COT3100 Application of Discrete Structures
Sample Test (120 min)

January 24, 2017

NOTE: This Sample test is only to indicate the rough formal and appearance of the test. DO NOT EXPECT the problems in the actual test to follow the pattern of the problems here. The GOAL of this course is to wean you off the idea that solving problems means following patterns and recipes. The GOAL is to get you to think about and solve problems whose "type" you may never have seen before. The GOAL is to get you to think analytically using your native skills and using first principles.

NOTE: All tests are CLOSED BOOK. NO CHEAT SHEETS ALLOWED. This class is not about memorization. However, if you do all the zybook reading and all the homeworks, you will automatically know certain definitions. Any definitions beyond those will be provided in the problem. You would have to read the definitions carefully, digest them, and use them in understanding the actual problem.

1. Prove or disprove that "$(\neg q \to \neg p) \wedge (\neg r \to \neg q) \to (p \to r)$ is a tautology". (6 points)

2. Are the following statements equivalent? "You can go to the movies only if you are over age or have permission" and "You can go to the movies whenever you are over age or have permission". Justify your answer. (5 points)

3. Prove or disprove that $[(p \to q) \wedge (p \to r)]$ and $[p \to (q \wedge r)]$ are logically equivalent. (5 points)

4. Consider the following predicates where the domain of x is students and the domain of y is courses. $U(y)$: y is an upper-level course. $M(y)$: y is a math course. $F(x)$: x is a freshman. $B(x)$: x is a full-time student. $T(x, y)$: student x is taking course y. If in English, convert to predicate logic. If in predicate logic, translate to English without using variables. (10 points)

   (a) All full time students are freshmen.

   (b) No freshman is taking an upper level math course.

   (c) $\forall x \; \exists y \; T(x, y)$.

   (d) $\forall x \; \forall y \; [(T(x, y) \wedge U(y)) \leftrightarrow B(x)]$.

5. Are the following system specifications consistent? The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is in interrupt mode. Justify your answer. (5 points)

6. On the island of knights, knaves and spies you encounter three kinds of people. Knights who always tell the truth, knaves who always lie and spies who can either tell the truth or lie. You encounter 3 people A, B and C. One of them is a knight, one of them is a knave and one is a spy. (8 points)

   (a) Suppose Person A says "I'm a knight", B says "A is not the knave" and C says "B is not the knave". Can you determine which of these persons is the knight, the knave and the spy? Justify your answer.

   (b) Suppose Person A says "I'm a knave", B says "A is not the knave" and C says "I'm a knight". Can you determine which of these persons is the knight, the knave and the spy? Justify your answer.

7. Are the following statements true? Justify your answer. (8 points)

   (a) $\forall x, x^2 \geq 0$, where the domain of x is the set of real numbers.

   (b) $\exists m\ \exists n\ m^2 + n^2 = 25$, where m and n are integers.

8. Are these valid arguments? Justify your answer. (5 points)

   (a) Suppose x is a real number. If x^2 is irrational, then x is irrational. Therefore if x is irrational, x^2 is irrational.

   (b) Suppose n is a real number. If $n > 1$, $n^2 > 1$. Suppose $n^2 > 1$, then $n > 1$.

9. Prove or disprove "The product of two irrational numbers is always irrational". (5 points)

10. Prove or disprove $min(a, min(b, c)) = min(min(a, b), c)$. Given that a, b, and c are integers and that $min(a, b)$ gives the minimum of the two numbers. (5 points)

11. Let A, B and C be sets. Prove or disprove that $A - (B - C) \subseteq A - B$. (5 points)

12. If A, B and C are sets, prove or disprove $A - (B \cap C) = (A - B) \cup (A - C)$. (5 points)

13. Consider the following statement. If you do problems, then you learn discrete math.
Give the (5 points)

(a) Converse

(b) Inverse

(c) Contrapositive

14. Suppose $P(x, y)$ is a predicate and the universe for the variables x and y is $\{1, 2, 3\}$. Suppose $P(1, 3), P(2, 1), P(2, 2), P(2, 3), P(3, 1), P(3, 2)$ are true, and $P(x, y)$ is false otherwise. Determine whether the following statements are true. (5 points)

(a) $\forall x \exists y P(x, y)$

(b) $\exists x \forall y P(x, y)$.

(c) $\exists x \exists y (P(x, y) P(y, x))$.

(d) $\forall y \exists x(P(x, y) P(y, x))$.

(e) $\forall x \forall y(x) = y(P(x, y) P(y, x))$.