COT 6315/CIS 4930 (Fall05,Sitharam)

Sample Test 3 and rough solutions – for detailed solutions, especially proofs, visit TA's office hours. Note also that this sample test does not cover stuff like hierarchy theorems, and general containments between deterministic and nondeterministic time classes – they are however required syllabus for the actual test

1. Classify the following languages as being: both r.e. and co-r.e; (co-)r.e but not enough information to tell where its complement is; r.e but not co-r.e; co-r.e, but not r.e; or neither r.e nor co-r.e. In all cases, fully justify your answer (the concept of reductions would be useful in some cases).

   NOTE 1: If the *question* concerns an entire class of languages G, your upper bound C should hold for the entire class G; for the lower bound D, give the largest D that does not contain *at least one member* of the class G. I.e., you are lowerbounding the complexity of the most difficult set in the class G.

   (a) Let $f : N \to N$ be a strictly increasing, computable function (i.e, there is an algorithm that computes it). Let $g : N \to N$ be any computable function.

   - Classify the problem of deciding emptiness of certain level sets of g. I.e, consider the set $\{(x, z) \in N \times N : \forall y$, if $f(y) \leq z$ then $g(y) \neq x\}$ (*Answer*: decidable, which is the same as saying: both r.e and co-r.e; need to check only finitely many y's; recall that f is an increasing function)
   - Consider the above set with the condition $f(y) \leq z$ removed. (*Answer*: co-r.e but not r.e; to show second part, reduce $H\bar{A}LT$ to this problem by a $\leq_m^{rec}$-reduction – we know $H\bar{A}LT$ is not in *r.e* by diagonalization proof given in class – why does this imply this problem is not *r.e*?).

   (b) Let L be a fixed regular language. The set EQUALTO(L) = $\{M$ pseudocode description: $L = L(M)\}$. (*Answer*: undecidable for *any* regular language, could be co-r.e. for some regular languages; for some regular languages, neither r.e. nor co-r.e. If the question said $L \subseteq L(M)$ and exclude $L = \emptyset$, then the set is still undecidable, it could be r.e. for some regular languages, and neither r.e. nor co-r.e. for others. If the question said $L(M) \subseteq L$ and exclude $L = \Sigma^*$, then the set is still undecidable, could be co-r.e. for some regular languages) and neither)

   (c) The set $\{(M_1, M_2, x) : M_1$ halts on x but M_2 does not halt on $x\}$. (*Answer:* neither r.e nor co-r.e – this is implied (why?) by the fact that both $HALT$ and $H\bar{A}LT$ are $\leq_m^{rec}$-reducible (how?) to this set).

2. Assuming P $\neq$ NP and NP $\neq$ co-NP (i.e, assume there is a set in NP, whose complement is not in NP). Classify the following problems as being: in P; NP but not in P; not in NP. Justify your answer in all cases (use the concepts of reduction and completeness).

   (a) The problem of deciding whether a city (set of houses) is unsiegable (cannot be seiged) by k soldiers. More formally, let G be a weighted graph with positive integer weights on the edges. $\{(G, k, t)$: for every set S of atmost k vertices in G, there is some vertex in G that cannot be reached from any of the vertices in S by a path of length atmost $t\}$

   (*Answer:* co-NP-complete under $\leq_m^P$ reductions (this is a complement of the dominating set problem, which inturn is NP-complete by reducing Vertex Cover to it); hence (why?) this problem is co-NP-complete and hence (why?) not in NP).

   (b) The above problem where k is fixed to be 25. (*Answer:* in P).

   (c) SubsetSum = $\{(S \subseteq N$, finite, $t \in N)$: some subset of S sums to $t\}$, when number of bits needed to represent t is bounded by $\log |S|$; when t is bounded by a polynomial in $|S|$; and finally when number of bits needed to represent t is bounded by $|S|$. (*Answer*: in P (simple algorithm exists); in P (simple algorithm exists); NP-complete (reduction in book still works)).