face fluctuating load and require proper load balancing. The relationship between the network structure and its load-balancing properties has not been fully understood. The main questions are: What type of network structure is convenient for load balancing? Given a particular network structure, what is the distribution of the load over the network nodes and what are suitable load-balancing techniques? If files should be replicated, where should we place the replicas?

This paper represents a step in the systematic study of such questions. The main contribution is that we have discovered a random, Plaxton-type network that has the self-routing property and a small network diameter, but in addition, has desirable load-balancing features. The Plaxton-type networks are broad enough to include Pastry, Tapestry and hypercube, and are fundamentally related to Chord [10]. We show that it is important to engineer the network structure with load balancing in mind, because, for some networks, transparent load balancing is impossible. By transparent load balancing, we mean that the requesting node is unaware that load balancing is taking place. For those networks where transparent load balancing is possible, the specific load-balancing techniques lead to drastically different performance. For instance, intuitive solutions such as replication at neighbors do not necessarily lead to balanced load, and can waste the resources of underloaded servers.

Our solution is to construct a Plaxton-type random network through a multiple-choice algorithm and to adopt a replica-placement strategy that replicates the popular file at nodes in decreasing order of the length of the common prefix with the original node (server) (known as LCP-replication). This leads to perfectly balanced load *in the average sense*, and does so fast and efficiently. Moreover, this replication strategy coincides with a simple on-demand replication/caching strategy based on the observed load. Due to the multiple-choice part, the strategy drastically reduces the statistical fluctuation of the load across the servers. The *maximum load* over all replication servers (a.k.a *cache servers* or *cache nodes*) is at most three times the *average load* for systems of practical sizes.

The intellectual motivation of the above design comes from two areas. The first is LCP-replication on the hypercube

network, which leads to perfectly balanced load quickly and efficiently. However, such a network can be restrictive for practical purpose. For instance, the ID space must be fully occupied by nodes. For more practical networks, we first show that the same conclusion holds for a *single-choice* Plaxton-type random network, when the load is averaged over many random network instances. For a fixed network instance, the maximum load can be up to ten times the average load across all cache servers. Such results come from the area of study about sending m balls into n bins by choosing the bin randomly for each ball, known as the balls-in-bins (BNB) problem.

Another of our contributions is to make the connection between a recursive BNB problem with the random network construction problem. The utility of making such a connection lies in that, a direct simulation study cannot handle a network with more than several tens of thousands of nodes, while one can simulate the BNB problem with up to 2^{26} balls. In addition, the known theoretical results about the BNB problem allow us to interpret the simulation results with confidence and to extrapolate them for even large network size. In this paper, we have made performance-related conclusions about networks with 2^{64} nodes. Perhaps more importantly, motivated by the multiple-choice BNB problem, we are able to invent the multiple-choice random network construction algorithm, which drastically reduces the maximum load for any fixed instance of the random network.

Effectively load balancing is extremely important in large-scale data-access applications since the popularity of files/data often exhibits a heavy tail distribution, and hence, the servers for popular files can be many times overloaded. We assume that the files themselves, instead of just the pointers to them, need to be replicated. We also assume that the files are large so that, when the demand is high, large-scale replication is necessary and incurs non-trivial cost, and hence, should be optimized. Every node and file has an ID, obtained by applying a uniform hash function [11] to certain attributes of the node or the file. The resulting IDs are distributed uniformly in a common ID space. *As a result, the origins of the queries for a file are uniformly distributed across the network, which is a key assumption of the paper.* A published file is stored in a node whose ID is "close" to the file ID, in the sense specified by the network designer.

We use file/data access on managed, infrastructure-based overlay networks as the main application scenario, which is fairly generic. Although file sharing in ad-hoc P2P networks using swarming techniques such as BitTorrent [12] are common, which can be very effective in resolving the server load issue, we expect infrastructure-based data access or distribution will coexist with the ad-hoc approach. Sensitive, dynamic or valuable data are often provided in an infrastructure-based mode by companies or other organizations, allowing accountability and service quality enforcement. Furthermore, caching sensitive data at arbitrary hosts could generate much anxiety.

The rest of the paper is organized as follows. In Section II, we introduce the LCP-replication scheme on the hypercube. This serves to illustrate the relationship between the load-balancing technique and the network structure and motivates the construction of a more applicable random network. The single or multiple-choice random networks are presented in Section III. In Section IV, we relate the recursive BNB problem to our random network construction algorithm and present a detailed study of LCP-replication on large networks. We conclude in Section V.

B. Related Work

File-replication strategies relevant to our work can be summarized into three categories, which are complementary rather than mutually exclusive: (i) caching, (ii) replication at neighbors or nearby nodes, and (iii) replication with multiple hash functions. The essential difference among them lies in where the replicas of the file is placed. In (i), the file can be cached at nodes along the route of the publishing message when it is first published, or more typically, at nodes along the routes of query messages when it is requested. In (ii), when a node is overloaded, it replicates the file at its neighbors, i.e., the nodes to which it has direct (virtual) links, or at nodes that are close in the ID space such as the successors or neighbor's neighbors.

CAN and Chord mainly use strategy (ii), complemented by (i) and (iii). Tapestry uses strategy (ii) and (iii). Following the suggestions in Chord, CFS [13] replicates a file at k successors of the original server and also caches the file on the search path. PAST [14], which is a storage network built on Pastry, replicates a file at k numerically closest nodes of the original server and caches the file on the insertion and the search paths. In the Plaxton network in [15], the replicas of a file are placed at directly connected neighbors of the original server and it is shown that the time to find the file is minimized. In [11], file replication is performed, in essence, through multiple hash functions. Hash-function-based replication has its own technical challenges, such as hash function management [16], and will not be considered in this paper. Several other works on load balancing are more distantly related to our work, including [17], [18], [19], [20], [21], [13], [22], [23].

The load-balancing performance of strategy (i) and (ii) is topology-dependent and this dependency has not been systematically studied. Replica-placement and network structure are jointly studied in [15] and [24]. However, the objective there is to reduce the access delay rather than server load balancing. The focus of our work is to discover different structured networks so that some simple replica-placement strategy delivers good load-balancing performance.

II. Deterministic Network: Perfect Load-Balancing on Hypercube

In this section, we define the structure and routing of the Plaxton-type networks and the hypercube. We describe LCP-replication and LCP-caching on the hypercube, which lead to perfectly balanced load at a fast speed, are easy to use, and are robust. The same load-balancing techniques will be used for the multiple-choice random network, also a Plaxton-type network, with similar performance. However, the intuitive justification is best seen on the hypercube.

A. Structure and Routing of Plaxton-Type Networks

The class of Plaxton-type networks have the prefix-based self-routing property and the path length is at most $\log_2 m$, where m is the size of the name (i.e., ID) space. Consider a network with n nodes, $n \leq m$, taking IDs from the name space 0 to $m-1$. Each file is mapped into a key value in the same name space $\{0, 1, \ldots, m-1\}$ through a uniform hash function, and is stored at the node whose ID is the "closest" to the file key. There is some flexibility in the definition of *closeness* [20]. Suppose $2^{e-1} < m \leq 2^e$, where e is a natural number. The node IDs and the file keys can all be expressed as binary numbers.

When a file is requested, the query message is routed from the requesting node, to the node that contains the file. Suppose the requesting node ID is $a_{e-1}a_{e-2}\ldots a_0$ and the file key is $b_{e-1}b_{e-2}\ldots b_0$, where $a_i, b_i \in \{0, 1\}$ for each i. In a Plaxton network, routing of the query from node $a_{e-1}a_{e-2}\ldots a_0$ to node $b_{e-1}b_{e-2}\ldots b_0$ can be viewed as converting the former number to the latter, one digit at each hop from the left to the right. More specifically, by moving to the i^{th} hop, $i = 1, 2, \ldots, e$, a_{e-i} is changed into b_{e-i} if they are not the same. This is known as *prefix routing*. As an example, suppose $m = n = 2^5$, and suppose node 10110 makes a query for file 00000, which resides in node 00000. A possible route in a Plaxton network may consist of the following sequence of nodes: $10110 \rightarrow 00101 \rightarrow 00101 \rightarrow 00011 \rightarrow 00001 \rightarrow 00000$. In practice, the repeated node, 00101, shows up on the path only once.

