Geometric Constraint Lecture(Feb 14–23)

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1 Combinatorial Property of Point-Line Matrix(Feb 14th)

Conjecture on the combinatorial property of Point-Line matrix.

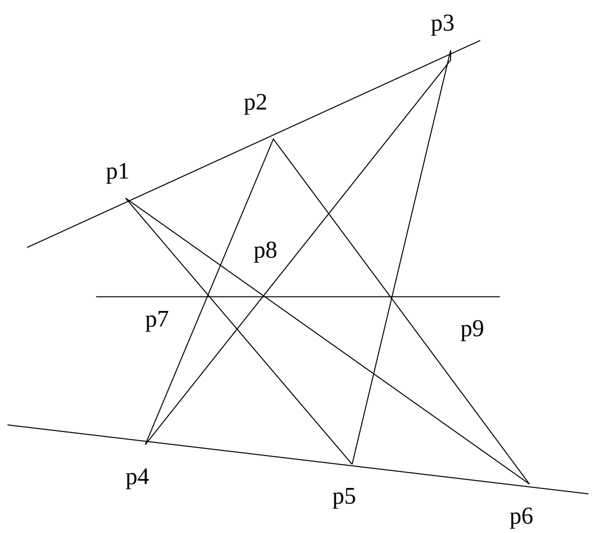

Figure 1: Point Lince example 1

The following matrix is for Figure 1

$$
\begin{pmatrix}
P1 & | & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
P2 & | & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
P3 & | & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
P4 & | & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
P5 & | & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
P6 & | & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
P7 & | & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
P8 & | & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
P9 & | & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}
$$

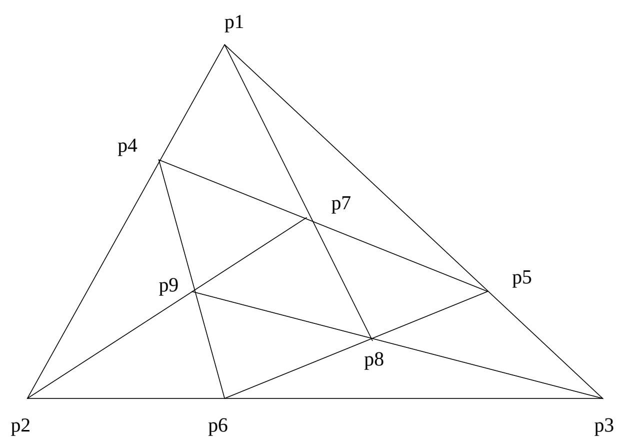

Figure 2: Point Line example 2

The following matrix is for Figure 2

$$
\begin{pmatrix}
P1 & | & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
P2 & | & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
P3 & | & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\
P4 & | & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
P5 & | & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
P6 & | & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
P7 & | & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
P8 & | & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
P9 & | & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
$$

For more details, please refer to the homework.

2 Feb 16th

Question:when viewed as incidence equations between points and lines in R^2, does it have dependence?

Can this question be answered using purely combinatorial properity($\in P$) I ? I.e. should not require solution of any system S_m (number of variables, number of equations or degree depending on m) of equations over R, N, Q.

2.1 Revisting generalicity

Define Generalicity regarding to Incidence Matrix (I_m)

1. Combinatorial characterization of I_m 2. Algebraic characterization of nongeneric inputs of I_m 3. Combinatorial properity P will be used to give an algorithm for deciding dependence of $I_m \in \Gamma$.

HDGC, for higher dimensions, GCP: approximate min dimension where a realization exists. (Bounds)

Recall: Given n, what is the min dimension in which $\exists m$ hyperplanes and m points s.t. every pair of points has exactly $m/2$ seperating it.

Application: communication complexility lower bound.

2.2 Questions

1. Problem 1 $\iff$ Problem 2?

Problem 1: Given a $n*n$ matrix M, find a real matrix M^* s.t. $sign(M^*) = sign(M)$ and $rank(M*)$ is minimized.

Problem 2: given M a $+1, -1$ matrix $n*n$, find the min dimension config C of n hyperplanes q_j and n points p_i s.t. $sign(< p_i, q_j >) = sign(M)$.

2.

(i) There exists not 5 points there exists not any set of m lines (hyperplanes) s.t. every pair of points is sperated by exactly $m/2$ (the same as $\geq m/2$) lines?

(ii) show d(5)=3; d(6)=?; $d(6) > 3$? Prove it.

3. List all 10-3 matrices and all 11-3 matrices? Any conjectured property P s.t. P implies ndependent?

4.

5. DOF counting for symmetry case. (For example, 2-D stewart platform, the length of any side of the inner triangle is $x1$, the length of any side of the outside triangle is $x2$ and the length of edges between the two triangles is $x3$, how is its DOF determined by $x1$,$x2$ and $x3$?)

