Homework assignment 3,

1. A set $U$ is $\leq^C_m$-complete for a class $D$ if every set $X$ in $D$ is reducible to $U$ by a $\leq^C_m$-reduction, i.e., there is a function $f$ in the class $C$, such that $x \in X \iff f(x) \in U$.

   (i) Show that the set $A_{\text{pseudocode}} = \{(M, x) : \text{the pseudocode } M \text{ accepts } x\}$ is $\leq_{\text{rec}}$-complete for the class of r.e. sets.

   (ii) Give a set that is $\leq_{\text{rec}}$-complete for the class co-r.e., and justify your answer.

   (iii) Can a set $A$ - that is $\leq_{\text{rec}}$-complete for the class of r.e. sets - be decidable. Why (not)?

2. The class $\text{Diff}$ consists of the intersections of sets in r.e with the sets in $\text{co} - \text{r.e.}$ i.e, $\text{Diff} = \{A \cap B : A \in \text{r.e.}, B \in \text{co} - \text{r.e.}\}$. (This is different from $\text{r.e.} \cap \text{co} - \text{r.e.}$ which we know to be the class of recursive or decidable sets).

   (i) Show that

   $$\text{r.e.} \cup \text{co} - \text{r.e.} \subseteq \text{Diff}$$

   and

   (ii) (Recalling the definition of complete given above) starting from a set $U$ that is known to be is $\leq_{\text{rec}}$-complete for r.e., construct a set, and show that it is $\leq_{\text{rec}}$-complete for $\text{Diff}$. Hint: how about $\{(x, y) : x \in U \land y \notin U\}$?

3. Read Chapter 6.4 in book. Answer 6.13 ($A_{TM}$ is our $A_{\text{pseudocode}}$), 6.16, 6.17 (Turing-recognizable = recursively enumerable).

4. (bonus) Show that the set $A_{\text{Hal}} = \{M : \text{the pseudocode } M \text{ halts on input } M \text{ (may or maynot accept) }\}$ is $\leq_{\text{rec}}$-complete for the class of r.e. sets.