Geometric Constraint Lecture

Instructor: Meera Sitharam, Recorded by Heping Gao

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1 Demo 1 from Solidwork software: Hexagon with three diagonals in 2D

Figure 1 is a Hexgagon with three diagonals in 2D and the whole graph is rigid(wellconstrainted, dense). The minimal rigid subgraph (wellconstrained subgraph, dense subgraph) is the whole graph. (Note: trival rigid subgraph, such as two vertices and one distance between them in 2D, is not considered!) So, the only DR-plan for this example is shown in the Figure 2.

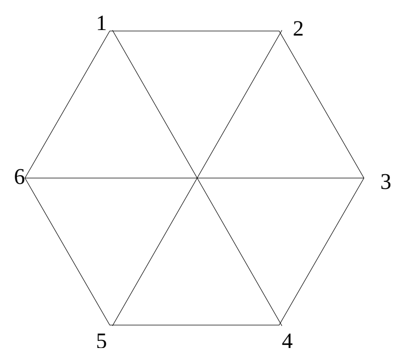

Figure 1: Hexagon with three diagonals in 2D

Figrue 3 has a DR-plan shown in Figure 4. The fan-in (the maximum number of the children of a cluster in the DR-plan) is 3 in Figure 4 while the fan-in in Figure 2 is 6, so intuitively hexagon is more difficult to solve. Please recall the difference bewteen Computational Geometry and Geometric Constraint Solving that we have introduced before: in Computational Geometry, the size of the minimum equation/inequality system we need to solve is bound by some constant, while in Geometric Constraint Solving, the

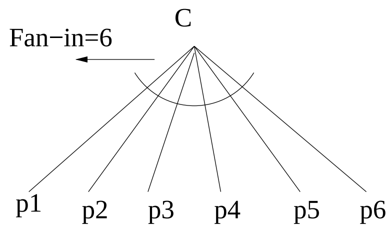

Figure 2: The only DR-plan for Hexagon with three diagonals in 2D

size can not be bound by some constant, for example it can be only bound by the input size.

Question:Can we give a 2D distance constraint graph G=¡V,E¿ (the only type of constraint is distance constraint) in which the minimum of fan-in of all possilbe DR-plans is $\theta(|V|)$?

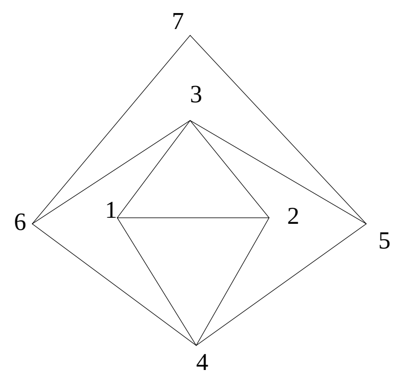

Figure 3: A Ruler Compass Constructable Example in 2D

2 Assembly example in Solidworks Software

Figure 5 is an assembly example in which C1 and C2 are two rigid bodies. C1 and C2 are assemblied by the constraints 1378 and 2456 are coplanar and 13 and 24 are colinear.

For rigid 3D cluster C1 and C2: $DOF(C1) = DOF(C2) = 6$. The coplanarity will remove 3 DOF, the colinarity will remove 2 more extra DOF and the zero distance between 1 and 2 will remove 1 more extra DOF. So, the whole system is still rigid because the DOF=6+6-3-2-1=6.

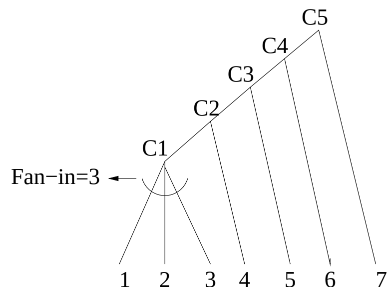

Figure 4: DR-plan for a RCC example

3 Demo of Frontier: solution navagation

In Figure 6, the point 1, 2 and 3 are the centers of three circles which are tangent to each other. Figure 7 is one of its DR-plan.

4 Banana in 3D

In Figure 8, there are two clusters C1 and C2 which are nontrivally rigid. By counting the DOF of the whole constraint graph, 2*6-6=6, so the whole graph should be rigid if the way counting DOF is correct! However, the whole system is either non-rigid and C1 and C2 can relatively rotate around hinge 13 if C1 and C2 provides a consistent distance for 13, or there is no real embedding if C1 and C2 provides a consistent distance for 13. This example also we can not check the rigidity of even a genric system by counting the DOF of its constraint graph.