The key to the above routing behavior in a Plaxton-type network is the choice of the (static) routing tables. In this paper, we define the Plaxton-type network as one whose routing table at every node has the form shown in Table I. Table I shows the routing table of node $a_{e-1}a_{e-2}\ldots a_0$. The search key is checked against the routing table when making the routing decision. The first column is called the *level*. During a route lookup, if the query message has traversed $i-1$ hops, then the i^{th} digit of the search key, counting from the left to the right, is checked against the first column of the routing table. The second column is the value of the digit. The third column is the ID of the next-hop node, i.e., a downstream neighbor. At each level, say level i, each next-hop node must satisfy the following requirement: (i) the i^{th} digit of the next-hop node must match the value in the second column; and (ii) the (i-1)-digit prefix of the next-hop node must match the (i-1)-digit prefix of the current node. Note that, at each level and for each value of the digit, there are possibly more than one next-hop nodes satisfying the above rules, all are called *eligible* next-hop nodes or eligible neighbors. Each "$*$" in the table is a wild card, which can be either 0 or 1.

The routing rule can be restated in a simpler form. At each node s and for the query destination (i.e., file key) t, the first bit position that s and t differs determines the level. For the next-hop node, choose the entry corresponding to the value of t at the i^{th} bit position. For instance, if $s = 00110$ and $t = 00000$, the routing table lookup will check level 3 and value 0. This yields the next-hop node of the form 000**.

Each particular choice for the wild cards in the routing table defines a specific version of the Plaxton-type networks, which all have the distinguishing prefix routing behavior, but other than that, may be very different networks with distinct properties. For each routing table entry, the original Plaxton network [15] chooses a next-hop node that is the closest eligible node to the current node, where the distance may be measured in the round-trip time (RTT). The objective is to minimize the file access delay. Tapestry tries to accomplish the same using incremental, distributed algorithms.

TABLE I
GENERAL ROUTING TABLE FOR NODE $a_{e-1}a_{e-2}\ldots a_0$

level/digit	value	next hop
1	0	$0*\ldots*$
	1	$1*\ldots*$
2	0	$a_{e-1}0*\ldots*$
	1	$a_{e-1}1*\ldots*$
...	...	...
e-1	0	$a_{e-1}a_{e-2}\ldots a_2 0*$
	1	$a_{e-1}a_{e-2}\ldots a_2 1*$
e	0	$a_{e-1}a_{e-2}\ldots a_1 0$
	1	$a_{e-1}a_{e-2}\ldots a_1 1$

B. Structure of the Hypercube

Suppose $m = n = 2^e$. If we let the wild cards in Table I take the same values as the current node ID at the corresponding digits, we get a hypercube (with fixed routing). The result is that each next-hop node matches the current node at all but at most one bit positions. The location of the special bit that can potentially be different coincides with the level of the routing table entry.

A helpful device for visualizing the hypercube is the embedded tree rooted at node 0, as shown in Figure 1, which is known as a binomial tree [25]. The tree paths to the root are allowed paths by prefix routing. The embedded binomial tree rooted at nodes other than 0, say i, can be derived from Figure 1 by XOR-ing each node ID with i. A key observation about the binomial tree is that it is imbalanced. For each fixed node, the sizes of the subtrees rooted at each of its children, when ordered increasingly, are $2^0, 2^1, 2^2, \ldots$. This property has important consequences on the load-balancing techniques and their performance.

C. Transparent Load Balancing

In some Plaxton-type networks, transparent load balancing is not possible. Suppose, for each routing table entry in Table I, we choose the largest node (in node ID) among all eligible nodes. Then, the two next-hop nodes in level i are $a_{e-1}\ldots a_{e-i+1}01\ldots 1$ and $a_{e-1}\ldots a_{e-i+1}11\ldots 1$. The result is that, no matter where the query for $00\ldots0$ originates, the routing takes the query to node $01\ldots1$ in one hop. It is not hard to see that the sequence of nodes traversed by any query is $01\ldots1 \rightarrow 001\ldots1 \rightarrow \ldots \rightarrow 0\ldots01 \rightarrow 0\ldots0$. Hence, there is no load balancing at all in this network regardless where the file is replicated. The example suggests that, to achieve properly balanced load, one must be very careful in constructing the routing tables, and hence, the structured

network itself. In Section II-D and II-E, we will show that transparent load balancing is possible in a hypercube and there exists a very simple and robust replica-placement algorithm that leads to perfectly balanced load.

D. LCP-replication in Hypercube

Upon close examination, a node that shares longer prefix with the search key receives more requests than a node that shares shorter prefix. Given a node s and a search key t, $s, t \in \{0, 1, ..., n-1\}$, let $l(s, t)$ be the length of the longest common prefix between s and t when expressed in binary numbers. We have the following lemma.

Lemma 1: Suppose a query for t originates from a random node, uniformly distributed on $\{0, 1, ..., n-1\}$. The probability that the query passes through node s on the way to t is $\frac{1}{2^{e-l(s,t)}}$.

Proof: A query goes through node s if and only if the starting node of the query shares the $(e - l(s, t))$-suffix with node s. There are exactly $2^{l(s,t)}$ such starting nodes. Hence, the probability that one such node is selected is $2^{l(s,t)}/2^e$, where $2^e = n$ is the total number nodes in the network. ■

One corollary of the lemma is that, given two node $s_1, s_2 \in \{0, 1, ..., n-1\}$ with the property $l(s_1, t) > l(s_2, t)$, node s_1 sees more queries on their way to t than s_2 does, if the queries are generated independently from each other and uniformly across all nodes. If we wish to reduce the load to node t, s_1 is the preferred location for file replication. This suggests the so-called *Longest-Common-Prefix-Based Replication* (LCP-replication), as shown in Algorithm 1. This algorithm can either be run by the original cache server in a centralized fashion or by each cache node distributedly.

Algorithm 1 LCP-replication

Replicate the file at nodes in decreasing order of $l(s, t)$.

The LCP-replication algorithm is also proposed in [24] for networks with the prefix-routing property. The concern there is to replicate files at the appropriate "prefix level" based on their popularity so that the average access time is constant. The load-balancing performance of LCP-replication and its relationship with the network routing structure have not been studied. We have the following key property of LCP-replication on the hypercube.

Lemma 2: Suppose independent queries for t originates from random nodes, uniformly distributed on $\{0, 1, \ldots, n-1\}$. After p replication steps, $p = 1, 2, ..., e$, the load to each cache node is $\frac{1}{2^p}$ of the total requests.

Proof: Without loss of generality, let us assume the destination is $t = 0$. If this is not true, we can rename each node ID or file key, say id, to $id \oplus t$, where $\oplus$ stands for XOR. We will prove the lemma by induction. After the first replication, node $0 \ldots 01$ has a copy of the file. By Lemma 1, this node receives $\frac{1}{2}$ fraction of the queries for file key 0. The other half of the queries end at node 0.

Now, suppose the lemma is true for p. The files are copied to nodes of the form $0 \ldots 0 * \ldots *$, where the last p digits are wild cards. At the $(p+1)^{th}$ step, the file is replicated to nodes of the form $0 \ldots 01 * \ldots *$, where the leading 1 occurs at the $(p+1)^{th}$ position from the right. Exactly 2^p new nodes are added as cache nodes. Again by Lemma 1, each of these 2^p new nodes receive $\frac{1}{2^{e-(p+1)}}$ fraction of the queries. Let us consider an arbitrary new node added in the $(p + 1)^{th}$ step, say node $0 \ldots 01 a_{p-1} a_{p-2} \ldots a_0$. The queries now received and served by the new node were originally received and served by node $0 \ldots 0 a_{p-1} a_{p-2} \ldots a_0$ just before the $(p + 1)^{th}$ replication. Hence, after the $(p + 1)^{th}$ replication, the load to node $0 \ldots 0 a_{p-1} a_{p-2} \ldots a_0$ is reduced by $\frac{1}{2^{e-(p+1)}}$. By the induction hypothesis, the load to node $0 \ldots 0 a_{p-1} a_{p-2} \ldots a_0$ after the $(p + 1)^{th}$ replication must be $\frac{1}{2^{e-p}} - \frac{1}{2^{e-(p+1)}} = \frac{1}{2^{e-(p+1)}}$. ■

Lemma 2 shows that LCP-replication has the nice properties that (i) in each step of replication, the number of nodes that contain the file is doubled, (ii) after each step of replication, the load to each original cache node is reduced by half, and (iii) the final loads to the cache nodes are all identical. Hence, LCP-replication leads to perfectly balanced load quickly and efficiently. Efficiency comes from the fact that, for serving the same number of requests, LCP-replication results in the smallest number of cache servers. This also leads to the least amount of replication traffic, and hence, is bandwidth-saving.

This process is illustrated in Figure 1, where the cache nodes are shaded. The directed edge indicates the source and destination of the replication process. The numerical label indicates the step at which the file is replicated to the corresponding node. At each replication step, a source node copies the file to only one other node. The end result is that each cache node handles exact $1/8$ of the query load. In contrast, if we place replicas at all neighbors of the overloaded nodes, much more nodes become cache servers. This leads to increased traffic volume. Under the same bandwidth, it increases the time required for the replication process to complete, and hence, increases the congestion duration at the current servers. After the replication process completes, the most loaded cache nodes each handle $1/8$ of all the queries, but some cache nodes, e.g., node 00001, do not handle any.

