

**COT 6315/CIS 4930 (Spr04,Sitharam)**

**Background Survey test (40 min)**

- Please write your name and ssn legibly on top of your answer sheet.
- The test will carry no points towards your final grade. It is meant to give you feedback on how well you are prepared for this class.
- If you receive less than 65%, you are definitely NOT adequately prepared to take this class. For undergrads, this number is 50%. If you nevertheless take this class, you should expect to spend double the time acquiring the required background on your own.
- The solution to this test will be posted on the web later today, you can look at them to get a rough idea of how you did. The graded tests will be returned at the next class.
- All questions are compulsory. Please **budget your time** so that you spend no more than an average of 1.5 min on each question.
- Use the space provided to write your answers. No explanations necessary for the answers, unless specifically asked for.

1. Consider the sequence  $S = 1, 3, 9, 27, 81 \dots$ . Denote the  $i^{th}$  element of the above sequence as  $S_i$ . Write an expression for  $S_i$ . Your expression should be entirely in terms of  $i$ .
2. Write an expression for  $\sum_{i=1}^n S_i$ . Your expression should be entirely in terms of  $n$ .
3. Consider the sequence in Question 1. For what  $i$  is  $S_i = m$ ? Your expression for this  $i$  should be entirely in terms of  $m$ .
4. What happens to  $(\log n)^4/n$  as  $n$  tends to infinity?
5. What happens to  $2^n/(12n^2)$  as  $n$  tends to infinity?

6. Describe in words succinctly (as best as you can), the following sets.

(i)  $\{2k + 1 : k \text{ integer}\}$

(ii)  $\{5k : k \text{ integer}\}$

7. Describe or draw as best as you can, the following sets. Let  $R$  represent the set of real numbers.

(i)  $\{(x, y) : x, y \in R \ \& \ x + y \geq 0\}$

(ii)  $\{(x, y) : x, y \in R \ \& \ x^2 + y^2 = 1\}$

(iii)  $\{(x, y, z) : x, y, z \in R \ \& \ |x|, |y|, |z| \leq 1\}$

8. Is it possible that  $b * c \pmod{a} = 0$ , but  $a$  does not divide  $b$ , and  $a$  does not divide  $c$ ? Assume that both  $|b|$  and  $|c|$  are atleast are large as  $|a|$ .

9. Answer the previous question for  $b + c$  instead of  $b * c$

10. Write the binary (base 2) representation (bit string) of the decimal number 7, and the ternary (base 3) representation of the decimal number 81.

11. What is the output of following algorithm when the inputs are  $m = 2$  and  $n = 4$ . The algorithm is written in an informal language.

```
Procedure test [n,m: positive integers]
temp:= 1;
For i from 1 to n
    temp = temp*i;
end{for}
```

Output temp\*m;

12. Give the output of the previous algorithm as an expression in  $m$  and  $n$ .

13. What does the following program output when the input array  $A$  is 3,7,2,1,2?

```
i := 1;
curr1 := 10;
curr2 := 10;
While i is less than or equal to the number of elements in A

    if A(i) < curr1
    then curr2 := curr1;
       curr1 := A(i);
    else continue
    end{if}

    i= i+1
end{while}

Output (curr2 - curr1) (i.e, the difference of curr2 and curr1)
```

14. What is the worst case sequential time complexity of sorting  $n$  arbitrary numbers? Briefly justify your answer. You can refer to well-known arguments by name without writing them down. In any case, one or two pointed sentences should be adequate to convey your understanding. (Not only do you need to argue that there exists an algorithm that achieves your stated time complexity, but also that no algorithm can do better than that in some sense).
15. What is the worst case sequential time complexity of the best algorithm you know for finding the minimum spanning tree of a weighted graph?

16. Consider  $N$  the universe of Natural numbers. Let  $C$  be the class of all sets which contain only even numbers or contain only odd numbers. Let  $C$  include the empty set. Is the union of any pair of sets in  $C$  also in  $C$ ? Is the intersection of any pair of sets in  $C$  also in  $C$ ? Is the complement of any set in  $C$  also in  $C$ ? I.e, is  $C$  closed under the operations of intersections, unions and complement respectively?
17. How many subsets are there of the set  $\{1, 2, 3, \dots, n\}$ . Your expression should be in terms of  $n$ .
18. When you toss 5 fair coins at random what is the probability that you will have atleast 3 heads? What is the probability that you will have exactly 3 heads and exactly 2 tails? What is the probability that you will have exactly 3 heads and exactly 2 tails in that sequence?
19. How many different ways can you distribute  $m$  identical pieces of candy to  $n$  distinct kids? You need not distribute equally, you are even allowed to let some kids go without candy.
20. For each of the following 5 pairs, decide whether
- $a$  is a sufficient condition for  $b$
  - $b$  is a sufficient conditions for  $a$
  - both of the above, i.e,  $a \iff b$
  - neither of the above

NOTE:  $B$  is an arbitrary predicate.

| $a$   | $b$   |
|---|---|
| $\forall x, \exists y, B(x, y)$   | $\forall y, \exists x, B(x, y)$                           |
| $\exists x, \forall y, B(x, y)$   | $\forall y, \exists x, B(x, y)$                           |
| $\exists x, \forall y, B(x, y)$   | $\exists x, \forall y, B(y, x)$                           |
| $\exists x, \forall y, B(x, y)$   | $\exists y, \forall x, B(y, x)$                           |
| $\exists x [ \forall y(x \leq y) \wedge \forall z [ (z \neq x) \rightarrow \exists w(z \not\leq w) ] ]$ | $\forall x(w \leq x \wedge z \leq x) \rightarrow (z = w)$ |

(1)

21. Suppose the following sentence is true. “If I slept well, I will pass this test.” What can I conclude logically from the following sentences:  
 (a) I passed the test  
 (b) I failed the test.
22. True or False. Justify your answer.  $\overline{(((\overline{A \cup B})) \cap C) \cap D)} = (A \cap D) \cup ((\overline{C} \cup B) \cap D)$
23. Every student is majoring in computer science or in math or in electrical engineering. There are 30 computer science, 20 mathematics and 25 electrical engineering majors. 10 of these are majoring in 2 or more disciplines. 5 of these are majoring in 3 disciplines. How many students are there?