

# Geometric Constraint Lecture - Higher Dimension

Instructor: Meera Sitharam, Recorded by Heping Gao

Thurseday, Feb 09, 2006

## 1 Introduction

Dimension is variable.

In which dimension the constraint graph is realizable?

What's the minimum dimension the constraint graph realizable?

What kind of constraint graph is realizable in any dimension?

## 2 Pointes and Distances

Considering points and distances, HGCS(Higer Dimension Constraint System): $(G, \overline{P})$

1. (Generically) What's the minimum  $d$  so that it is realizable in  $R^d, C^d$ ?

2. Is  $(G, \overline{P})$  realizable in any finite dimension  $d$ ?

a. If  $(G, \overline{P})$  is not a metric space (Geometry  $\triangle$  inequality doesn't hold), there doesn't exist  $d$  s.t.  $(G, \overline{P})$  is realizable in  $R^d$ .

b. Exact characterization?

— Caley Menger (Algebraic)

— Bnurgain/Milmau/Kigie/Sehechtman /Jolnson-Hindenstruss

3. Is  $G$   $d$ -realizable?

Def: A  $(G, \overline{P})$   $[G]$  is  $d$ -realizable (generically) iff any Euclidean realization in any dimension implies it can be realizable in  $R^d$ .

4. Fixed  $d$ , generically classify : no solution, unique solution or finitely many solution.

Note, in 3d, the above question is still open. Particularly, for what special class of graphs in  $d$  Dimension, we can classify it?

For question 1, a randomized algorithm with high probability  $(1 - \epsilon(|a|))$ , finds an appropriate  $\overline{P} + \overline{\delta}$  realization in approximately  $(1 + \gamma)d_{min}$  dimension  $d$  by running time  $T(|a|, 1/\epsilon, 1/\gamma, 1/\delta)$ .

### 3 Non-Distance Constraint

Very specific graphs (regular graphs)

### 4 Questions and Discussion

1. How to generate a random subspace of dimension  $d$  in  $R^n$ ? For example, we can sample two angles for 3D. How to sample 2-d in  $R^4$ ?

2. In 2-d, the largest  $m$  s.t.

(1).  $\exists m$  points and  $m$  lines s.t. every pair of points is separated by exactly  $m/2$  lines;

(2). For maximum  $m$ , find all such realization? Modulo what?