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Single Bend Wiring*

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The problem of wiring pairs of points with wires having at most one bend is considered. We develop an $O(n^2)$ algorithm to determine whether or not a set of n point pairs can be wired in this manner on a single layer. When this is possible, our algorithm generates the layout. We show that determining the maximum number of point pairs that can be wired in with at most one bend is NP-hard. The problem of determining the minimum number of layers needed to wire a set of n point pairs is also NP-hard. © 1986 Academic Press, Inc.

1. SINGLE BEND WIRING

Wire routing (equivalently, *wiring*, *wire layout*, etc.) is the problem of defining precisely the conductor paths in a given *wiring medium* for a set of *wires*. A wire is a specification of a pair of points to be connected.[1]

In the most general wire routing case, the conductor paths can be curves. In fact, it is not unusual to find curved conductor paths in manual layouts. However, in automated systems, conductor paths are usually constrained to contain horizontal and vertical segments only. The resulting loss in generality is offset by the following advantages:

(1) The conductor paths can now be represented far more easily. General curves, on the other hand, are not amenable to compact representation on a computer. Note, however, that in the case of river routing, Tompa [TOMP80] has shown that general wiring need consider only straight line segments and arcs of circles of different radii. In this case, the restriction to horizontal and vertical segments does not significantly enhance the representation of wires in a computer.

(2) The length calculations are considerably simplified.

(3) Fabrication equipment such as plotters is usually capable of horizontal and vertical movements only. Curves and diagonal lines are simulated by a series of small, alternating horizontal and vertical movements, and take a long time to draw.

Wiring where each path is made up of horizontal and vertical segments only is called *rectilinear wiring*. It is often thought of as wiring on a grid. All the wire endpoints are grid points; each conductor path is constrained to lie along grid lines only; and conductor paths are not allowed to cross. The separation between grid lines reflects the minimum conductor separation necessary to avoid inductive cross-talk. Thus, the concept of wiring on a grid neatly captures a number of practical constraints inherent in wire routing on a single layer. Figure 1.1 shows an example of wiring on a grid.

Any point on a grid may be specified by its x- and y-coordinates. Assuming that consecutive grid lines are one unit apart, the coordinates of grid points are always natural numbers. Let $S = \{(u_i, v_i, x_i, y_i) | 1 \le i \le n\}$ be a set of n wires to be connected. Wire i is specified by requiring that the point (u_i, v_i) is to be connected to the point (x_i, y_i), $1 \le i \le n$. S is called a *wire set*. For the example of Fig. 1.1, $S = \{(2,3,6,3), (3,2,5,5), (2,4,3,4), (3,4,1,5)\}$.

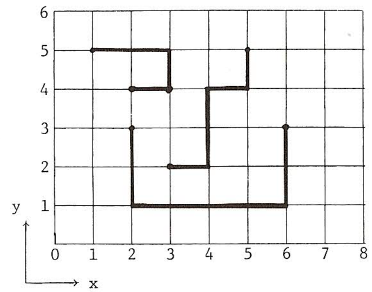

FIG. 1.1. Wiring on a grid.

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[1]With this terminology, the context of usage ensures that, in our discussion, there is no confusion between the problem specification (i.e., the wire) and the problem solution (i.e., the wiring or the wire layout). Another point is that in general, a *signal net* (a set of points to be interconnected) may be specified, rather than a wire. However, in fast circuit technologies (e.g., ECL), transmission line considerations impose rigid restrictions on the topology of the interconnection tree. Specifically, Steiner trees may not be permitted. In such cases, the nets are effectively decomposed into wires (before wire routing even begins) in ways that guarantee that all restrictions will be satisfied. We shall assume such an environment.

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With the constraint that conductor paths are not allowed to cross, it should be clear that not all wire sets S can be wired in a rectilinear fashion on a single layer. Hence, we need to consider multilayered wiring media. A multilayered wiring medium has k layers on which wires can be run. Typically, the points to be connected are the terminals of modules located on or in the wiring surface. It is assumed that these wire end points are available on all the layers. Interlayer connections are accomplished by means of holes called *vias*. There may be rules that restrict just where vias may be located. Each layer on which wiring is permitted is treated as a grid, and the same restrictions apply as before.

