1. Lectures 1 and 2

1.1. **Motivation 1.** CAD part and assembly design,
Teaching Geometry and automated theorem proving,
Robotics,
Graphics Animation

References for CAD applications. www.solidworks.com
www.dcubed.com
www.cise.ufl.edu/∼sitharam - FRONTIER opensource software

Formalization.

- What are geometric constraint satisfaction problems?
- Roughly what are the questions:
  -Realizability - Genericity;
  -Generic classification: rigidity (wellconstrainedness)/ underconstrainedness/ overconstrainedness;
  -Finer classification of generic realization space structure;
  -Finding generic realizations;
  -Finer classification of nongeneric realization space structure;

  My surveys and Andrew's thesis.
  www.cise.ufl.edu/∼sitharam/drone.pdf
  www.cise.ufl.edu/∼sitharam/dimacs.pdf
  www.cise.ufl.edu/∼sitharam/andrew-thesis.pdf
- Overall framework for GCS questions:

  -What is the input constraint system: Complete or partial graph/metric space (usually) or a matroid, coxeter matroid?
  www.math.umn.edu/∼reiner/Papers/rigidity.ps
  -Realization in: Euclidean, Projective, Lp, Other metric space?
  -Realization in: fixed or variable (min possible) dimension
  -Exact or approximate realization (allow some distortion)
  -Input is General or Special (regular, symmetric)
  -Algorithmic or Characterization(construction/existence), or -Extremal/negative constructions - non-realizability, non-embeddability etc.
  -For fixed dimension and exact realization: generic or non-generic
  -Defining generic
  -When are combinatorial characterizations possible? What does

"combinatorial" mean?
-Inverse problem: Given desired realizations, find "easiest" input - partial constraint system

Grad class (first few) lecture notes taken by students.
www.cise.ufl.edu/∼sitharam/COURSES/GC/lec1.pdf
www.cise.ufl.edu/∼sitharam/COURSES/GC/lec2.pdf
www.cise.ufl.edu/∼sitharam/COURSES/GC/lec3.pdf
www.cise.ufl.edu/∼sitharam/COURSES/GC/lec4.pdf
www.cise.ufl.edu/∼sitharam/COURSES/GC/lec5.pdf
www.cise.ufl.edu/∼sitharam/COURSES/GC/lec6.pdf

1.2. **Easy-to-state formal Questions.**

(1) optimal decomposition/ construction plan
-given construction method.
Example: sequential ruler and compass constructibility

-allowing (varying level of) choice of construction method
Example: existence of size 3 Euclidean DR-plans
Example: find optimum DR-plan

(2) combinatorial detection of generic rigidity, unique realizability, algebraic dependence (big question: what can be decided combinatorially?, what does combinatorial mean?)
Example: For sequential rcc: rigidity, dependence, unique realizability
Example: 2D sequential ruler-and-compass: "combinatorial" characterization of realization existence
Example: general 2D rigidity using dofs? 3D rigidity and bananas?
Example: angle constraint system (detecting dependences)

(3) Same as above for non-generic (big question: what can be decided combinatorially?, what does combinatorial mean?)
Example: all distances equal, sequential rcc: for which graphs is this generic? in general, same questions as above
Example: point-line incidence structures - detecting dependence (Pappus)

(4) Describing, sampling - points and paths in realization space Collision avoiding paths from one realization to another (big question: what can be decided combinatorially?
Example: For which 1 dof graphs is there some completion edge

for which realization space is an interval
Example: Polygon straightening without collisions

(5) Inverse problem: given desired realizations, designing "easy or low complexity" constraint system

1.3. **Mathematical directions, conjectures, known results.**

Combinatorics: graphs, matroids and oriented matroids.

(i) Combinatorial classification of generic realization space structure: 2D Quadratic solvability
What is known: Characterization of sequential ruler and compass constructibility
What is known: Planar case of quadratic solvability
http://www.maths.lth.se/matstat/ecmi/modw/reportgroup4/reportgroup4.pdf

(ii) Combinatorial classification of generic realization space complexity:
- Minimum dense subgraph problem
what is known: NP-completeness, Frontier Vertex Algorithm
http://www.cise.ufl.edu/∼sitharam/drtwo.pdf
- *what is known:* optimal algebraic complexity
http://www.cise.ufl.edu/∼sitharam/jsc-skeleton.pdf

(iii) Combinatorial characterization of rigidity
what is known:
Laman's theorem, Henneberg construction, Rigidity matrix independence; Module rigidity; Seam graphs.
http://www.math.cornell.edu/∼connelly
http://www.math.cornell.edu/∼connelly/BasicsI.BasicsII.pdf
http://www.cise.ufl.edu/∼sitharam/overlap-new.pdf
http://www.cise.ufl.edu/∼sitharam/module.pdf
http://biophysics.asu.edu/banf$f_f$iles/jackson/banffprobs.pdf

(iv) Global rigidity, Unique realizability
what is known: Hendrickson-Jackson-Jordan's theorem
http://www.math.cornell.edu/∼connelly/global-6.pdf
http://www.cs.elte.hu/egres/tr/egres-02-12.pdf

Algebraic geometry, varieties, semi-algebraic sets, invariant theroy.

(i) "Roadmapping" varieties (Cayley-Menger, Grassmanian) corresponding to classes of geometric constraint systems
(finding an efficient projection/representation, then characterizing connected components, Betti numbers etc.)

Why is this important: solution picking (chirality, variety with grassmanian stratification), sampling; singularities; motion

what is known: Heping's theorem, 2-trees, 2-realizability
http://www.math.cornell.edu/

what is known: Cayley-Menger conditions and properties (low-rank etc) of distance matrices
http://www.maths.lth.se/matstat/ecmi/modw/reportgroup4/reportgroup4.pdf
web.mit.edu/tfhavel/www/Public/dg-review.ps.gz
and Cayley-Menger varieties
http://arxiv.org/P$S_c$ache/math/pdf/0207/0207110.pdf

what is known: Grassmanians, When panel and hinge structures collapse
http://maven.smith.edu/∼streinu/Papers/megaProc.pdf

what is known: Carpenter's rule
http://www.ams.org/featurecolumn/archive/links1.html
http://citeseer.ist.psu.edu/connelly01straightening.html
http://citeseer.ist.psu.edu/streinu00combinatorial.html

(ii) Is "generic" classification of all (isolated) solutions of a well-constrained system any easier than fully roadmapping the big variety corresponding to the given set of constraints with variable parameters?
what is known: www.cise.ufl.edu/∼sitharam/jsc-skeleton.pdf

(iii) Finding generators for a "hidden" ring of polynomial invariants (finding corresponding group of transformations)
Why is this important: choosing optimal solution method (generalizing DR-plan). Very little is known

Geometric groups, representations and algebras.

(i) "Roadmapping" a variety corresponding to a classes of geometric constraint systems (analyzing local(tangentspace) group structure)
Why is this important: completely characterizing "motions," points and paths used a lot in robotics, elsewhere. Look at the first two books by Selig and Bayro-Corrochano et al in
http://books.google.com/books?ie=UTF-8& q=geometric
++groups++and++robotics& btnG=Search+all+books

(ii) Detecting incidence geometry dependences using Grassman-Cayley algebra
http://www.danroozemond.nl/site/archief/ProvingCinderella.pdf
http://www.math.ufl.edu/∼white/antw2.ps

http://www.math.ufl.edu/∼white/hbz.ps
http://www.math.ufl.edu/∼white/tutorfig.ps

Efficient algorithms (algebraic-numeric) for solution of systems of polynomial equations and inequalities. Vast area