Maximizing the Lifetime of Wireless Sensor Networks with Mobile Sink in Delay-Tolerant Applications

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Abstract—This paper proposes a framework to maximize the lifetime of the wireless sensor networks (WSN) by using a mobile sink when the underlying applications tolerate delayed information delivery to the sink. Within a prescribed delay tolerance level, each node does not need to send the data immediately as it becomes available. Instead, the node can store the data temporarily and transmit it when the mobile sink is at the most favorable location for achieving the longest WSN lifetime. To find the best solution within the proposed framework, we formulate optimization problems that maximize the lifetime of the WSN subject to the delay bound constraints, node energy constraints, and flow conservation constraints. We conduct extensive computational experiments on the optimization problems and find that the lifetime can be increased significantly as compared to not only the stationary sink model but also more traditional mobile sink models. We also show that the delay tolerance level does not affect the maximum lifetime of the WSN.

Index Terms—Wireless Sensor Network, Lifetime Maximization, Linear Programming, Delay-Tolerant Applications, Mobile Sink

1 INTRODUCTION

A wireless sensor network (WSN) consists of sensor nodes capable of collecting information from the environment and communicating with each other via wireless transceivers. The collected data will be delivered to one or more sinks, generally via multi-hop communication. The sensor nodes are typically expected to operate with batteries and are often deployed in not-easily-accessible or hostile environments, sometimes in large quantities. It can be difficult or impossible to replace the batteries of the sensor nodes. On the other hand, the sink is typically rich in energy. Since the sensor energy is the most precious resource in a WSN, efficient utilization of the energy to prolong the network lifetime has been the focus of much of the research on WSNs.

Although the lifetime of a WSN can be defined in many ways, we adopt the widely used definition, which is the time until the first node exhausts its energy. Much work has been done during recent years to increase the lifetime of a WSN. Among them, in spite of the difficulties in realization, taking advantage of mobility in the WSN has attracted much interest from researchers [1], [2], [3], [4], [5], [6], [7], [8], [9]. We can take the mobile sink as an example of mobility in a WSN. Communication in a WSN often has the many-to-one property in that data from a large number of sensor nodes needs to be concentrated to one or a few sinks. Since multi-hop routing is generally needed for distant sensor nodes to send data to the sink\(^1\), the nodes near the sink can be burdened with relaying a large amount of traffic from other nodes. This phenomenon is sometimes called the “crowded center effect” [10] or the “energy hole problem” [11], [12], [13]. It results in early energy depletion at the nodes near the sink, potentially disconnecting the sink from the remaining sensors that still have plenty of energy. However, by moving the sink in the sensor field, one can avoid or mitigate the energy hole problem and expect an increased network lifetime.

This paper proposes a framework to maximize the lifetime of a WSN by taking advantage of sink mobility. Compared with other mobile-sink proposals, the main novelty is that we consider the case where the underlying applications tolerate delayed information delivery to the sink. One of the application examples is battlefield surveillance, where sensor nodes are deployed to monitor the movement of enemy vehicles or troops. A mobile sink attached to an unmanned aerial vehicle flies over the monitored region regularly to harvest the collected intelligence. To avoid being intercepted or detected by enemy forces, the mobile sink needs to operate in only a few safe locations within a limited operation time. Another example is habitat monitoring where a mobile robot is used to collect information from the sensor nodes in the field. If much of the habitat area is not accessible by the robot or if it is desirable to minimize disturbance to the targeted animal species, the mobile robot will trace predetermined paths and stop by a set of pre-arranged locations regularly for data collection.

In our proposal, within a prescribed delay tolerance level, each node does not need to send the data im-
mediately as it becomes available. Instead, the node can store the data temporarily and transmit it when the mobile sink is at the location most favorable for achieving the longest network lifetime. To find the best solution within the proposed framework, we formulate optimization problems that maximize the lifetime of the WSN subject to the delay bound constraints, node energy constraints and flow conservation constraints. Another one of our contributions is that we compare our proposal with several other lifetime-maximization proposals and quantify the performance differences among them. Our computational experiments have shown that our proposal increases the lifetime significantly when compared to not only the stationary sink model but also more traditional mobile sink models without delay tolerance.

Our proposal is more sophisticated than most previous lifetime-improvement proposals that we know of. It integrates the following energy-saving techniques, multipath routing, a mobile sink, delayed data delivery and active region control, into a single optimization problem. Such sophistication comes at a cost. Whether the proposal should be adopted in practice will depend on the tradeoff between the lifetime gain and the actual system cost. The latter includes all costs/complexity when compared to the unconstrained problem by properly dividing the problem can be easily formulated into linear programming. They proposed an approximation algorithm to circumvent the complexity of the proposed mathematical formulation.

The authors formulated a linear programming problem of determining how to move the mobile sink and how long to park the mobile sink at each stop along the path of the sink so as to maximize the lifetime of the WSN. However, in their model, data flows are not decision variables of the lifetime optimization problem. On the contrary, in our formulations, not only the sink sojourn times at different sink stops but also the routing scheme are decision variables. The analysis and experiments in [1] were conducted under a simple structured network topology where the sensor nodes are deployed in a grid-like pattern. In [5] and [14], the authors further extended the research of [1]. The model proposed in [5] [14] includes the cost of moving the mobile sink (such as nodal energy consumption for route establishment/release when the sink moves to a new stop) and the sink mobility rate determined by the minimum sink sojourn time at the sink stops. Furthermore, the model incorporates a hop-length limit when the sink moves to next stop. This restricts the packet latency, which is related to the traveling time of the sink between stops. The authors proposed an MILP (Mixed Integer Linear Programming) problem formulation to obtain the optimal travel route of the sink and the sojourn times at the sink stops for maximizing the lifetime of the system. They also suggested a distributed heuristic algorithm to circumvent the complexity of the proposed mathematical formulation.

The authors of [3] showed that the network lifetime can be extended significantly if the mobile sink moves around the periphery of the WSN. They assumed that, if the mobile sink can balance the traffic load of the nodes, the lifetime of the network can increase. Therefore, they proposed an optimization problem for choosing a mobility strategy that minimizes the maximum traffic load of the nodes. However, they assumed the shortest path routing, which, in general, does not produce the best lifetime.

