COT 6315/CIS 4930 (Spr01,Sitharam)

Solution of Background Survey test

1. Consider the sequence S = 1,3, 9, 27, 81 .. Denote the i^{th} element of the above sequence as S_i. Write an expression for S_i. Your expression should be entirely in terms of i.

   Solution: $S_i = 3^{i-1}$

2. Write an expression for $\sum_{i=1}^{n} S_i$. Your expression should be entirely in terms of n.

   Solution: $\sum_{i=1}^{n} S_i = \frac{3^n - 1}{2}$

3. Consider the sequence in Question 1. For what i is $S_i = m$? Your expression for this i should be entirely in terms of m.

   Solution: $i = (\log_3 m) + 1$

4. What happens to $\log^4 n/n$ as n tends to infinity?

   Solution: it goes to 0.

5. What happens to $2^n/(12n^2)$ as n tends to infinity?

   Solution: it goes to ∞.

6. Describe in words succinctly (as best as you can), the following sets.

   (i) $\{(2k + 1 : k\ integer\}$

   Solution: set of odd integers.

   (ii) $\{(5k : k\ integer\}$

   Solution: set of integers divisible by 5.

7. Describe or draw as best as you can, the following sets. Let R represent the set of real numbers.

(i) $\{(x, y) \in R : x, y \in R \& x + y \geq 0\}$

Solution: consider a straight line in 2D through points (-1,1) and (0,0). Then the set consists of all points to the right of this line.

(ii) $\{(x, y) : x, y \in R \& x^2 + y^2 = 1\}$

Solution: all points on 2D unit circle centered at the origin.

(iii) $\{(x, y, z) : x, y, z \in R \& |x|, |y|, |z| \leq 1\}$

Solution: all points within cube in 3D whose vertices are in $\{-1, 1\}^3$.

8. Is it possible that $b * c \pmod{a} = 0$, but a does not divide b, and a does not divide c? Assume that both $|b|$ and $|c|$ are at least are large as $|a|$.

   Solution: yes

9. Answer the previous question for $b + c$ instead of $b * c$

   Solution: yes.

10. Write the binary(base 2) representation (bit string) of the decimal number 7, and the ternary (base 3) representation of the decimal number 81.

    Solution: 111; 10000

11. What is the output of following algorithm when the inputs are $m = 2$ and $n = 4$. The algorithm is written in an informal language.

```
Procedure test [n,m: positive integers]
temp:= 1;
For i from 1 to n
    temp = temp*i;
end{for}

Output temp*m;
```

    Solution: 48

12. Give the output of the previous algorithm as an expression in m and n.

    Solution: m *n!

13. What does the following program output when the input array A is 3,7, 2,1,2?

```
i := 1;
curr1 := 10;
curr2 := 10;
While i is less than or equal to the  number of elements in A

   if A(i) < curr1
   then curr2 := curr1;
        curr1 := A(i);
   else continue
   end{if}

   i= i+1
end{while}

Output (curr2 - curr1) (i.e, the difference of curr2 and curr1)
```

Solution: 1. (difference between the second smallest and the smallest number).

14. What is the worst case sequential time complexity of sorting n arbitrary numbers? Briefly justify your answer. You can refer to well-known arguments by name without writing them down. In any case, one or two pointed sentences should be adequate to convey your understanding. (You need to not only argue that there exists an algorithm that achieves your stated time complexity, but also that no algorithm can do better than that).

    Solution: $\Omega(n \log n)$. There are several (say merge sort) algorithms that sort n arbitrary numbers in $O(n \log n)$ time and height of decision tree of comparison sort algorithm is $\Omega(n \log n)$.

15. What is the worst case sequential time complexity of the best algorithm you know for finding the minimum spanning tree of a weighted graph?

    Solution: $\Omega(E + V \log V)$.

16. Consider N the universe of Natural numbers. Let C be the class of all sets which contain only even numbers or contain only odd numbers. Let C include the empty set. Is the union of any pair of sets in C also in C? Is the intersection of any pair of sets in C also in C? Is the complement of any

set in C also in C? I.e, is C closed under the operations of intersections, unions and complement respectively?

Solution: set C is closed under intersection, it is not closed under union and complement.

17. How many subsets are there of the set $\{1, 2, 3, ..., n\}$. Your expression should be in terms of n.

    Solution: 2^n.

18. When you toss 5 fair coins at random what is the probability that you will have at least 3 heads? What is the probability that you will have exactly 3 heads and exactly 2 tails? What is the probability that you will have exactly 3 heads and exactly 2 tails in that sequence?

    Solution: $\frac{16}{32}, \frac{10}{32}, \frac{1}{32}$

19. How many different ways can you distribute m identical pieces of candy to n distinct kids? You need not distribute equally, you are even allowed to let some kids go without candy, but all the pieces of candy need to be distributed.

    Solution: $\binom{m+n-1}{n-1}$.

20. For each of the following 4 pairs, decide whether

    - a is a sufficient condition for b
    - b is a sufficient condition for a
    - both of the above, i.e, $a \iff b$
    - neither of the above

    NOTE: B is an arbitrary predicate.

a	b
$\forall x, \exists y, B(x,y)$	$\forall y, \exists x, B(x,y)$
$\exists x, \forall y, B(x,y)$	$\forall y, \exists x, B(x,y)$
$\exists x, \forall y, B(x,y)$	$\exists x, \forall y, B(y,x)$
$\exists x, \forall y, B(x,y)$	$\exists y, \forall x, B(y,x)$
$\exists x, \forall y\ [\ (x \leq y) \wedge$ $\forall z, (z \neq x) \rightarrow \exists w (z \not\leq w)\ ]$	$\forall z, \forall w\ [\ (\ \forall x (z \leq x) \wedge$ $\forall x ((z \leq x)\ \wedge\ (w \leq x)\ ) \rightarrow (z = w)$

(1)

Solution: neither, a is sufficient for b, neither, both, a is sufficient for b.

21. Suppose the following sentence is true. "If I slept well, I will pass this test." What can I conclude logically from the following sentences:
(a) I passed the test
(b) I failed the test.

Solution: (a) nothing, (b) I did not sleep well.

22. True or False. Justify your answer. $(\overline{((\overline{(A \cup B)}) \cap C)} \cap D) = (A \cap D) \cup ((\overline{C} \cup B) \cap D)$

True. To the LHS, starting with the inner most parantheses, apply, in order: demorgan's law, associative and commutative laws, demorgan's again, and finally the distributive law.

23. Every student is majoring in computer science or in math or in electrical engineering. There are 30 computer science, 20 mathematics and 25 electrical engineering majors. 10 of these are majoring in 2 or more disciplines. 5 of these are majoring in 3 disciplines. How many students are there?

Solution: 30+20+25-5-10=60