In contrast, Figure 2 shows the process of replication-at-neighbors. Much more nodes become caches, roughly $\frac{1}{2} \log n$ times more nodes than necessary. This leads to increased traffic volume. Under the same bandwidth, it increases the time required for the replication process to complete, and hence, increases the congestion duration at the current servers. After three replication steps, the most loaded cache nodes each handle $1/8$ of all the queries, but some cache nodes, e.g., node 00001, do not handle any.

E. Automatic Load Balancing by Caching

With active file replication, an overloaded server "pushes" a copy of the file to other nodes. In contrast, file caching is typically a "pull" strategy, where a node automatically retrieves and saves a copy of the file upon seeing many requests. These are complementary approaches for rapid, scalable, and reliable load balancing. LCP-replication has a natural counterpart: an on-demand caching algorithm that automatically achieves load balancing in the spirit of LCP-replication.

Suppose, for each file f, each node keeps a load threshold, θ_f. The node measures the rate of requests for file f, denoted

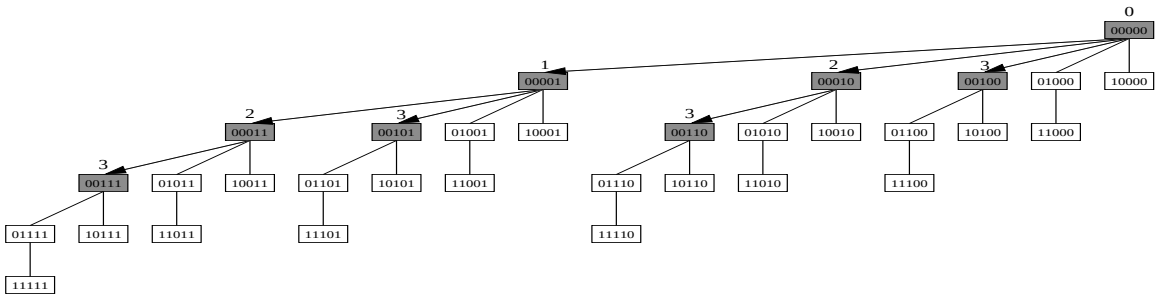


Fig. 1. An example of LCP-replication. The shaded nodes are cache nodes.

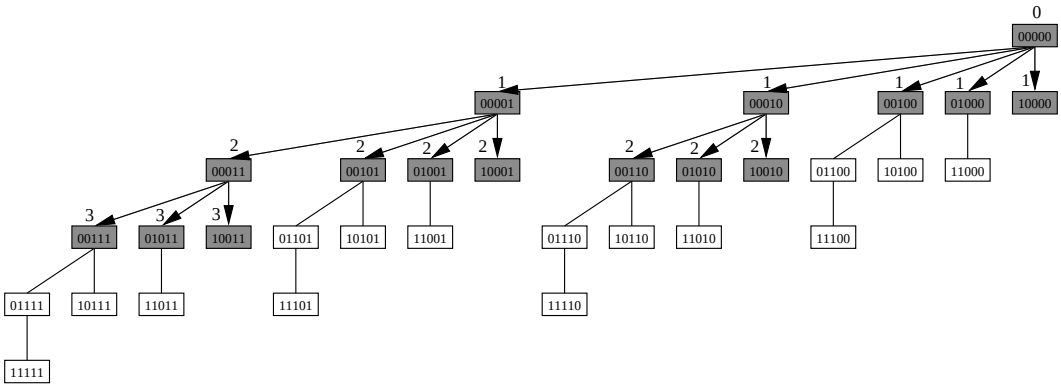


Fig. 2. An example of replication-at-neighbors. The shaded nodes are cache nodes.

by $r(f)$. The caching algorithm is listed in Algorithm 2, known as *LCP-caching*. The parameter $\epsilon \geq 0$ is a safety margin that controls the oscillation of the algorithm.

Algorithm 2 LCP-caching

if $r(f) > \frac{1}{2}\theta_f + \epsilon$ **then**
  The node caches a copy of f.
else if $r(f) \leq \frac{1}{2}\theta_f - \epsilon$ **then**
  The node removes f (or marks it as being absent).
end if

There is no novelty in the LCP-caching algorithm itself. It is similar to many on-demand caching algorithms based on the observed request load at the current node, for instance, [11] and [13]. However, caching (even LCP-replication) in general does not necessarily work at all in some Plaxton networks, as shown in the example of section II-C. The key is that hypercube possesses the nice property that the simple, distributed, on-demand caching algorithm works very well: It mimics LCP-replication on the hypercube and has the same optimal load-balancing performance. We state the following lemma about LCP-caching. The proof and additional facts about LCP-caching are given in Appendix A.

Lemma 3: Given any initial set of nodes that cache the file, LCP-caching (with $\epsilon = 0$) eventually converges to the LCP-replication outcome.

III. LCP-Replication in Randomized Plaxton Network

The hypercube requires $m = n = 2^e$. While this can be satisfied in managed networks, in large public networks, the nodes may only sparsely populate the name space. We will handle this situation by constructing a random network, which has the desired load-balancing properties as the hypercube.

A. Single-Choice Randomized Routing Tables

Starting with the generic routing table as shown in Table I, for each entry, we choose a node uniformly at random from all eligible nodes for the entry. For instance, suppose the current node is $a_{e-1}a_{e-2}\ldots a_0$ and $a_{e-2} = 1$. The next-hop node at level 2 for digit value 0 is chosen uniformly at random from all *available* nodes of the form $a_{e-1}0*\ldots*$. The entry at level 2 for digit value 1 can be ignored since route lookup will never use that entry. The resulting network is an instance of a random network. We can show that, whether or not the name space is fully populated by nodes, the random network has the desirable load-balancing properties similar to the hypercube. Without loss of generality, consider the queries for file 0.

Theorem 4: Suppose the file is replicated at all nodes $0\ldots0*\ldots*$ with p wild cards, $0 \leq p \leq e$. Suppose a query originates from a node chosen uniformly at random among all nodes. It is equally likely to be served by any of the cache nodes.

Proof: First, consider a query from a node of the form $0\ldots01a_{p-1}\ldots a_0$. At the first hop of the routing, it will be routed to one of the cache nodes with equal probability. Next, suppose the lemma is true for queries originated from any node of the

form $0...01*...*$, with the leading 1 at the i^{th} position from the right, where $p+1 \leq i \leq q < e$. Consider nodes of the form $0...01*...*$ with the leading 1 at the $(q+1)^{th}$ position. After the first hop, the query is routed to a node of the form $0...0a_{q-1}...a_0$ with equal probability. Either this node is one of the cache node, or it isn't. If it isn't, then it is as if the query starts from that node, and by the induction assumption, will be served by one of the cache nodes with equal probability. Hence, the original query will be served by one of the cache nodes with equal probability. ■

We now present preliminary simulation results to show the effectiveness of LCP-replication on the random network, but also point out room for further improvement. Let the name space size $m = 5000$. Suppose the network has 500 nodes sparsely populating the name space. This type of situations is the main reason for moving from the hypercube to the random network. Suppose the nodes are distributed uniformly in the name space, and every node in the network generates one query for file 0. Suppose the requested file is cached according to the LCP-replication scheme at the nodes whose IDs are less than 128. The number of such nodes is a random variable, which is a function of the network instance.

Figure 3 (a) shows the query count at each node, averaged over 1000 instances of the random network. The horizontal axis shows the nodes in increasing order of their IDs. We see that the cache nodes each serve nearly the same number of queries, about 32 queries. As a comparison, without caching, the only server would serve 500 requests. Non-cache nodes also see the queries but do not serve them. As expected, the query count decreases roughly by half between consecutive groups of nodes. In summary, LCP-replication achieves perfect load balancing in the average sense, where the average is taken over instances of the network.

Figure 3 (b) shows the query count at each node in an instance of the random network. The load to any cache server is significantly less than 500 queries. However, the number of queries served by different cache servers varies between 4 to 67. The *maximum load over all cache servers* is 67, about twice as much as the *average load across all cache servers*.

The average load-balancing performance is an inadequate metric for our case, because we do not repeatedly draw different network instances. Once a network instance is generated, it will be put to use. Except the gradual node arrivals and departures, the network remains more or less fixed for a long time. In the actual network instance to be used, the discrepancy between the maximum load and the average load is a cause of concern. We can show that, for larger but practical system sizes, the maximum load can be up to 10 times the average load, but is highly unlikely to be more than that. The methodology for such result on larger networks is described in Section IV. Our next task is to reduce this discrepancy to be less than 3 in virtually all network instances using a multiple-choice algorithm, which will be explained.