The problem of finding the trajectory of the mobile sink so as to optimize the lifetime of the WSN is hard to solve due to its infinite search space when the locations for the sink stops are not constrained. In [9], the authors studied how to find the optimal sink stops and the schedule of visit to each of the stops. If the candidate locations for the stops are unconstrained, this problem is also NP-hard. However, if the stops are constrained to be selected from a finite set of known locations, the problem can be easily formulated into linear programming. They proposed an approximation algorithm to the unconstrained problem by properly dividing the
whole sensor field into a finite number of disjoint small areas, and then, converted the unconstrained problem into a constrained problem. However, to obtain a good approximation ratio, the number of small areas can potentially be very large, making the linear programming computation time-consuming. Therefore, in this paper, we restrict the set of potential sink stops to be from a small number of given locations rather than from arbitrary locations.

The WSN model proposed in [7] is close to ours. The authors studied the maximum lifetime problem of the WSN where the mobile sink can visit only a small number of locations. They showed that the lifetime can be further increased by optimizing not only the schedule of sink visits but also routing of the traffic. However, they did not consider applications where delayed information delivery is allowed.

The rest of the paper is organized as follows. Section 2 describes various related lifetime maximization problems that we will compare against. The mathematical formulations of the models are provided for the purpose of comparison. In section 3, we propose two novel models with a mobile sink and delayed information delivery. We show some nice properties that our models possess. In section 4, we compare our models with others by simulation and numerical experiments. The conclusions are given in Section 5.

2 RELATED LIFETIME MAXIMIZATION PROBLEMS

In this section, we discuss related lifetime maximization problems that have been published in the literature. We will later compare their performance with our new proposal.

First, we will describe the general assumptions about the WSN models. Let the set of sensor nodes be denoted by \( \mathcal{N} \). For experimental convenience, we suppose they are uniformly randomly deployed into a circular area with radius \( R \). Let the center of the disk be the origin. Each node \( i \) is assumed to generate data at a constant rate of \( d_i \) during its life span and the initial energy of \( i \) is denoted by \( E_i \). Furthermore, the nodes have the ability of adjusting their transmission power level to match the transmission distance. Similar to [15], the energy required per unit of time to transmit data at the rate of \( x_{ij} \) from node \( i \) to \( j \) can be determined as follows.

\[
E'_{ij} = C^t_{ij} \cdot x_{ij},
\]

where \( C^t_{ij} \) is the required energy for transmitting one unit of data from node \( i \) to \( j \) and it can be modeled as follows [16].

\[
C^t_{ij} = \alpha + \beta \cdot d(i,j)^e,
\]

where \( d(i,j) \) is the Euclidean distance between node \( i \) and \( j \), \( \alpha \) and \( \beta \) are nonnegative constants, and \( e \) is the path loss exponent. Typically, \( e \) is in the range of 2 to 6, depending on the environment. Here, the energy cost per unit of data does not depend on the link rate, and this is valid for the low rate regime. Hence, we need to assume that the traffic rate \( x_{ij} \) is sufficiently small compared to the capacity of the wireless link.

The energy consumed at node \( i \) per unit of time for receiving data from node \( k \) is given by [15]

\[
E^r_{ki} = \gamma \cdot x_{ki},
\]

where \( \gamma \) is a given constant. Hence the total energy consumption per unit time at node \( i \) is

\[
\sum_{j \in \mathcal{N}} E^r_{ij} + \sum_{k \in \mathcal{N}} E^r_{ki} = \sum_{j \in \mathcal{N}} C^t_{ij} \cdot x_{ij} + \sum_{k \in \mathcal{N}} \gamma \cdot x_{ki}.
\]

We assume that each sensor node has the same transmission range. Let \( l \) denote the sink. In this paper, we take the convention that the sink is a special node different from the sensor nodes and \( l \notin \mathcal{N} \). The required energy for transmitting one unit of data from a sensor node \( i \) to the sink \( l \) is denoted by \( C^u_{il} \), and it is given by (2) with \( j \) replaced by \( l \). We define the (downstream) neighbors of node \( i \) as \( N(i) = \{ j \in \mathcal{N} \cup \{ l \} | d(i,j) \leq d, j \neq i \} \), when the transmission range is \( d \). Note that the neighbors may include the sink.

The paper does not consider MAC-layer contention. It is assumed that contention is resolved by some MAC-layer protocol. The operation of the MAC-layer protocol determines the link rates, which are assumed to be large enough so that they do not impose a constraint on the data rates. Future work may try to relax these assumptions. Conversely, if the data rates are small, then even simple MAC-layer protocols will be able to deliver the required link rates. In other words, it can be easy to design such a protocol.

2.1 Static Sink Model

In the static sink model (SSM), the sink is located at the origin and remains stationary during the operation of the WSN. Data originated from the sensor nodes flows into the sink in a multi-hop fashion. As soon as the data becomes available at a node, it gets transmitted toward the sink. Typically, the rate at which each sensor node \( i \) harvests data from the outside world is a constant. We denote it by \( d_i \). The data generated by a source is sometimes called a commodity or a sub-flow [17], [18]. Let \( x^c_{ij} \) be the rate assignment from node \( i \) to node \( j \) for the traffic generated by node \( c \) (commodity \( c \)). The problem of maximizing the lifetime in this model is formulated as follows [19], [20].
The constraint (7) is the “flow conservation constraint”, which states that, at a node \(i\), the sum of all outgoing flows for a commodity \(c\) is equal to the sum of all incoming flows for the commodity \(c\). If \(i = c\), the incoming flows should include the flows generated at node \(i\) itself, or \(d_i\). The inequality (8) is the energy constraint and it means that the total energy consumed by a node during the lifetime \((Z)\) cannot exceed the initial energy of the node. With this formulation, the routing is dynamic and allows multipath communications. There is no assumption on fixed-path routing, such as the shortest path routing. The above optimization problem can be easily converted into a linear programming (LP) problem.

The particular formulation above is equivalent to the following formulation, where the flows of the commodities are aggregated into a single arc flow. The new formulation has much reduced complexity and is useful for finding numerical solutions quickly. However, it is less generalizable.