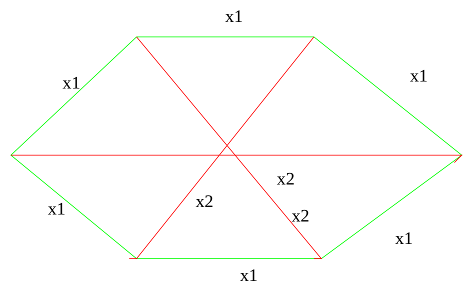

Figure 3: Hexgon example

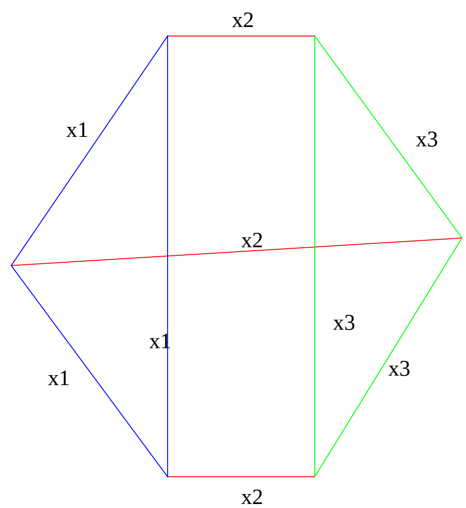

Figure 4: 2-D Stewart Platform

6. The max number of orthorgonal line pairs (l_{i1}, l_{i2}) s.t. (i) $\forall i$, $| < l_{i1}, l_{i2} > | = 0$; (ii). the vales of $| < l_{i1}, l_{j2} > |$ and $| < l_{i2}, l_{j1} > |$ are the same $\forall i \ \forall j \ i \neq j$.

7. Senthil's Conjecture.

Please refer to Figure 5.

3 Feb 21th

Pleae refer to Problem 1 listed in the next section.

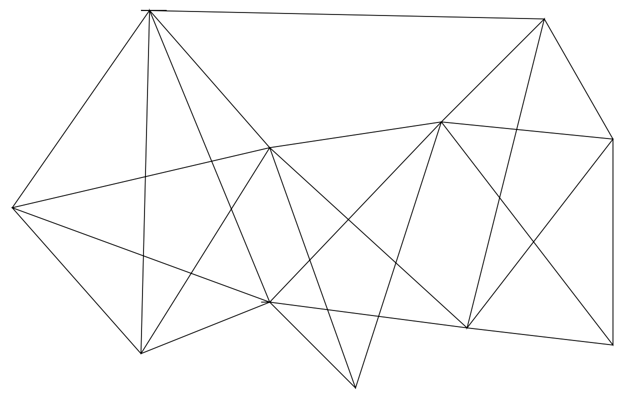

Figure 5: An example to Senthil's conjecture

4 Feb 23th

4.1 $d(\epsilon, \delta, k, m)$

Problem: want k sets of m lines each, $l_{ij}, 1 \leq i \leq k; 1 \leq m$ s.t.

(i). $\forall i$, $| < l_{ij_1}, l_{ij_2} > | = \epsilon, j_1 \neq j_2$;

(ii). $\forall i_1, i_2, i_1 \neq i_2$, $\forall j_1, j_2$, $| < l_{i_1 j_1}, l_{i_2 j_2} > | = \delta$;

Min dimension, let $d(\epsilon, \delta, k, m)$ be the min dimension where such a collection exists in $R^{d(\epsilon,\delta,k,m)}$ $(C^{d(\epsilon,\delta,k,m)})$?

A general question is $d(\epsilon, \delta, k, m, \epsilon_1, \delta_1)$

Problem: want k sets of m lines each $l_{ij}, 1 \leq i \leq k; 1 \leq m$ s.t.

(i). $\forall i$, $| < l_{ij_1}, l_{ij_2} > | \in (\epsilon - \epsilon_1, \epsilon + \epsilon_1), j_1 \neq j_2$;

(ii). $\forall i_1, i_2, i_1 \neq i_2$, $\forall j_1, j_2$, $| < l_{i_1 j_1}, l_{i_2 j_2} > | \in (\delta - \delta_1, \delta + \delta_1)$;

Min dimension, let $d(\epsilon, \delta, k, m)$ be the min dimension where such a collection exists in $R^{d(\epsilon,\delta,k,m)}$ $(C^{d(\epsilon,\delta,k,m)})$?

An equivalent question is to maximize $k(\epsilon, \delta, d, m, \epsilon_1, \delta_1)$ where d is fixed.

For example, $\epsilon = 0$, orthorgonal; $\epsilon_1 = 0, \delta_1 = 0$, that's the former question.