B. Multiple-Choice Randomized Routing Tables

In this section, we consider the *multiple-choice random network construction algorithm*, an improvement to the single-choice algorithm. The motivation is to reduce the statistical

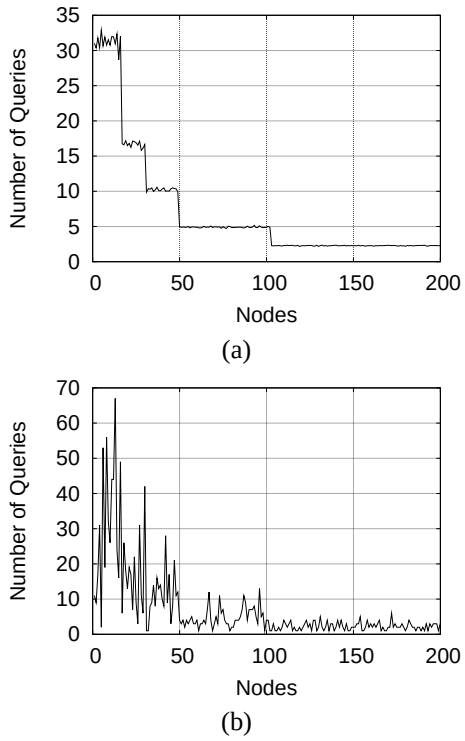


Fig. 3. Number of queries seen by each node in the random network. $m = 5000$, $n = 500$. With LCP-replication. The file is at all nodes whose IDs are less than 128. (a) Averaged over 1000 network instances; (b) in an instance of the random network

fluctuation in the load to the cache nodes. We defer the discussion about its rationale and about how to evaluate its performance in large networks to Section IV. One related work that also uses the multiple-choice strategy for random network construction is [21]. The goal there is to make the distribution of the nodes in the name space more uniform.

Each node of the network keeps e counters. Consider an arbitrary node, denoted by v, whose ID is $b_{e-1}...b_0$. The counters are denoted by $v[1], ..., v[e]$. Each $v[i]$ counts the number of upstream nodes of the form $b_{e-1}...b_{e-i+1}*...*$, i.e., the nodes that share at least $(i-1)$-digit prefix with v, that make v a next-hop node at level i in their routing tables.

For an arbitrary node, $a_{e-1}...a_0$, let us consider how to fill its routing table that conforms to the generic routing table shown in Table I. Recall that each next-hop entry in Table I specifies the format of the so-called *eligible nodes* for the corresponding level and the corresponding digit value. We will focus on the next-hop entry for level-i and digit value c, where $1 \leq i \leq e$ and $c = 0$ or 1. If c is equal to the i^{th} digit of the current node's ID, i.e., $c = a_{e-i}$, we leave that entry unfilled since it won't be used by the routing algorithm.

Let us denote the complement of the binary digit a_{e-i} by $\bar{a}_{e-i}$. If $c = \bar{a}_{e-i}$, we choose d nodes, where $d \geq 1$, uniformly at random from all eligible nodes for the entry, that is, nodes of the form $a_{e-1}...a_{e-i+1}\bar{a}_{e-i}*...*$. The d choices are independent from each other and with replacement, i.e., there can be repetition among the choices. Let us denote the d choices by $v_1, ..., v_d$. We then select among the d nodes the one with the least i^{th}-counter value, $v_j[i]$, and make it the next-

hop node in the routing table entry being considered. In other words, the node selected to fill the routing table entry, denoted by v_l, satisfies $v_l[i] = \min_{1 \le j \le d} v_j[i]$. Finally, we increment $v_l[i]$ by 1. The above procedure is summarized in Algorithm 3. Each node can run it independently and distributedly. The notation assumes that the current node ID is $a_{e-1}...a_0$ and that $T[i][c]$ is the next-hop node in the routing table entry corresponding to level i and digit value c. The statement $T[i][c] \Leftarrow v_l$ sets v_l as the next-hop node for this routing table entry.

Algorithm 3 Multiple-Choice Random Network Construction

```
for each level i from 1 to e do
  for each value c ∈ {0, 1} do
    if c ≠ a_{e−i} then
      choose d eligible nodes independently, uniformly at
      random and with replacement, denoted by v_1, ..., v_d
      l ⇐ argmin_{1≤j≤d} v_j[i]
      T[i][c] ⇐ v_l
      v_l[i] ⇐ v_l[i] + 1
    end if
  end for
end for
```

Figure 4 shows the query counts at the nodes in a network instance. The parameters are $m = n = 5000$, $k = 32$, where k is the number of cache servers. The file is contained at cache nodes 0 to 31. The number of multiple choices is $d = 1, 5, 10$ and 50 for different curves. The case of $d = 1$ corresponds to the single-choice network. The average load over all cache nodes is equal to $n/k = 156.25$ for all network instances, since we assume every node generates one query for file 0. The maximum load for $d = 1$ is about 390. It can be reduced as d increases. At $d = 5$ or 10, the load to the cache nodes is fairly well balanced. At $d = 50$, the load to the cache nodes, as well as all other nodes, starts to approach the average behavior (averaged over network instances).

The remaining important concern is: How does the above results change as the random network becomes large? This will be the subject of Section IV.

IV. Model of the Random Network: Recursive Balls in Bins (RBNB)

When queries are generated uniformly by the nodes, Theorem 4 gives no information about the variation of the load across the cache nodes and across different network instances. Of particular interest is the load of the most loaded cache node, the maximum load. In this section, we will recast the random network into a balls-in-bins (BNB) model for easy understanding of such issues.

One benefit of making such a connection is that we can simulate much larger BNB problem due to its simplicity. In addition, the known theoretical results [26], [27], [28], [29] about the BNB problem allow us to interpret the simulation results with confidence and to extrapolate them for even large network size. For instance, we will eventually make performance-related conclusions about networks with 2^{64} nodes. More importantly, the multiple-choice BNB problem has motivated the invention of the multiple-choice random network construction algorithm, which drastically improves the load-balancing performance for any fixed instance of the random network.

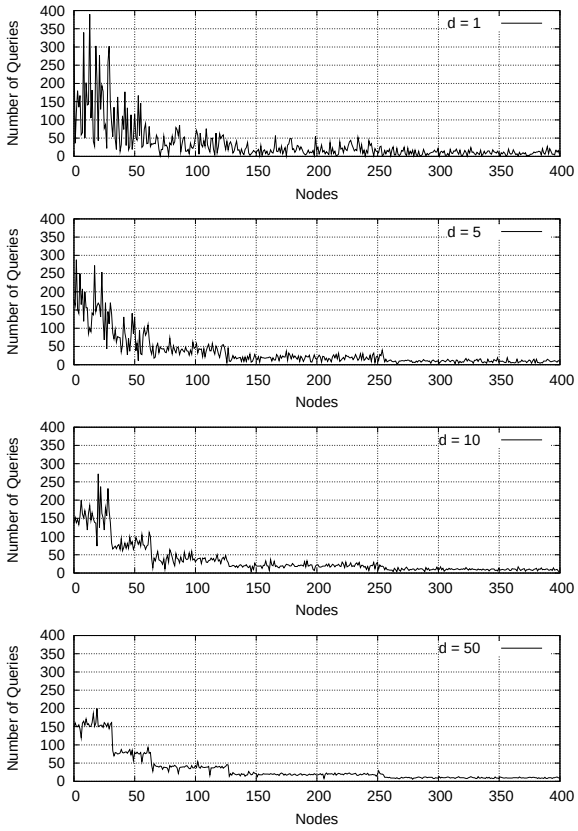


Fig. 4. Query counts at the nodes in a network instance. $m = 5000$, $n = 5000$. With LCP-replication, $k = 32$. The file is contained at node 0 to node 31.

A. Equivalence of the RBNB Problem and the Single-Choice Random Network Construction Problem

To simplify the presentation, let us assume the name space size, m, is the same as the number of nodes, n, and that $m = n = 2^e$ for some natural number e. Also assume every node generates exactly one query for file 0. With k cache nodes, the average load is always a constant, n/k. Theorem 4 says that the *expected number of queries* received by each cache node is n/k, where the expectation is taken over all network instances.