### Aggregate-Traffic SSM

\[
\text{max} \quad Z \quad \text{subject to} \quad \sum_{j \in N(i)} x_{ij} - \sum_{k: i \in N(k)} x_{ki} = \begin{cases} d_i, & \text{if } i = c, \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in N \quad (7)
\]

\[
\left( \sum_{j \in N(i)} \sum_{c: i \in N(k)} C_{ij} x_{ij} + \sum_{k: i \in N(k)} \gamma : x_{ki} \right) \cdot Z \leq E_i \quad \forall i \in N \quad (8)
\]

\[
x_{ij} \geq 0, \forall i \in N; \forall j \in N(i) \quad (9)
\]

\[
Z \geq 0. \quad (10)
\]

The equivalence is only true for the particular constraints considered here. The two formulations are not usually equivalent in more general settings, for instance, if the costs of the commodities (energy per unit of data transmitted or received) are different, or if some individual commodity rate at some link is upper bounded or lower bounded by a non-zero value, which in turn might be the result of assigning different importance levels to different commodities. The per-commodity formulation (5) is more generalizable. The aggregate-traffic formulation (11) is not necessarily useful if one wishes to incorporate more constraints. But it is useful in this paper because it is easier to compute.

### 2.2 Mobile Sink Model

In the mobile sink model (MSM), we assume that the sink can move around within the sensor field and stop at certain locations to gather the data from the sensor nodes. Let \(L\) be the set of possible locations where the sink can stop (also known as sink stops). The sink does not necessarily stop at (i.e., stays for a positive duration) all locations in \(L\) in the interest of maximizing the network lifetime \([1, 9]\).

As previous authors \([9]\), throughout the paper, we make the assumption that the traveling time of the sink between locations is negligible. This way, the resulting problem formulations are simple enough for us to obtain precise numerical solutions for evaluation purpose. The assumption is appropriate when the traveling time is much smaller than the time spent by the sink to collect data in each location.

In this model, the order of visit to the stops has no effect on the network lifetime and can be arbitrary. The sink sojourn time at a location \(l \in L\) is denoted by \(z_l\); it is the time that the sink spends at \(l\) to collect data from the sensor nodes. The overall network lifetime is \(Z = \sum_{l \in L} z_l\). When the sink is at stop \(l\), we denote the (downstream) neighbors of node \(i\) as

\[
N(i, l) = \{ j \in N \cup \{l\} | d(i, j) \leq \bar{d}, j \neq i \}. \quad (17)
\]

To find the optimal network lifetime, we need to consider the routing of the traffic as well as the duration of the sink’s sojourn time at each stop (also see \([9, 2, 1, 7]\)).

Similar to the case of the static sink model in Section 2.1, there is a per-commodity-based formulation of the lifetime-maximization problem, and there is an equivalent, simpler, aggregate-traffic-based formulation. For brevity, we will only present the latter. However, we re-iterate that, if additional constraints are present, the
Aggregated-Traffic Mobile Sink Model (MSM)  

\[
\text{max } Z = z_1 + z_2 + \cdots + z_{|L|}
\]

\[
\text{s. t. } \sum_{j \in N(i,l)} x_{ij}^{(l)} - \sum_{k \in N(k,l)} x_{ki}^{(l)} = d_i, \quad \forall i \in N; \forall l \in L (20)
\]

\[
\sum_{l=1}^{|L|} z_l \left( \sum_{j \in N(i,l)} C_{ij}^{l} x_{ij}^{(l)} + \sum_{k \in N(k,l)} \gamma x_{ki}^{(l)} \right) \leq E_i, \quad \forall i \in N (21)
\]

\[
x_{ij}^{(l)} \geq 0, \quad \forall i \in N; \forall l \in L; \forall j \in N(i,l) (22)
\]

\[
z_l \geq 0, \quad \forall l \in L. (23)
\]

Constraint (20) denotes the flow conservation for all nodes when the sink is at \(l\). Constraint (21) says that the total energy consumed at the node \(i\) can not exceed the initial energy \(E_i\). By multiplying (20) with \(z_l\) and substituting \(x_{ij}^{(l)} \cdot z_l\) with a new variable \(y_{ij}^{(l)}\), we can replace (20) with the following new constraint.

\[
\sum_{j \in N(i,l)} y_{ij}^{(l)} - \sum_{k \in N(k,l)} y_{ki}^{(l)} = z_l \cdot d_i, \quad \forall i \in N; \forall l \in L. (24)
\]

Similarly, constraint (21) can be changed into

\[
\sum_{l=1}^{|L|} \left( \sum_{j \in N(i,l)} C_{ij}^{l} \cdot y_{ij}^{(l)} + \sum_{k \in N(k,l)} \gamma \cdot y_{ki}^{(l)} \right) \leq E_i, \quad \forall i \in N. (25)
\]

With the constraints (24), (25), and the non-negativity constraints for \(y_{ij}^{(l)}\), the above optimization problem is converted into an LP problem. Here, \(y_{ij}^{(l)}\) is interpreted as the total traffic volume that node \(i\) sends to node \(j\) while the sink stays at \(l\).

**3 Lifetime Maximization in Delay Tolerant Mobile Sink Model**

In this section, we consider how to maximize the lifetime of WSN with the mobile sink in applications that can tolerate a certain amount of delay. We call the resulting WSN model delay tolerant mobile sink model (DT-MSM). In this setting, each node can postpone the transmission of data until the sink is at the stop most favorable for extending the network lifetime. This way, the nodes can collectively achieve a longer network lifetime. In contrast, the SSM and MSM do not exploit this possibility.

Let \(D\) be the maximum tolerable delay, or the delay tolerance level. We assume that the sink finishes one round of visit to all the stops (where the sink stays for a positive duration to collect data) in \(D\) time units, and then, repeats with another round again and again. Note that two consecutive visits to the same stop takes a time \(D\).