Usually, the cost of a realization goes up sharply with the number of layers used for the wiring. So it is generally desirable to minimize the number of layers used. Also, engineering considerations might limit the number of layers available to do the wiring. For example, in printed circuit boards with more than 10 or 12 layers, vias tend to be unreliable. As another example, integrated circuits can often have just 2 or 3 layers of metallization.

There are a number of applications where it is necessary to limit the number of bends on each conductor path. For example, when fabricating microwave or millimetric wave integrated circuits, the conductors, called *microstrip lines*, function as transmission lines. Bends on a microstrip line result in reflections, and this reduces the transmission efficiency of the line. Thus, it is desirable to limit the number of bends on each microstrip line.

So, a wiring problem worth considering is that of wiring S on a grid subject to the added constraint that no wire may bend more than k times. This problem has been studied, for example, by Pomentale [POME65]. In the wiring of Fig. 1.1, the wire joining (2, 3) and (6, 3) has two bends. When the wiring is restricted such that no wire has more than one bend, it is called a *single bend wiring*.

Two points (u, v) and (x, y) can be wired with no bends if and only if either $u = x$ or $v = y$. If $u \neq x$ and $v \neq y$, then there are exactly two ways to wire this pair using only one bend. These two ways are shown in Fig. 1.2, and are respectively called the *upper* and *lower* wirings.

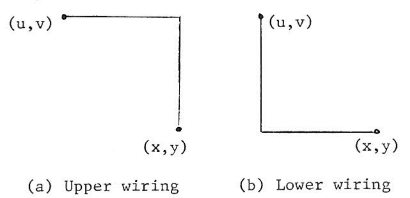

FIG. 1.2. Possible single bend wirings.

In Section 2, we show that it is possible to efficiently determine whether S is single bend wireable on one layer. Our algorithm has time complexity $O(n^2)$, where $n = |S|$. For wire sets S that are single bend wireable on one layer, our algorithm also obtains the layout in the stated time bound.

For those wire sets that cannot be single bend wired on one layer, the following questions are of interest:

(1) What is the minimum number of layers needed to single bend wire S? This is also called the layering problem. Minimizing the number of layers needed reduces the cost of the realization, as discussed earlier.

(2) What is the maximum subset of S that can be single bend wired on one layer? One heuristic that has been suggested [BREU72] for the layering problem is to assign as many wires as possible to the first layer; then assign as many of the remaining wires as possible to the next; and so on. It is reasonable to expect that confining the layout of each wire to any single layer and maximizing the number of wires laid out on any given layer will help reduce the number of feedthroughs required to other layers as well as the number of layers used. Since feedthroughs reduce the reliability of the system, their use is to be avoided wherever possible. Hence, the relevance of this question.

In Section 3, we show that determining the answer to question (2) is NP-hard (the reader unfamiliar with the terms NP-hard and NP-complete is referred to Garey and Johnson [GARE79], Sahni *et al.* [SAHN80], and Ullman [ULLM84]). In Section 4, we show that determining whether S is single bend wireable on two layers is NP-complete. This implies that determining the answer to question (1) is NP-hard. Note that the result obtained in Section 3 rules out the most obvious greedy approach to the layering problem (i.e., put as many wires as possible on the first layer; put as many of the remaining wires as possible on the second layer; etc.).

Further complexity results concerning design automation problems can be found in Johnson [JOHN82], Ullman [ULLM84], and Sahni *et al.* [SAHN80].

2. THE FEASIBILITY OF SINGLE BEND WIRING ON ONE LAYER

In this section, we outline how to determine whether a given wire set S can be single bend wired feasibly (i.e., without intersections) on one layer. In general, there are two ways to lay out a wire in single bend fashion, as shown in Fig. 2.1. However, the presence of other wires may preclude one or even both these layouts (see Fig. 2.2).