The problem of balls-in-bins (BNB) is to place n balls into k bins by selecting one destination bin uniformly at random and with replacement for each ball, and independently across different balls. Suppose k is also a positive integer of some power of 2. The *recursive balls-in-bins* (RBNB) problem is as follows. Initially, there are n bins, numbered $0, 1, ..., n-1$, each with exactly one ball. In the first step, for each ball in bin i, where $n/2 \le i \le n-1$, a destination bin is chosen for it from bins 0 through $n/2-1$, uniformly at random and with replacement. The ball in bin i is placed into the chosen destination bin. This is repeated independently for each i,

$n/2 \leq i \leq n-1$. At the end of the first step, all balls are in bins from 0 to $n/2-1$. At the beginning of the general j^{th} step, for $1 \leq j \leq \log_2(n/k)$, all balls are in bins from 0 to $n/2^{j-1}-1$. For each bin i in the second half, i.e., $n/2^j \leq i \leq n/2^{j-1}-1$, we randomly choose a destination bin from the first half, i.e., from 0 through $n/2^j-1$, and place all balls in the former bin into the destination bin. The process repeats $\log_2(n/k)$ steps and all balls end up in bins 0 through $k-1$ (See Figure 5.).

The RBNB problem is the same as our single-choice random network construction problem. To see this, consider the subnetwork that consists of all the nodes and edges on the routes to node 0, which is an embedded tree rooted at node 0, with all edges directed toward the root. The construction of this subnetwork can be described as follows. In the first step, every node of the form $1a_{e-2}...a_0$ selects a node of the form $0a_{e-2}...a_0$ randomly as its next-hop neighbor. In the j^{th} step, every node of the form $0...01a_{e-1-j}...a_0$ selects a node of the form $0...00a_{e-1-j}...a_0$ randomly as its next-hop neighbor. The process repeats $\log_2(n/k)$ times. Describing the network construction in synchronized, ordered steps is not essential but serves to identify the network construction with the RBNB problem. Any asynchronous and distributed network construction algorithm following the same neighbor-selection rule results in the same network.

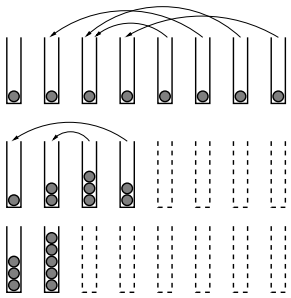


Fig. 5. Example of RBNB with two recursion steps. $m = n = 8$, $k = 2$.

B. Multiple-Choice RBNB

An advantage of recasting the random network construction problem into the RBNB problem is that, the BNB problem and its variants have been well studied. For instance, it is well known that, with $k = n$, the number of balls in the bin with the most balls, which will also be called the maximum load, is $(1+o(1))\frac{\ln n}{\ln \ln n}$ with high probability. Note that the average number of balls per bin, which will be called the average load, is 1 in this case. One of the variants of the BNB problem is the multiple-choice BNB problem. In such a problem, for each ball, d bins are independently and randomly selected, with replacement, and their contents are examined. The ball is placed into the bin with the fewest balls. Rather surprisingly, for $d > 1$, the maximum becomes $(1 + o(1))\frac{\ln \ln n}{\ln d}$ with high probability. With multiple choice, there is an exponential reduction in the maximum load.

We are not aware of similar theoretical results for the RBNB or the multiple-choice RBNB problems. The techniques for proving the aforementioned results in the BNB case do not seem to apply to the RBNB case. However, the apparent relatedness of the RBNB and the BNB problems leads us to conjecture that, in RBNB, the ratio between the maximum load and the average load is not large, and that, multiple-choice RBNB can reduce the ratio significantly. We will substantiate the conjectures with simulation experiments.

We may distinguish two versions of the multiple-choice RBNB algorithm. Recall that, in a general step of the RBNB algorithm, some of the bins are source bins and some are destination bins and the objective is to move the balls in each source bin into a destination bin. In the *ball-count* version, each destination bin keeps track of the number of balls it currently contains. The balls in a source bin are assigned to the destination bin with the least number of balls among d choices. In the *bin-count* version, each destination bin keeps track of the number of *source bins* it has already consolidated. The balls in a source bin are assigned to the destination bin with the least number of consolidated bins among d choices. The ball-count RBNB algorithm apparently should be more effective in reducing the maximum load than the bin-count algorithm. However, the bin-count algorithm is more easily applicable to the random network construction problem. Our algorithm in Section III-B (Algorithm 3) is equivalent to bin-count RBNB. The performance difference of the two versions is observable but not significant. Subsequently, the multiple choice algorithm refers to the bin-count version unless otherwise mentioned.

In the more general setting where $m > n$, the initial number of bins is m. Only n of them contains one ball each and the rest are empty. The RBNB algorithm needs a minor and straightforward modification. For brevity, we omit the details.

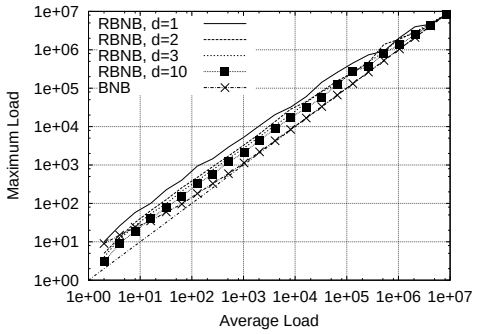
C. Evaluation of Single-Choice and Multiple-Choice RBNB

We have conducted extensive simulation-based study characterizing the maximum load in the RBNB problem. This is the same as studying the maximum server load in the multiple-choice random network.

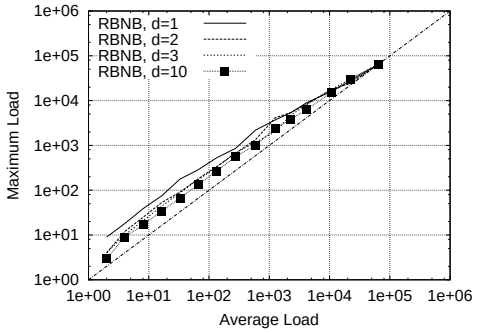
1) The maximum load in a single run of RBNB: Figure 6 (a) shows the maximum load vs. the average load, n/k, for the bin-count RBNB algorithm on log-log scale. Here, $m = n = 2^{23}$ and k takes all possible values from $2^{22}, 2^{21}, ..., 2^0$. The different curves correspond to $d = 1, 2, 3$ and 10, where d is the number of choices. For comparison purpose, we also plot the result for the BNB algorithm, and the straight line with slope 1 corresponding to the case that the maximum load is equal to the average load. The data for each curve is collected over a single run of the algorithm, and hence, each maximum load is an instance of a random variable.

The curves for the RBNB algorithm are not as smooth as the one for the BNB algorithm, indicating higher statistical fluctuation in the maximum load. But the fluctuation is small. For the plain RBNB algorithm ($d = 1$), the maximum load is at most 7 times of the average load. Multiple choice is very effective in further reducing the difference. For instance, for $k = 2^{16}$, the maximum loads are 939 or 475 when $d = 1$ or 2, respectively. The average load is 128. Overall, the maximum load is 1 to 4 times the average load when $d = 2$. When $d = 10$, the maximum load is no more than 2.5 times the average load.

Figure 6 (b) shows similar plots for the case where the name space is not fully populated, with $m = 2^{23}$ and $n = 2^{16}$. For $d = 1$, the ratio of the maximum load to the average load is less than 10 for all experimented parameters. With a small number of multiple choices, the ratio substantially decreases. In particular, with $d = 3$, the ratios are all less than 3. An additional result not shown here is that, for fixed m, the ratio increases as n increases. The implication is that we can take the case of fully populated name space (i.e., $m = n$) as the worst-case scenario.



(a)

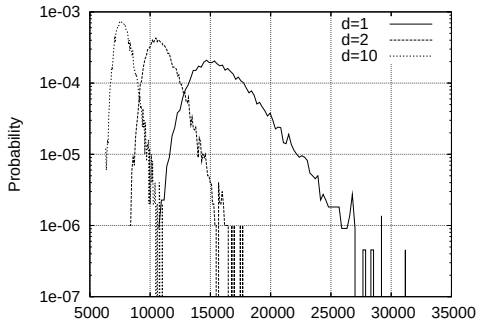


(b)

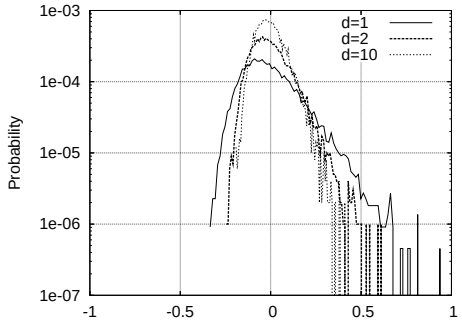
Fig. 6. Maximum load vs. average load for RBNB under the bin-count multiple-choice method. (a) $m = n = 2^{23}$; (b) $m = 2^{23}$, $n = 2^{16}$.