Let’s take an example to show how our framework can outperform other ones. Consider the two-node example shown in Figure 1. \(N_1\) and \(N_2\) are two sensor nodes and \(L_1\) and \(L_2\) are the candidate stops of the mobile sink. Suppose we ignore the receiving energy requirement and suppose the transmission energy per unit of data is equal to the square of the distance between the sender and the receiver. Both nodes \(N_1\) and \(N_2\) generate data at 1 bps and have 100 units of energy initially. If the sink is located at \(O\) in the SSM, both nodes spend 4 units of energy for sending a bit of data. It is obvious that the optimal lifetime is 25 seconds. In the MSM with sink locations \(\{L_1, L_2\}\), due to the symmetry of the structure, the sink stays at both \(L_1\) and \(L_2\) for the same amount of time to achieve the maximum lifetime. Each node spends 1 or 9 units of energy for sending 1 bit of data depending whether the sink is at \(L_1\) or \(L_2\). The average energy consumption per bit is 5 units. Thus, the lifetime is 20 seconds. In the DT-MSM, we assume that the sink alternates between the two stops and stays for 1 second at each stop in each cycle. Hence, this is the case that \(D = 2\) seconds. When the sink stays at \(L_1\), only \(N_1\) sends 2 bits of data to the sink; when the sink moves to \(L_2\), only \(N_2\) transmits 2 bits of data (\(N_2\) keeps its data while the sink is at \(L_1\)). Both nodes spend 2 units of energy every 2 seconds or 1 unit of energy per second on average. Thus, the lifetime is 100 seconds, a significant increase compared to the SSM and MSM. This is because, in the DT-MSM, the nodes do not always participate in communication for all the sink stops; they each wait until the sink’s location is most favorable for energy saving, and then send data at the higher rate. Recall that we have assumed that the traffic rate is sufficiently small compared to the capacity of the wireless link, and hence, sending data at a higher rate does not alter the per-bit energy consumption.

Unlike the MSM or SSM, the sink in the DT-MSM can collect data from only a subset of the set of all
sensor nodes, $N$, at each stop. Let $R_l$ be the subset of $N$ such that only nodes in $R_l$ can participate in the communication toward the sink when the sink is at $l \in L$. We call $R_l$ the coverage of the sink location $l$. Note that the union of $R_l$ over $l \in L$ must be the set of all sensor nodes, $N$. In other words, any sensor node should be covered by at least one sink location. When the node $i$ is in $R_l$, node $i$ is said to be active at $l \in L$. Although we can construct $R_l$ in many ways depending on the application of interest, in this paper, a very simple method of constructing $R_l$ is considered. Fix a positive number $r$. We call $r$ the radius of coverage of the sink. For each $l \in L$, if $d(i, l) \leq r$, where $i \in N$, then $i \in R_l$. Here, the radius of coverage of the sink ($r$) should be large enough so that every sensor node belongs to at least one $R_l$. Note that the minimum $r$ depends on the locations of the sink stops.

**Remark:** In our problem formulations, we take $R_l$ as parameters, although we will show that the larger $R_l$, the longer the network lifetime is. However, when $R_l$ is small, the routing and rate allocation decisions are confined within a small area around the current sink location. This is likely to reduce the protocol complexity for handling routing and rate allocation. In the simplest case, the coverage may be within the broadcast range of the sink so that routing and rate allocation can be done locally with the help of a broadcast-based control protocol. By incorporating the coverage in our formulations, we can evaluate the tradeoff between the lifetime performance and protocol complexity, e.g., how much the loss in lifetime is when we reduce $R_l$.

In both SSM and MSM, the sink collects data from each node $i$ at the same rate at which node $i$ generates the data. However, in the DT-MSM, the data transmission rate at node $i$ during the collection time is no longer the same as the constant data generation rate $d_i$. When node $i$ is not active (i.e., not covered by the current sink location), it continues to gather data and should store the newly generated data. Hence, data buffering is required by our framework. Within a cycle of $D$ time units, the total stored data at each node $i$ is at most $D \cdot d_i$. For ease of presentation, we assume the sink visits all locations in $L$ in the order of $1 \rightarrow 2 \rightarrow \cdots |L| \rightarrow 1 \cdots$. The sink may stay at some location for zero time. With slight abuse of terminology, we define the network lifetime $T$ to be the number of cycles made by the sink until the first node dies due to energy exhaustion. The actual lifetime is $T \cdot D$.

Once traffic is allowed to be buffered, there are different strategies on whose traffic is buffered. Which strategy gets adopted in practice probably depends on the application, other practical concerns, and the designer’s preference. Since we do not know these factors in advance, we next describe two strategies, or two variants of the model: the sub-flow-based model and the queue-based model. The main purpose is to illustrate that choices exist and they lead to different performance-complexity tradeoffs.

### 3.1 Sub-Flow-Based Model

In the sub-flow-based model, the nodes in the current coverage $R_l$ are not allowed to buffer the relayed traffic from other nodes; as soon as a node in $R_l$ receives the data from other nodes, it immediately forwards the data to its neighboring nodes. To model this constraint at each node $i$, we need to differentiate the data generated by node $i$ itself and the data originally generated by other nodes but forwarded to node $i$. Again, let $x_{ij}^{(c,l)}$ be the rate assignment from node $i$ to node $j$, while the sink is at $l$, for the traffic generated by node $c$ (commodity $c$). Let $x_{ij}^{(l)}$ be the aggregated rate of traffic that needs to be forwarded to node $j$ from node $i$ when the sink is at $l$. That is,

$$x_{ij}^{(l)} = \sum_{c \in R_l} x_{ij}^{(c,l)}, \quad \forall l \in L; \forall i, \forall j \in N_i(i). \quad (26)$$

Here, we define $N_i(i) = R_l \cap N(i, l)$, where $N(i, l)$ is as given in (17). It is the set of the downstream neighbors of node $i$ that are in the coverage $R_l$.

