Geometric Constraints II

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Embeddability, Localizability, A Compendium of Facts and Definitions to be proved subsequently

Question: For which metric spaces does isometric embedding into L_p automatically imply isometric embedding into ${L_p}^d$

Definition: A partial metric space (graphs) is d-realizable if embeddability into $L_2 \Rightarrow$ embeddability into ${L_2}^d$

Definition: A graph is d-realizable if $\forall$ possible metric value assignments to the edges of the graph, embeddability into $L_2 \Rightarrow$ embeddability into ${L_2}^d$

Definition: A minor is obtained by taking a subgraph and replacing simple paths by edges.

Theorem: A graph is 3-realizable iff it does not have K_5 or an octahedron as a minor.

Theorem: A graph is 2-realizable iff it does not have K_4 as a minor.

Fact: If n-point metric space is embeddable in L_p then it is embeddable in ${L_p}^n$

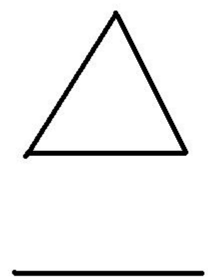
Notice: Embeddability of n-point metric space d in $L_2 \Rightarrow$ metric matrix M is positive semidefinite, has rank at most n, so embeddable in ${L_2}^n \Leftrightarrow$ $rank of M is at most d \Leftrightarrow$ embeddable in ${L_2}^n$. Rank of M is at most d $\Leftrightarrow$ embeddable in ${L_2}^d$

Definition: The class of n-point partial metric spaces are embeddable in $L_p \Leftrightarrow$ they are embeddable in ${L_p}^n$

Definition: The class of graphs G on n vertices such that $\forall$ "distance" values assigned to edges of the graph, the corresponding (partial) metric space $\in G$

localizablility: A partial metric space is strongly localizable in d-dimensions if the complete set of realizations (embeddings) in any higher dimension can be embeded in ${L_2}^d$ without any modifications.

Eg: The D simplex is d-strongly localizable of graphs that are d-strongly realizable (d = 2,3)



For example consider a triangle or a line embedded in 2d, in whichever dimension it is embedded its going to be the same configuration as its in 2d.

Open Questions: Exact relationship between these 3 class of graphs. 1.d-realizability. 2.d-strong localizability. 3.unique realizability in d-dimensions (global rigidity).