2) Statistical fluctuation of the maximum load: Previous RBNB experiments show only instances of the maximum load. What allows us to draw conclusions based on them is that the statistical fluctuation of the maximum load is not large. Multiple choice further reduces the fluctuation significantly. We now substantiate these claims.

We will focus on the cases where $m = n = 2^{20}$ as an example. For each fixed k and d, we collect samples of the maximum load from 10000 different runs of the RBNB algorithm, and then derive the empirical probability mass function (pmf) from the histogram of the samples. In Figure 7 (a), we compare the pmf's of the single-choice and multiple-choice RBNB. We see that, with or without multiple choice, the tail of the pmf decreases at least exponentially fast and the statistical fluctuation is not large compared to the respective mean of each case. For instance, the largest ratio observed between a sample of the maximum load and its mean is no more than 3, and this occurs for $d = 1$. With multiple choices, significant reduction occurs not only in the mean of the maximum load, but also in its statistical fluctuation.



(a)



(b)

Fig. 7. Probability mass function of the maximum load for RBNB. Comparison of the single-choice and multiple-choice RBNB. $m = n = 2^{20}$, $k = 2^8$. (a) Maximum load; (b) normalized maximum load.

Furthermore, the tail of the pmf also decays faster.

Figure 7 (b) shows the pmf for the normalized maximum load, which demonstrates the normalized fluctuation of the maximum load. The normalization is done by subtracting the sample average of the maximum load from each sample, and dividing the result by the sample average. The expectation of the normalized sample is now at 0, and the fluctuation is relative to the expected maximum load, which decreases as d increases. We see that the normalized fluctuation also decreases as d increases, indicating that the fluctuation of the un-normalized maximum load decreases even faster than its expected value. Furthermore, the chance that the maximum load is greater than twice of its expected value is very small.

An important question is how the statistical fluctuation of the maximum load varies with n. In order for the comparison to be fair, we must keep n/k constant. This keeps the number of recursive steps in the RBNB algorithm constant. As Figure 8 shows, the statistical fluctuation decreases as n increases. This is not surprising, since it is known that, in the case of BNB, the maximum load is more and more concentrated at $(1 + o(1)) \ln n / \ln \ln n$ as n approaches infinity.

3) Statistical Fluctuation across Bins: Figure 9 (a) demonstrates how the sample mean of the maximum load changes with the number of multiple choices, d, for both the bin-count RBNB. We choose $n = 2^{20}$ for all curves and let k vary among $2^2, 2^8, 2^{14}$ and 2^{19}. Each of the horizontal lines is the average load across the bins for each different value of k. It can be seen that the expected maximum load quickly comes within a factor of 1 to 2 of the average load for all cases. The

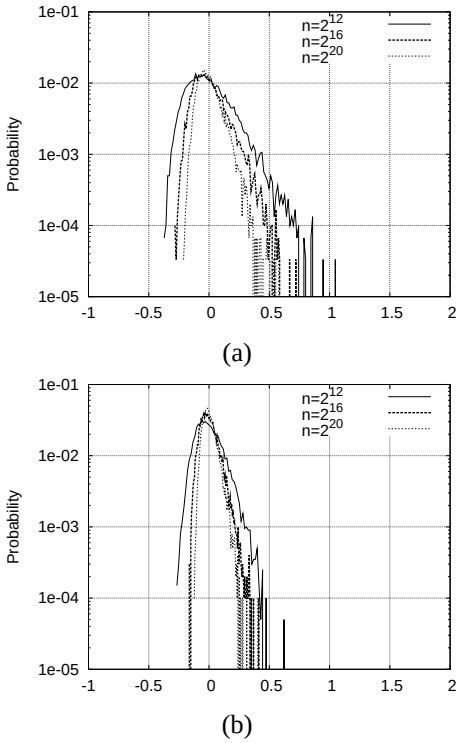


Fig. 8. Probability mass function of the normalized maximum load for single or multiple-choice RBNB. Different curves in each plot are for $(n = 2^{12}, k = 2^6)$, $(n = 2^{16}, k = 2^{10})$, and $(n = 2^{20}, k = 2^{14})$. (a) Single-choice RBNB ($d = 1$); (b) multiple-choice RBNB ($d = 3$).

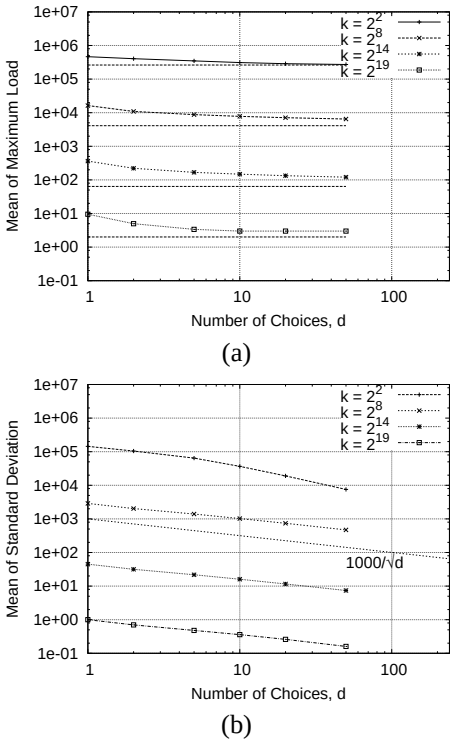


Fig. 9. Sample mean of the maximum load and sample mean of the standard deviation across bins versus the number of multiple choices, in bin-count RBNB. $n = 2^{20}$. (a) Maximum load; (b) standard deviation.

largest decrease occurs for small values of d, the number of choices. The decrease slows down or stops when d exceeds 10. This result is desirable since it ensures that a small value of d is effective enough to reduce the expected maximum load to be within a factor of two of the average load.

Figure 9 (b) shows the sample mean of the standard deviation of the loads across the bins, which is an indicator of how varied the loads are across the bins after the final step of the recursion (Note that the standard deviation is with respect to the loads to the bins in a single simulation run. The sample mean is an average over all simulation runs.). When compared with Figure 9 (a), we see that the standard deviation is less than the average load at $d = 1$ and decreases as d increases. Moreover, the decrease remains at a significant rate even after the mean of the maximum load nearly stops decreasing at larger values of d. We conclude that the effectiveness of multiple choice for load balancing not only lies in reducing the maximum load, but also in reducing the overall variations of the loads across the bins. In Figure 9 (b), we plot the function $1000/\sqrt{d}$ and find that the decrease in standard deviation is nearly proportional to $1/\sqrt{d}$.

Figure 10 shows the number of balls in each bin after the final step of the bin-count RBNB algorithm for a single simulation run. The parameters are $n = 2^{20}$ and $k = 2^{10}$. In this case, the average load across the bins should be 1024. The five plots correspond to $d = 1, 2, 5, 10$, and 50, respectively. It can be seen that the maximum load is reduced significantly when d increases from 1 to 2. At $d = 2$, the maximum load is about 2 to 3 times the average load. It becomes more difficult to further reduce the maximum load by increasing d. However, the overall variation of the number of balls across the bins continues to decrease. At $d = 50$, the numbers of balls in most bins are very close to the average load, with a small number of exceptions.

4) Trend of the expected maximum load versus n: We next discuss how the expected maximum load grows with n, the number of balls. We assume $m = n$ throughout. The goal is to extrapolate the trend and make prediction about the size of the maximum load for cases with much larger n, too large to be observed directly by simulation.

Figure 11 (a) shows how the expected maximum load grows with $\log_2 n$ for the bin-count RBNB algorithm. To be able to compare the results for different n, we keep n/k constant at 2^6, so that the resulting average load across the bins at the end of the algorithm is kept at 64. The reason we choose such a value 2^6 is that, according to another study not shown here, the ratio of the expected maximum load to the average load is the largest when k is close to n. We observe that, for each d, the growth of the expected maximum load is slower than a linear function of $\log_2 n$. Using multiple choices ($d > 1$) in the RBNB algorithm can drastically reduce the maximum load. This is true for even small d, e.g., $d = 2$ to 5.