Since at node $i \in N$, the commodity or sub-flow of other nodes $c \in R_l$, $c \neq i$ must be forwarded as soon as it has been received, we should have

$$\sum_{k : i \in N_i(k)} x_{ki}^{(c,l)} = \sum_{j \in N_i(i)} x_{ij}^{(c,l)}. \quad (27)$$

The flow conservation at node $i$ can be expressed as follows, which is the same as in the MSM except that the amounts of traffic originated from node $i$, $w_{ij}^{(l)}; l \in L; i \in R_l$, are now decision variables.

$$z_l \left( \sum_{j \in N_i(i)} x_{ij}^{(l)} - \sum_{k : i \in N_i(k)} x_{ki}^{(l)} \right) = w_{ij}^{(l)}. \quad (28)$$

The data buffered during the previous sink-movement cycle must be cleared in the current cycle. This requirement can be written as

$$\sum_{l : i \in R_l} w_{ij}^{(l)} = D \cdot d_i. \quad (29)$$

Due to (27), it may appear that we need a per-commodity-based formulation of the problem. Similar to the case of the SSM problem (5) in Section 2.1, there is also a simpler, equivalent, aggregate-traffic formulation, using only the aggregate arc flow variables $x_{ij}^{(l)}$, as given in (26).
Aggregate-Traffic Sub-Flow-Based DT-MSM

\[ \begin{align*}
\text{max} \quad & T \\
\text{s. t.} \quad & z_l \left( \sum_{j \in N_i(l)} x_{ij}^{(l)} - \sum_{k \in N_i(k)} x_{ki}^{(l)} \right) = w_i^{(l)}, \\
& \sum_{l=1}^{\left| L \right|} z_l \left( \sum_{j \in N_i(i)} C_{ij} \cdot x_{ij}^{(l)} \right) + \sum_{k \in N_i(k)} \gamma \cdot x_{ki}^{(l)} \right) \left. \right|_{l \in \mathcal{L}; \forall i \in R_l} = T \leq E_i, \quad \forall i \in \mathcal{N} \\
& \sum_{l=1}^{\left| L \right|} w_i^{(l)} = D \cdot d_i, \quad \forall i \in \mathcal{N} \\
& x_{ij}^{(l)} \geq 0, \quad \forall l \in \mathcal{L}; \forall i \in R_l; \forall j \in N_i(i) \\
& w_i^{(l)} \geq 0, \quad \forall l \in \mathcal{L}, \forall i \in R_l \\
& z_l \geq 0, \quad \forall l \in \mathcal{L} \\
& T \geq 0.
\end{align*} \] (30) (31) (32) (33) (34) (35) (36) (37) (38)

3.2 Queue-Based Model

In the queue-based model, each sensor node can buffer data originated from any node. Let \( q_i^{(l)} \) be the queue length at node \( i \) just before the sink moves from location \( l \) to \( l+1 \). Assume that each node \( i \) has \( D \cdot d_i \) amount of data at the beginning of a cycle, which is denoted by \( q_i^{(0)} \). When the sink finishes a cycle of visit, the queue at node \( i \) must be cleared. Thus we have \( q_i^{(|\mathcal{L}|)} = 0 \). In this model, the flow conservation constraint is replaced by the queue length dynamics, which is expressed as follows.

\[ z_l \left( \sum_{k \in N_i(k)} x_{kj}^{(l)} \right) + q_i^{(l-1)} - z_l \left( \sum_{j \in N_i(i)} x_{ij}^{(l)} \right) = q_i^{(l)}, \quad \forall l \in \mathcal{L}; \forall i \in \mathcal{N}. \] (39)

The energy constraints can be expressed in the same way as in the sub-flow based MSM. From the above discussion, we have the following optimization problem for maximizing the lifetime.

**Queue-Based DT-MSM**

\[ \begin{align*}
\text{max} \quad & T \\
\text{s. t.} \quad & z_l \left( \sum_{j \in N_i(k)} x_{kj}^{(l)} \right) + q_i^{(l-1)} - z_l \left( \sum_{j \in N_i(i)} x_{ij}^{(l)} \right) = q_i^{(l)}, \\
& \forall l \in \mathcal{L}; \forall i \in \mathcal{N} \\
& \sum_{l=1}^{\left| \mathcal{L} \right|} z_l \left( \sum_{j \in N_i(i)} C_{ij} \cdot x_{ij}^{(l)} \right) + \sum_{k \in N_i(k)} \gamma \cdot x_{ki}^{(l)} \right) \left. \right|_{l \in \mathcal{L}; \forall i \in \mathcal{N}} = T \leq E_i, \quad \forall i \in \mathcal{N} \\
& \sum_{l=1}^{\left| \mathcal{L} \right|} w_i^{(l)} = D \cdot d_i, \quad \forall i \in \mathcal{N} \\
& q_i^{(0)} = D \cdot d_i, \quad \forall i \in \mathcal{N} \\
& q_i^{(\left| \mathcal{L} \right|)} = 0, \quad \forall i \in \mathcal{N} \\
& x_{ij}^{(l)} \geq 0, \quad \forall l \in \mathcal{L}; \forall i \in R_l; \forall j \in N_i(i) \\
& q_i^{(l)} \geq 0, \quad \forall l \in \mathcal{L}; \forall i \in \mathcal{N} \\
& z_l \geq 0, \quad \forall l \in \mathcal{L} \\
& T \geq 0.
\end{align*} \] (40) (41) (42) (43) (44) (45) (46) (47) (48) (49)

The problem shown above can be converted into an LP problem by substituting \( y_{ij}^{(l)} \) for \( z_l \cdot x_{ij}^{(l)} \) and introducing the new variable \( u = 1/T \). This linearization method can also be applied to the sub-flow based MSM. Once \( y_{kj}^{(l)} \) is solved, we can assign an arbitrary positive value to \( z_l \), as long as \( \sum_{l=1}^{\left| \mathcal{L} \right|} z_l \leq D \), and assign \( x_{ij}^{(l)} = y_{kj}^{(l)}/z_l \). The root reason that we can do this is that there is no upper bound on the link rate \( x_{ij}^{(l)} \). If such an upper bound were in our formulations, the situation would be very different and the problem would become very hard.

**Discussion on the two delay-tolerant models:**

- The two delay-tolerance formulations represent two strategies on what data to buffer. The sub-flow-based formulation allows buffering of only self-generated traffic; the queue-based formulation allows buffering of any traffic, which naturally leads to the best lifetime performance among different strategies. The two models can be considered as two “extreme cases”, and various intermediate strategies can be similarly formulated.
- In the sub-flow-based formulation, the maximum required buffer size at node \( i \) is \( Dd_i \). In the queue-based case, the maximum buffer size at a node may depend on the total number of other nodes in the same coverage area, which can be much larger.
- The sub-flow-based formulation looks more similar to the standard multi-commodity flow problem. It can be easier to find fast, specialized algorithms to solve this problem.