In Figure 9, an extra distance between 24 can prevent the relative rotation between C1 and C2.

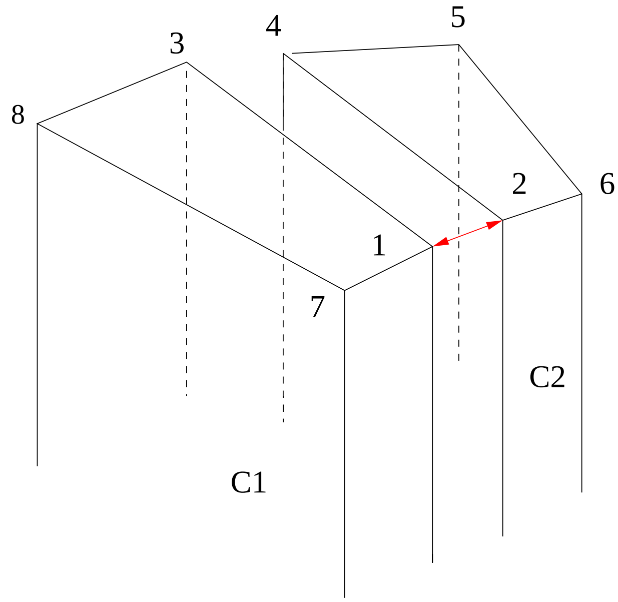

Figure 5: Assembly example of coplanarity and colinearity

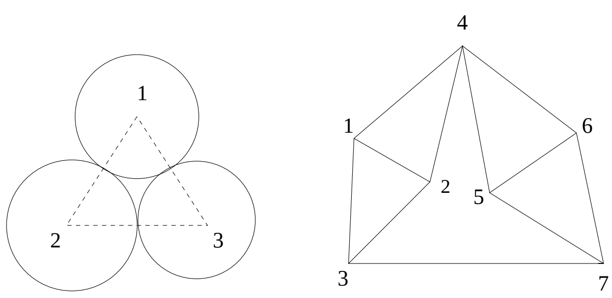

Figure 6: Frontier Demo

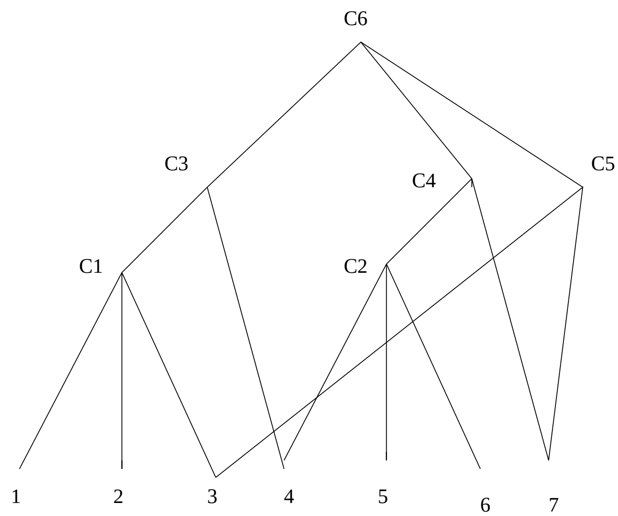

Figure 7: DR-Plan for the Frontier Demo

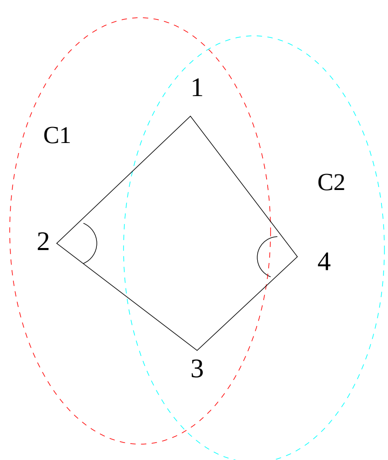

Figure 8: Banana example

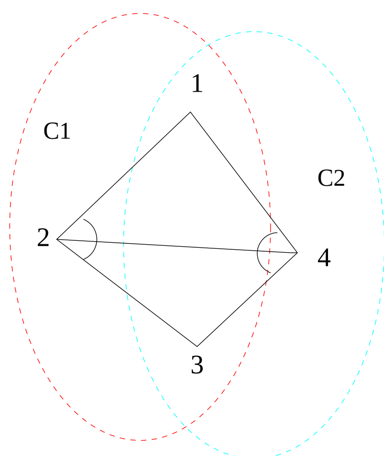

Figure 9: Banana example