The real significance of multiple choice reveals itself when we plot the same set of results with the x-axis in log scale, as in Figure 11 (b). A straight line in this plot corresponds to a linear function in $\log \log n$. We see that, for $d = 1$, the growth of the expected maximum load is faster than $\log \log n$, while for $d > 2$, the growth is no faster than $\log \log n$. This

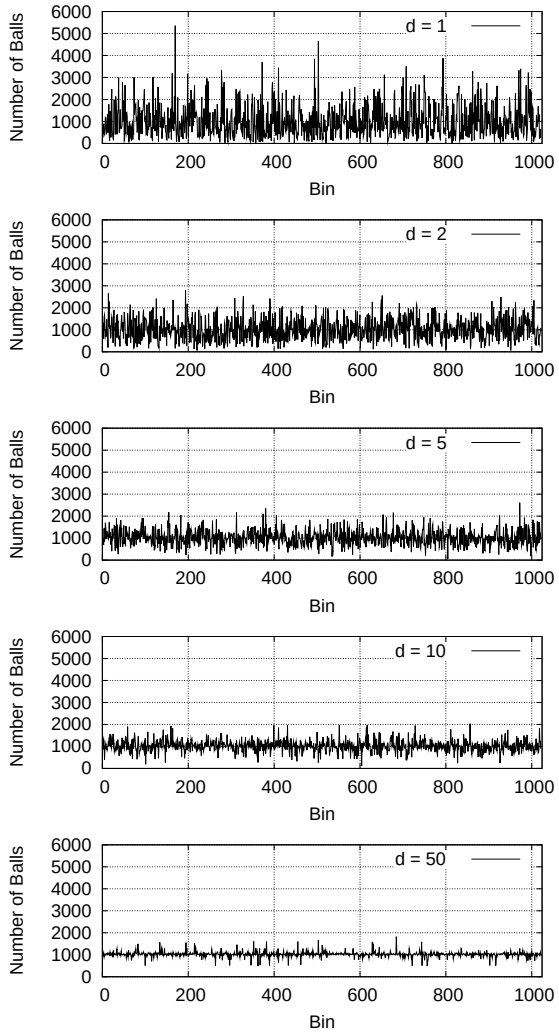


Fig. 10. Number of balls in the bins after the final step of bin-count RBNB. $n = 2^{20}$, $k = 2^{10}$. $d = 1, 2, 5, 10$ and 50, respectively

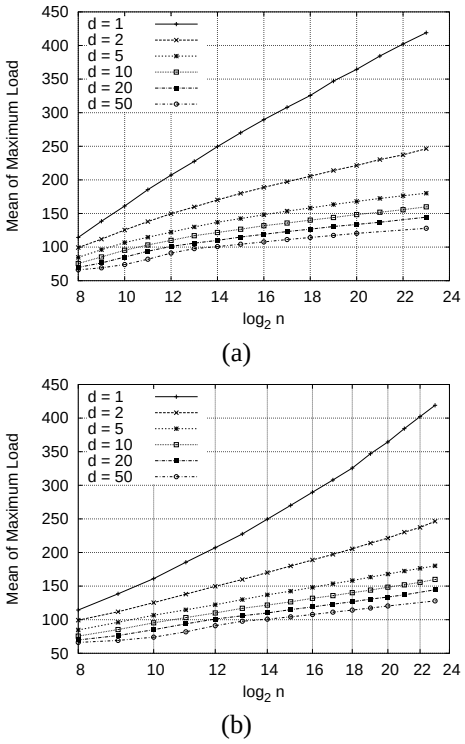


Fig. 11. Expected maximum load versus $\log_2 n$ for bin-count RBNB. We keep $n/k = 2^6$ for all cases. The average load across the bins is 64. (a) x-axis in linear scale; (b) x-axis in log scale.

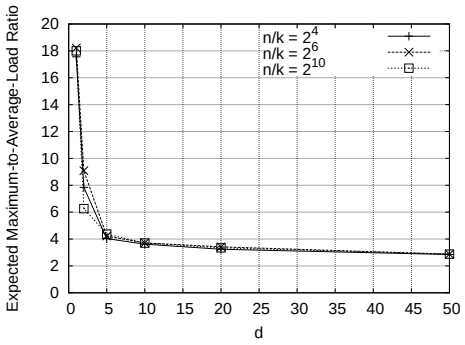


Fig. 12. Ratio of the expected maximum load to the average load for bin-count RBNB. $n = 2^{64}$.

phenomenon is believable in view of the similarity between the multiple-choice RBNB problem and the multiple-choice BNB problem.

With Figure 11 (a) and (b), if the trend of the expected maximum load continues, we can find its upper bound for larger values of n by fitting a linear function that grows no slower than the respective curve. We have done this for the case of $n = 2^{64}$, a large enough number for most practical systems, but too large to be obverved by direct simulation. The result is presented in Figure 12, where we normalize the expected maximum load against the average load. The expected maximum load is bounded from above by 1166, 582, 270, 238, 219, and 184 for $d = 1, 2, 5, 10, 20$ and 50, respectively. The corresponding ratios of the expected maximum load to the average load are 18.2, 9.1, 4.2, 3.7, 3.42 and 2.9. For n up to 2^{64}, a small value of d, e.g., $d \leq 5$, gives us most of the reduction in the expected maximum load. In addition, the ratio is not sensitive to n/k, indicating that similar results hold for a wider range of values for k.

V. Conclusions

This paper presents several Plaxton-type networks that have excellent load-balancing features, in addition to the usual properties of allowing prefix routing and having small network diameters. Our most original and important contribution is the invention of the multiple-choice random network. It was first motivated by our study of load balancing on the hypercube network. We observed that LCP-replication on the hypercube leads to perfectly balanced load across the cache servers. For the same number of requests, this requires the smallest number of cache servers, the smallest number of replicas, the smallest amount of replication traffic, and the replication process takes the shortest time. We also observed that there is a simple, distributed, on-demand caching algorithm, LCP-caching, that achieves the same performance results as LCP-replication.

However, the hypercube can be restrictive for practical purpose. It requires the size of the name space to be a power of 2, and also fully populated by nodes. We discover that a far more practical and flexible Plaxton-type network with randomized routing tables can accommodate the same LCP-replication and caching algorithms and enjoys similar load-balancing performance, except that the load is perfectly balanced after averaged over many network instances. For fixed network instances, the maximum load over all cache servers can be up to 10 times the average load.

The multiple-choice random network drastically reduces this ratio in virtually any network instance. Its invention is inspired by our discovery of its connection with the recursive balls-in-bins problem. The connection also allows us to conduct simulation experiments on large networks, to interpret the simulation results with confidence, and to extrapolate them for even larger network sizes, e.g., with 2^{64} nodes.

Due to the complexity of any load-balancing system, many system-level details that we have considered are not discussed in this paper. These include, for instance, the joining-and-departure dynamics of the nodes, the case of multiple files, and the case of heterogeneous server capacities. We expect no fundamental difference between our basic design and other published designs with respect to the degree of difficulty in incorporating these details.

REFERENCES

[1] D. Li, X. Lu, and J. Wu, "FISSIONE: A scalable constant degree and low congestion DHT scheme based on Kautz graphs," in *Proceedings of IEEE Infocom 2005*, Miami, FL, March 2005.

[2] I. Stoica, R. Morris, D. Karger, M. F. Kaashoek, and H. Balakrishnan, "Chord: A scalable peer-to-peer lookup service for Internet applications," in *Proc. ACM SIGCOMM '2001*, San Diego, CA, August 2001, pp. 149–160.

[3] B. Y. Zhao, L. Huang, J. Stribling, S. C. Rhea, A. D. Joseph, and J. D. Kubiatowicz, "Tapestry: a resilient global-scale overlay for service deployment," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 1, pp. 41–53, Jan. 2004.

[4] A. Rowstron and P. Druschel, "Pastry: Scalable, distributed object location and routing for large-scale peer-to-peer systems," in *Proceedings of the 18th IFIP/ACM International Conference on Distributed Systems Platforms (Middleware '01)*, Heidelberg, Germany, November 2001.

[5] S. Ratnasamy, P. Francis, M. Hanley, R. Karp, and S. Shenker, "A scalable content-addressable network," in *Proc. ACM SIGCOMM '2001*, San Diego, CA, August 2001, pp. 161–172.

[6] F. Kaashoek and D. R. Karger, "Koorde: A simple degree-optimal hash table," in *Proceedings of the Second International Workshop on Peer-to-Peer Systems (IPTPS '03)*, Berkeley, CA, Feb. 2003.

[7] D. Loguinov, A. Kumar, V. Rai, and S. Ganesh, "Graph-theoretic analysis of structured peer-to-peer systems: Routing distances and fault resilience," in *Proceedings of ACM SIGCOMM 2003*, Karlsruhe, Germany, August 2003.

[8] D. Malkhi, M. Naor, and D. Ratajczak, "Viceroy: A scalable and dynamic lookup network," in *ACM Symposium on Principles of Distributed Computing (PDOC 2002)*, Monterey, CA, July 2002.

[9] A. Kumar, S. Merugu, J. Xu, and X. Yu, "Ulysses: A robust, low-diameter, low-latency peer-to-peer network," in *Proceedings of the 11th IEEE International Conference on Network Protocols (ICNP03)*, Atlanta, Georgia, USA, Nov. 2003.