### 3.3 Properties of Delay-Tolerant Mobile Sink Model

Both delay-tolerant models include the coverage of each sink location in the formulation. This is motivated by practical concerns, in particular, how easy it is to design
practical protocols for coordinating the communication. When at a sink location, it is far easier for the sink to coordinate with the nearby sensors and set up the data collection process. Hence, a small radius of coverage is preferable from the protocol complexity point of view. However, the radius of coverage can affect the network lifetime, which we will explore next.

For illustration, consider the optimization problem for the sub-flow-based model. Depending on the radius of coverage, we may obtain different instances of the optimization problem. Thus, we can parameterize these instances according to the radius of coverage. Let $P(N, L, r)$ be the optimization problem when the radius of coverage of the sink is $r$, the set of sensor nodes is $N$, and the set of sink locations is $L$. The value $r$ must be large enough so that all sensor nodes can be covered by at least one sink location and we denote this minimum radius of coverage for connectivity by $r_0$. Under the same configuration with $N$ and $L$, different $r$ values only affect $R_l$ and $N_l(i)$. We will use the notations $R_l(r)$ and $N_l(i, r)$ if it is necessary to specify the radius of coverage. In the next theorem, we prove that the bigger the radius of coverage, the longer the optimal lifetime is.

**Theorem 1.** If $r_0 \leq r_1 < r_2$, then the optimal objective value for the problem $P(N, L, r_2)$ is greater than or equal to that for the problem $P(N, L, r_1)$.

**Proof:** Consider the two optimization problems $P(N, L, r_1)$ and $P(N, L, r_2)$ with $r_1 < r_2$. It is obvious that $N_l(i, r_1) \subseteq N_l(i, r_2)$ for all $i \in N, l \in L$. Therefore, we can split the larger set $N_l(i, r_2)$ into two sets $A$ and $\bar{A}$, where $A = N_l(i, r_1)$ and $\bar{A} = N_l(i, r_2) \setminus A$. Similarly, we can also split the upstream neighbor set for node $i$ into $B = \{k \in N_i|i \in N_l(k, r_1)\}$ and $\bar{B} = \{k \in N_i|i \in N_l(k, r_2)\} \setminus B$. In other words, $A$ and $\bar{B}$ are the sets of additional downstream and upstream neighbors for node $i$, respectively, as the radius of coverage increases from $r_1$ to $r_2$.

Suppose that $(\hat{x}, \hat{w}, \hat{z}, \hat{T})$ is a feasible solution to the problem $P(N, L, r_1)$. Now, consider equation (32) for the optimization problem $P(N, L, r_2)$. For all $i \in L, \forall i \in R_l(r_2)$,

$$z_l \left( \sum_{j \in N_l(i, r_2)} x_{ij}^{(l)} - \sum_{k : i \in N_l(k, r_2)} x_{ki}^{(l)} \right) = w_i^{(l)}. \quad (50)$$

We have the following by separating the neighbor sets into $A, \bar{A}, B$, and $\bar{B}$.

$$z_l \left( \sum_{j \in A} x_{ij}^{(l)} + \sum_{j \in \bar{A}} x_{ij}^{(l)} - \sum_{k : i \in N_l(k, r_2)} x_{ki}^{(l)} - \sum_{k \in B} x_{ki}^{(l)} \right) = w_i^{(l)}. \quad (51)$$

Fix $l \in L$. Suppose $i \in R_l(r_1)$. We extend the vector $\hat{x}$ so that $\hat{x}_{ij}^{(l)} = 0$ when $j \in A$ and $\hat{x}_{ij}^{(l)} = 0$ when $k \in B$. Then, for such $l$ and $i$, the extended vector $(\hat{x}, \hat{w}, \hat{z}, \hat{T})$ satisfies (51) since the original vector satisfies (32) for $i \in R_l(r_1)$.

Next suppose $i \in R_l(r_2) \setminus R_l(r_1)$. Then, we can extend the vector $\hat{x}$ further by setting $\hat{x}_{ij}^{(l)} = 0$ for all $j \in N_l(i, r_2)$ and $\hat{x}_{ij}^{(l)} = 0$ for all $k$ such that $i \in N_l(k, r_2)$. Furthermore, we extend $\hat{w}$ by setting $w_i^{(l)} = 0$. After such extension, $(\hat{x}, \hat{w}, \hat{z}, \hat{T})$ satisfies (51) for $i \in R_l(r_2) \setminus R_l(r_1)$ (since all terms are zero).

For the energy constraint (33), we can apply a similar procedure. Hence, we can conclude that any feasible solution to the problem $P(N, L, r_1)$, after suitable extension, is also a feasible solution to the problem $P(N, L, r_2)$.

Next, the queue-based model is less constraining than the sub-flow-based model; this results in lifetime gains in the former model. The following theorem formalizes the fact that the queue-based model always outperforms the sub-flow-based model.

**Theorem 2.** Let $\hat{T}$ be the optimal objective value to problem (30), and $T^*$ be the optimal objective value to problem (40) with the same configuration $(N, L)$ and the same radius of coverage $r$. Then $\hat{T} \leq T^*$.

**Proof:** Let $((\hat{x}_i^{(l)}), (\hat{w}_i^{(l)}), (\hat{z}_k), \hat{T})$ be the optimal solution to problem (30). We will prove this theorem by constructing a feasible solution to problem (40) with $((\hat{x}_i^{(l)}), (\hat{w}_i^{(l)}), (\hat{z}_k), \hat{T})$ and showing that under this feasible solution, the objective value of problem (40) is $\hat{T}$.

