Geometric Constraints April 4, 6 & 11

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Objects in discussion

- Infinitesimal Rigidity Matroids.
- Generic Rigidity Matroids.
- Abstract Rigidity Matroids.

Some Observations

1. *Recall from previous lectures: Every greedy algorithm has an underlying matroid, and every matroid has a greedy algorithm on top of it*
2. *Just like looking at Independent sets in Matroids, we look for rigid sets in Abstract Rigidity Matroids*
3. *Rigidity is the same as Infinitesimal Rigidity for Generic embeddings*
4. *In order to define a Generic Rigidity Matroid we need an embedding, but this is not the case with Abstract Rigidity Matroids, Thus we are moving for a purely combinatorial characterization with ARMs*
5. *Class of Infinitesimal Rigidity Matroids on general embeddings is the same as the class of IRMs on generic embeddings*
6. *Every Infinitesimal Rigidity Matroid and Generic Rigidity Matroid is an Abstract Rigidity Matroid. In fact the relationship looks like Fig 1*

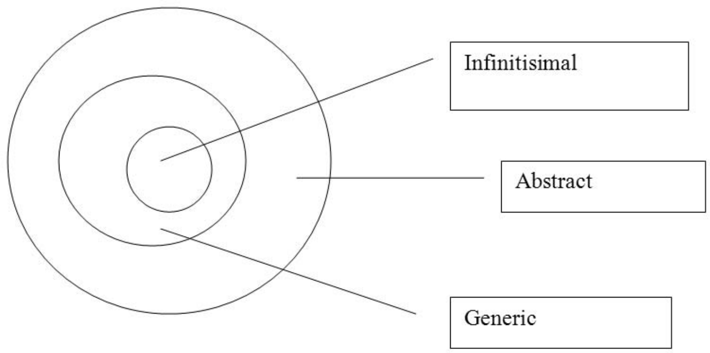

Figure 1:

Definitions And Pointers:

- Isostatic Sets: The edge sets which uniquely determine an Abstract Rigidity Matroid. (Basically rigid sets);

– *Look at Lem 2.6.1 for some useful properties of Isostatic sets*

- *0-Extension: Let the finite set V be given and consider an m-dimensional abstract rigidity matroid* $A_m forV, letE \subseteq$ *K = K(V), let U* $\subseteq$ *V(E) where* $|U| = m$ *and let i* $\in$*(V-V(E)). The edge set F = E* $\cup$ *(i,j)* $|j \in U$ *is called a 0-extension of E (in dimension m)*
  - *The relationship between isostatic sets and 0-extensions is described by Theorem 2.6.1.*

- *Recall: Abstract Rigidity Matroids on a complete graph G(V,E) is when the closure operator satisfies all the matroid conditions and the following condition:*
  - *if* $Q_1, Q_2 \subseteq E$ *and* $|V(Q_1)| \cap |V(Q_2)| < k$,
    then $< Q_1 \cup Q_2 > \subseteq Completion(V(Q_1)) \cup Completion(V(Q_2)$
  - *Theorem 3.11.3 tells you what is true for independence,counting and abstract rigidity in ARMs*

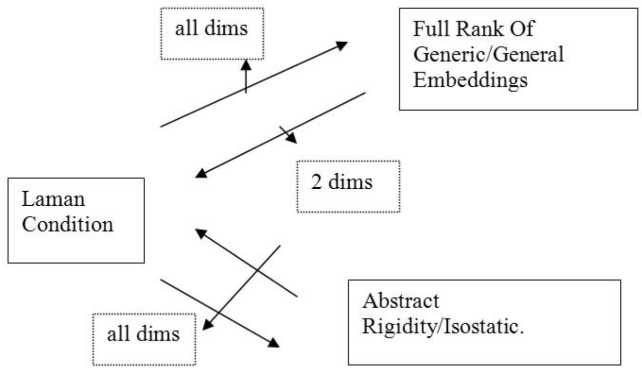

Figure 2: the big picture

- 1-Extension: Let the finite set V be given and consider an m-dimensional abstract rigidity matroid $A_m forV.LetF \subseteq$ K = K(V), let U $\subseteq$ V(F) where $|U| = m + 1$, let (h,k) $\in$ F(U) and let i $\in$ (V - V(F)). The edge set E = F - (h,k) $\cup$ (i,j)$j \in$ U is a 1-extension of F (in dimension m)
  - *Theorem 4.1.1 tells about 2 dimensional ARMs and 1 extendability*