[10] J. Xu, A. Kumar, and X. Yu, "On the fundamental tradeoffs between routing table size and network diameter in peer-to-peer networks," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 1, pp. 151–163, Jan. 2004.

[11] D. Karger, E. Lehman, T. Leighton, M. Levine, D. Lewin, and R. Panigrahy, "Consistent hashing and random trees: Distributed caching protocols for relieving hot spots on the world wide web," in *Proceedings of the Twenty-Ninth Annual ACM Symposium on Theory of Computing (STOC '97)*, El Paso, TX, May 1997.

[12] BitTorrent Website, http://www.bittorrent.com/.

[13] F. Dabek, M. F. Kaashoek, D. Karger, R. Morris, and I. Stoica, "Wide-area cooperative storage with CFS," in *Proceedings of the 18th ACM symposium on Operating systems principles (SOSP '01)*, Banff, Alberta, Canada, October 2001, pp. 202–215.

[14] A. Rowstron and P. Druschel, "Storage management and caching in PAST, a large-scale, persistent peer-to-peer storage utility," in *Proceedings of the 18th ACM Symposium on Operating Systems Principles (SOSP '01)*, Banff, Alberta, Canada, Oct 2001, pp. 188–201.

[15] C. Plaxton, R. Rajaraman, and A. Richa, "Accessing nearby copies of replicated objects in a distributed environment," in *Proceedings of the 9th Annual ACM Symposium on Parallel Algorithms and Architectures (SPAA '97)*, Newport, Rhod Island, June 1997, pp. 311–320.

[16] Y. Xia, S. Chen, and V. Korgaonkar, "Load balancing with multiple hash functions in peer-to-peer networks," in *Proceedings of The Twelfth International Conference on Parallel and Distributed Systems (ICPADS 2006)*, Minneapolis, USA, July 2006.

[17] M. Adler, E. Halperin, R. M. Karp, and V. V. Vazirani, "A stochastic process on the hypercube with applications to peer-to-peer networks," in *Proceedings of the thirty-fifth annual ACM symposium on Theory of computing*, San Diego, CA, June 2003, pp. 575–584.

[18] D. R. Karger and M. Ruhl, "Simple efficient load balancing algorithms for peer-to-peer systems," in *Sixteenth ACM Symposium on Parallelism in Algorithms and Architectures (SPAA 2004)*, Barcelona, Spain, June 2004.

[19] J. W. Byers, J. Considine, and M. Mitzenmacher, "Geometric generalizations of the power of two choices," in *Sixteenth ACM Symposium on Parallelism in Algorithms and Architectures (SPAA 2004)*, Barcelona, Spain, June 2004.

[20] X. Wang, Y. Zhang, X. Li, and D. Loguinov, "On zone-balancing of peer-to-peer networks: Analysis of random node join," in *Joint International Conference on Measurement and Modeling of Computer Systems (ACM SIGMETRICS 2004)*, New York, June 2004.

[21] K. Kenthapadi and G. S. Manku, "Decentralized algorithms using both local and random probes for P2P load balancing," in *Proceedings of the 17th annual ACM symposium on Parallelism in algorithms and architectures*, Las Vegas, Nevada, July 2005.

[22] A. Rao, K. Lakshminarayanan, S. Surana, R. Karp, and I. Stoica, "Load balancing in structured P2P systems," in *2nd International Workshop on Peer-to-Peer Systems (IPTPS '03)*, Berkeley, CA, Feb. 2003, pp. 311–320.

[23] B. Godfrey, K. Lakshminarayanan, S. Surana, R. Karp, and I. Stoica, "Load balancing in dynamic structured P2P systems," in *Proceedings of IEEE Infocom 2004*, Hong Kong, March 2004.

[24] V. Ramasubramanian and E. G. Sirer, "Beehive: O(1) lookup performance for power-law query distributions in peer-to-peer overlays," in *Symposium on Networked Systems Design and Implementation (NSDI 2004)*, San Francisco, USA, March 2004.

[25] F. T. Leighton, *Introduction to Parallel Algorithms and Architectures: Arrays, Trees, Hypercubes*. Morgan Kaufmann, 1991.

[26] N. L. Johnson and S. Kotz, *Urn Models and Their Application*. John Wiley & Sons, 1977.

[27] G. H. Gonnet, "Expected length of the longest probe sequence in hash code searching," *Journal of the ACM*, vol. 28, no. 2, pp. 289–304, April 1981.

[28] Y. Azar, A. Z. Broder, A. R. Karlin, and E. Upfal, "Balanced Allocations," *SIAM Journal on Computing*, vol. 29, no. 1, pp. 180–200, February 2000.

[29] M. Mitzenmacher, "The Power of Two Choices in Randomized Load Balancing," Ph.D. dissertation, University of California, Berkeley, 1996.

APPENDIX A
MORE ABOUT LCP-CACHING

To understand LCP-caching, let us consider the following idealized situation. Suppose each node in the network sends queries for the file at rate λ_f. Hence, the total query rate is $n\lambda_f$. Suppose $n\lambda_f/\theta_f = 2^p$, for some $p \in \{0, 1, ..., e-1\}$. Let us consider the static outcome of the algorithm with $\epsilon = 0$. Clearly, one possible outcome is that the file is replicated at nodes $0...0a_{p-1}a_{p-2}a_0$, where $a_i \in \{0, 1\}$ for $i = 0, 1, ..., p-1$, and nowhere else. Each of these cache nodes receives and serves the queries at exactly the rate θ_f. Each of

the other nodes receives the queries at a rate no greater than $\frac{1}{2}\theta_f$ and does not cache the file. This is exactly the outcome of the LCP-replication algorithm, which is what we wish to see. One question is: does any other outcome exist? The following lemma says that the answer is no.

Lemma 5: The outcome of the LCP-replication algorithm is the only static outcome of the caching algorithm with $\epsilon = 0$.

Proof: (of Lemma 5) We first show that no node other than the nodes of the form $0...0a_{p-1}a_{p-2}...a_0$ may cache the file. Suppose node s is not of that form, but caches a copy of the file. It must have $l(s,t) \leq e-p-1$. By the same reasoning as in lemma 1, the fraction of queries for 0 that passes through s must be no greater than $\frac{1}{2^{p+1}}$. Hence, the total rate of query served by s cannot be greater than $\frac{n\lambda_f}{2^{p+1}} = \frac{\theta_f}{2}$. However, by the algorithm, node s should remove the cached copy in this case, which is a contradiction.

Next, consider nodes of the form $0...01a_{p-2}...a_0$. The total rate of queries that reach one such node is precisely θ_f because there are no other upstream nodes on the query paths that cache the file. Hence, each such node must cache the file.

Next, consider nodes of the form $0...001a_{p-3}...a_0$. The total rate of queries that potentially can reach one such node is precisely $2\theta_f$, out of which, θ_f are intercepted by the corresponding node $0...011a_{p-3}...a_0$. No other upstream nodes cache the file. Therefore, node $0...001a_{p-3}...a_0$ receives queries at rate θ_f, and hence, caches and serves the file. This argument can be applied inductively, and we can conclude that every node of the form $0...0a_{p-1}a_{p-2}...a_0$ receives queries at rate θ_f, and hence, caches and serves the file. ■

Lemma 5 says, if the caching algorithm ever reaches an equilibrium, the outcome must be that of the LCP-replication algorithm. The equilibrium is actually stable in the sense of Lemma 3.

Proof: (Lemma 3) Nodes not of the form $0...0a_{p-1}a_{p-2}...a_0$ first remove their cached copies of the file, if they initially have any, due to insufficient queries that pass through them. Then, nodes of the form $0...01a_{p-2}...a_0$ must cache the file for the same reason as in the proof of Lemma 5. Next, nodes of the form $0...001a_{p-3}...a_0$ must cache the file, again for the same reason as in the proof of Lemma 5. Inductively, all nodes of the form $0...0a_{p-1}a_{p-2}...a_0$ must cache the file and each handles the queries at rate θ_f. ■

We see that the caching algorithm is in fact robust. The above results do not require $n\lambda_f/\theta_f$ is an integer power of 2. Suppose $2^p < n\lambda_f/\theta_f < 2^{p+1}$, for some integer $0 \leq p \leq e-1$. Then, the result of the caching algorithm is that the file is cached at the 2^{p+1} nodes of the form $0...0a_pa_{p-1}...a_0$ and each of them handles the queries at rate $n\lambda_f/2^{p+1}$, which is less than θ_f but greater than $\frac{1}{2}\theta_f$. The proofs for Lemma 5 and Lemma 3 in this case are essentially the same.