We now define a vector $w$ as follows. For each $l \in L$, we let $w_i^{(l)} = \hat{w}_i^{(l)}$ if $i \in R_l$, and $w_i^{(l)} = 0$ otherwise. Then, we let $q_i^{(0)} = D \cdot d_i$, and $q_i^{(l)} = q_i^{(l-1)} - w_i^{(l)}$ for all $i \in N$. We have the following sequence of assignments for the $q_i^{(l)}$:

$$q_i^{(1)} = q_i^{(0)} - w_i^{(1)} = D \cdot d_i - w_i^{(1)}$$
$$q_i^{(2)} = q_i^{(1)} - w_i^{(2)}$$
$$\vdots$$
$$q_i^{(l)} = q_i^{(l-1)} - w_i^{(l)}$$

By summing up above assignments for all $l \in L$, we have $q_i^{(l)} = D \cdot d_i - \sum_l w_i^{(l)} = D \cdot d_i - D \cdot d_i = 0$ by (34) and the construction of $w$. Hence, (45) is satisfied. Since the configuration and radius of coverage $r$ for problem (30) are the same as those for problem (40), $N_l(i), i \in N, l \in L$ are the same for both problems. Because of this and by (32) and $w_i^{(l)} = q_i^{(l-1)} - q_i^{(l)}$, (42) is satisfied. The energy constraints (33) and (43) are identical. Hence, given the optimal solution $((\hat{x}_i^{(l)}), (\hat{w}_i^{(l)}), (\hat{z}_k), \hat{T})$ to problem (30), we just constructed a feasible solution $((\hat{x}_i^{(l)}), (q_i^{(l)}), (\hat{z}_k), \hat{T})$ to problem (40) with the same objective value $\hat{T}$. Hence, $T^* \geq \hat{T}$.

In the following theorem, we show that the maximum lifetime of the system is the same for all values of $D$. Here, the maximum lifetime of the system is equal to the product of $D$ and the corresponding optimal objective value $T^*(D)$.

**Theorem 3.** Define $P(D)$ as the lifetime optimization problem parameterized by the value $D$, for some fixed network configuration. Let $T^*(D)$ and $T^*(D')$ be the optimal objective
values for the problem \( P(D) \) and \( P(D') \), respectively. Then, 
\[ T^*(D) \cdot D = T^*(D') \cdot D'. \]

**Proof:** Consider the queue-based model. Let \( (x^*(D), q^*(D), z^*(D), T^*(D)) \) be the optimal solution to the problem \( P(D) \), and let \( (x^*(D'), q^*(D'), z^*(D'), T^*(D')) \) be the optimal solution to the problem \( P(D') \).

Let \( x = \left( \frac{D'}{D} \right) x^*(D'), q = \left( \frac{D'}{D} \right) q^*(D'), z = z^*(D'), T = \left( \frac{D'}{D} \right) T^*(D') \). We want to show that \((x, q, z, T)\) satisfies the constraints (42)-(49). Since it is obvious that the solution \((x, q, z, T)\) satisfies the constraints (46), (47), (45), (48), and (49), we focus here on constraints (42), (43), and (44) only. Since the optimal solution \((x^*(D'), q^*(D'), z^*(D'), T^*(D'))\) is feasible to the problem \( P(D') \), it must satisfy constraint (42). Next, let us plug \( \left( \frac{D'}{D} \right) x, z, \) and \( \left( \frac{D'}{D} \right) q \) into constraint (42) in the places for \( x^*(D'), z^*(D'), \) and \( q^*(D') \), respectively. Then, we have

\[
z_i \sum_{k \in N_i(k)} x_{ki}^{(l)} + q_{ij}^{(l-1)} - z_i \sum_{j \in N_i(i)} x_{ij}^{(l)} = q_i^{(l)}. \tag{52}
\]

If we put \( \left( \frac{D'}{D} \right) x, z, \) and \( \left( \frac{D'}{D} \right) T \) in the places for \( x^*(D'), z^*(D') \) and \( T^*(D') \) on the left hand side of constraint (43), we have

\[
\left\{ \sum_{i=1}^{\mid L \mid} z_i \left( \sum_{j \in N_i(i)} C_{ij}^l \cdot x_{ij}^{(l)} \left( \frac{D'}{D} \right) \right) + \sum_{k \in \mathit{N}_k(k)} \gamma \cdot x_{ki}^{(l)} \left( \frac{D'}{D} \right) \right\} \cdot T \left( \frac{D'}{D} \right).
\tag{53}
\]

After canceling \( D \) and \( D' \), it is easy to see that the new solution \((x, q, z, T)\) satisfies the energy constraint of the problem \( P(D) \).

From the constraint (44) for the problem \( P(D') \), we have \( q_i^{(0)}(D') = D'd_i \). Since \( q = \left( \frac{D'}{D} \right) q^*(D') \), \( q_i^{(0)}(D') = \left( \frac{D'}{D} \right) q_i^{(0)}(D) = D'd_i (\forall i \in L) \). Therefore we have

\[
q_i^{(0)} = Dd_i. \tag{54}
\]

From above argument, we have shown that new solution \((x, q, z, T)\) is feasible to the problem \( P(D) \). Hence, we have

\[ T^*(D) \geq T = \left( \frac{D'}{D} \right) T^*(D'). \]

Thus, it must be that \( T^*(D)D \geq T^*(D')D' \).

Using a similar argument, we can also conclude that \( T^*(D)D \leq T^*(D')D' \). Hence, \( T^*(D)D \) must equal to \( T^*(D')D' \).

4 **Experimental Results**

In this section, we will present the results from numerical experiments. In particular we have compared the network lifetimes of the following models.

- **Static Sink Model (SSM):** The stationary sink is located at the origin. We take the performance of this model as the reference for comparison.
- **Mobile Sink Model (MSM):** The sink can move to several locations to collect data. When the sink is at each location, all sensors participate in the communication, sending and relaying traffic to the sink.
- **Delay-Tolerant Mobile Sink Model (DT-MSM):** When the mobile sink is at a stop, a subset of the sensor nodes can participate in the communication.

We use the queue-based variant of this model to evaluate the performance.

We have experimented with different parameters extensively, such as the number of nodes, the number of possible sink locations and the parameters for the energy consumption model. Only a small subset of the results are reported here for brevity. In Table 1, we provide the system parameters and their values for the reported experiments in this paper. We adopt the data for the last four parameters from [21]. In all experiments, we use GLPK for solving the linear programming problem.