4.2 Some questions

1. $\delta = \frac{1}{\sqrt{d}}$
2. Equal δ
3. Fix i_1, i_2, $max(min_{j_1,j_2}|l_{i_1 i_2}, l_{j_1 j_2}|)$, δ is the optimizer.
4. Fix i_1, i_2, $min(max_{j_1,j_2}|l_{i_1 i_2}, l_{j_1 j_2}|)$, δ is the optimizer.
6. $k(0, \delta, 2, 2, 0, 0)$? $k(0, \delta, 3, 2, 0, 0)$? The answer is 2. Prove it.

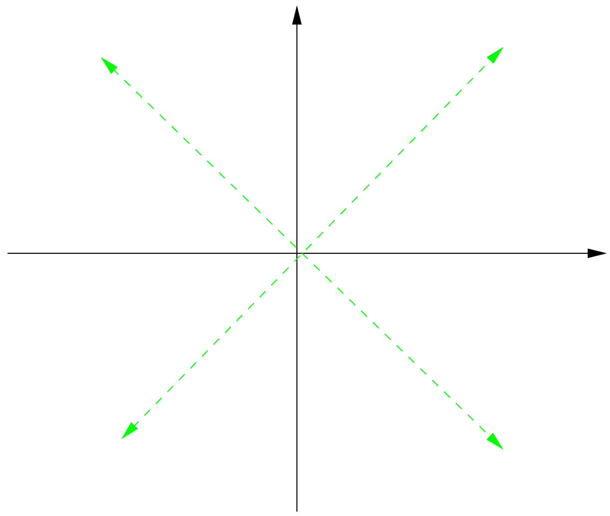

Figure 6: A graph for 2D

Proof: Please refer to Figure 6. Suppose orthorgonal l_{11} and l_{12} are fixed. l_{21} is orthorgonal to both l_{11} and l_{12}, it must be in one of the four red directions. Then, we have no way to place l_{31} so that it is orthorgonal to l_{11}, l_{12} and l_{21}.

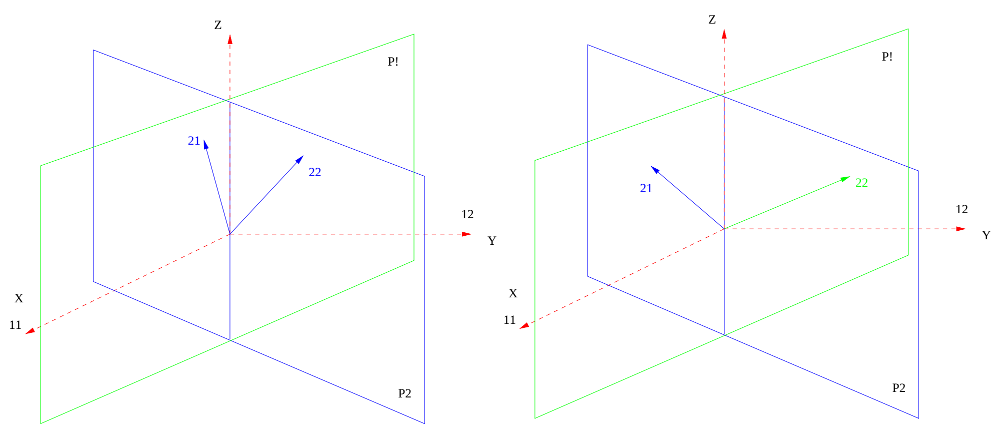

Figure 7: A graph for 3D

Let's consider the 3D cases in Figure 7. Firstly, any pair of lines l_{i1} and l_{i2} are orthorgonal and so span a plane. For l_{21} amd l_{22}, they must be in the "bisection" (actually it is two perpendicular plances) of l_{11} and l_{12}. Please refer to Figure 7. We have two bisection planes for l_{11} and l_{12}: P1 in green and P2 in blue. l_{21} and l_{22} must be in P1 or P2. Let's consider the case one of l_{21} and l_{22} is in P1 and the other is in P2 and they are not both in P1 or P2. l_{21} and l_{22} are orthorgonal, so this case can only happen when one of l_{21} and l_{22} must be $axis-X$ or $axis-Y$ (thus in the same plane determined by l_{11} and l_{12}). However, we know if three lines are in a plane, we can not add l_{31}. Another case is l_{21} and l_{22} are both in P1 (or P2). Without loss of genericity, we assume that l_{21} and l_{22} are both in P1. In the same reason, l_{31} and l_{32} have to be in P2(in P1 is impossilbe). We can easily check that l_{11} are perpendicular to l_{21}, l_{22} , l_{31} and l_{32} but l_{31} can not be perpendicular to both l_{21} and l_{22}. So, the answer is 2.

6': Is there any sequence of d's s.t. strictly increasing k? The answer is NO from 6.

6": Does k always increase with d?

7. what is $k(0, \frac{1}{\sqrt{d}}, 3, 3, 0, 0)$

7': Does k always increase with d?