We first would like to mention the impact of the radius of coverage of the sink on the performance of the DT-MSM. For this experiment, the positions for 100 nodes and 20 mobile-sink locations are randomly generated \(|L| = 100, |N| = 20\) in a circular area with radius 25 meters. We use a simple algorithm to find the minimum radius of coverage (denoted by \( r_0 \)). At each sink location, we increase the radius of coverage from 0

<table>
<thead>
<tr>
<th># of Experimental Parameters and Their Values</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td># of sensor nodes</td>
<td>100, 200</td>
</tr>
<tr>
<td># of possible sink locations</td>
<td>{5, 6, 7, 8, 9, 10, 15, 20, 30, 40}</td>
</tr>
<tr>
<td>path loss exponent (e)</td>
<td>{2.0, 3.0}</td>
</tr>
<tr>
<td>transmission range</td>
<td>{5, 6, 7, 8, 9, 10, 15, 20, 30, 40, 50} m</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>50 nJ/bit</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.0013 pJ/bit/(m², m³)</td>
</tr>
<tr>
<td>Initial Energy ( (E_i) )</td>
<td>500 J</td>
</tr>
<tr>
<td>Data generation rate ( (d_i) )</td>
<td>500 bps</td>
</tr>
</tbody>
</table>

Fig. 2. Comparison of lifetimes of MSM and DT-MSM under the various radii of coverage.

---

3. Note that the proof can be adapted to the sub-flow-based model.

4. The choice of unit for \( \beta \) depends on the path loss exponent \( (e) \) used in the simulation.
simultaneously until the union of all coverages contains all sensor nodes. At that point, we have reached the minimum radius of coverage required to cover all nodes. After that, we increase the radius of coverage in 0.1 increments. The result of the experiment is plotted in Figure 2. Note that, in the figure, the lifetime is normalized with respect to the optimal lifetime of the MSM. As shown in the figure, the lifetime of the DT-MSM increases as the radius of coverage increases, which is consistent with Theorem 1. The increase is the sharpest when the radius just exceeds the minimum radius required to cover all nodes. After that, further increase of the radius has a negligible effect. Recall that, when the mobile sink reaches one of the stops, say $l$, only those sensor nodes in the coverage of $l$ (i.e., $R_l$) can communicate. It is generally desirable for $R_l$ to have as few nodes as possible, since this reduces the communication and coordination complexity. The aforementioned behavior of lifetime increase is desirable.

Next, we compare the lifetimes of models under various numbers of the sink locations. The number of nodes is set to 100 or 200, and the path loss exponent $e$ is 2.0. The coverage is set large enough to always cover the entire sensor field. We ran the experiment 100 times for each configuration. The lifetimes of the MSM and DT-MSM are again normalized with respect to the optimal lifetime of the SSM. As shown in Figure 3, the lifetime of the MSM is about $100\% \sim 200\%$ greater than that of the SSM. However, the DT-MSM is $200\% \sim 1000\%$ better than the SSM. Moreover, the curves all look linear; the performance gap can grow even larger with more sink locations.

Interestingly, the lifetime of the MSM increases very slowly with the number of sink locations. As explained in [9], in the optimal solution, only a few locations from the set of sink locations are chosen as the true stops for the sink. However, in DT-MSM, the rate of lifetime increase is substantial as $|L|$ increases. This is because each node can have better and better sink location as $|L|$ increases, and it is not forced to participate in the communication when the current coverage is not the most favorable for energy saving, even if the node may belong to that coverage. This is not possible in the MSM because no matter where the sink stops, every node must participate in the communication.

We wish to make the following remarks. First, our formulations and reported experiments all use the optimal routing with respect to maximizing the system lifetime. The routing strategy is important for increasing the system lifetime. For instance, based on our experiences, when the shortest path routing is used in the static sink model (results not shown), the lifetime performance is quite inferior to the case of optimal routing. Second, in our model, the locations of the sink stop candidates are randomly chosen. We expect more performance gain if these candidate locations are carefully selected.
We conduct similar experiments with the same configuration but minimum coverage. The result is shown in Figure 4. Although the slope of lifetime increase of the DT-MSM is lowered when compared to the maximum coverage case, the increase pattern is similar. Although a larger set of sink locations increases the network lifetime, it can be undesirable if the sink-traveling time cannot be ignored. The longer traveling time may exceed the delay tolerance level $D$. Therefore, there is a tradeoff between the gain from more sink locations and the delay or other system costs.

In Figure 5, we show the lifetimes of the three models under various values for the transmission range. The transmission range determines whether a link exists between a pair of nodes. Whether an existing link is useful or not depends on the radius of coverage. A node cannot use a link to another node if the two nodes are not in the common coverage area.

Both the MSM and the DT-MSM exhibit a sharp lifetime increase when the transmission range is small but increasing. However, as the transmission range becomes large, the lifetime increase comes to a stop for all three models. This is because the energy cost increases with the transmission distance, and hence, in an optimal solution, a node does not pick far-away nodes as the next-hop neighbors even if the transmission range allows it. The observed fluctuation in the curves is due to statistical fluctuation in the samples of the random network topologies.

5 CONCLUSION AND FUTURE WORK

In this paper, we proposed a new framework for improving the network lifetime by exploiting sink mobility and delay tolerance. It is expected to be useful in applications that can tolerate a certain amount of delay in data delivery. We presented the mathematical formulations for optimizing the network lifetime under the proposed framework. We identified several properties that our models possess. To validate the framework’s ability for improving network lifetime, we conducted extensive experiments and found that the framework is superior to the models published previously, including the static sink model and the mobile sink model without delay tolerance. The lifetime gain of the proposed model is significant when compared to the previous models. Furthermore, as the number of sink locations increases, the optimal network lifetime increases substantially. The results of the paper can both be applied to practical situations and be used as benchmarks for studying energy-efficient network designs.

We can point out three interesting future work directions. The paper has not touched upon the issue of finding efficient algorithms to solve the optimization problems, but has relied on standard, centralized algorithms. The first direction is to find simpler, preferably distributed algorithms, which are clearly more generally applicable. The goal is likely to be attainable since the problems formulated in this paper are extension of the network-flow problems and many efficient algorithms are known for such problems. The second direction is to relax some of the simplifying assumptions of the formulations. For instance, we can bring the non-zero traveling time by the sink and/or the finite link transmission rate into the formulations. Either one seems to make the problems very difficult, but more relevant at the same time. The third direction is to consider where to choose for sink stops so that the network lifetime can be optimized.

REFERENCES


