Enhanced Interval Trees for Dynamic IP Router-Tables *

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Abstract

We develop an enhanced interval tree data structure that is suitable for the representation of dynamic IP router tables. Several refinements of this enhanced structure are proposed for a variety of IP router tables. For example, the data structure called BOB (binary tree on binary tree) is developed for dynamic router tables in which the rule filters are nonintersecting ranges and in which ties are broken by selecting the highest-priority rule that matches a destination address. Prefix filters are a special case of nonintersecting ranges and the commonly used longest-prefix tie breaker is a special case of the highest-priority tie breaker. When an n-rule router table is represented using BOB, the highest-priority rule that matches a destination address may be found in $O(\log^2 n)$ time; a new rule may be inserted and an old one deleted in $O(\log n)$ time. For general ranges, the data structure CBOB (compact BOB is proposed). For the case when all rule filters are prefixes, the data structure PBOB (prefix BOB) permits highest-priority matching as well as rule insertion and deletion in $O(W)$ time, where W is the length of the longest prefix, each. When all rule filters are prefixes and longest-prefix matching is to be done, the data structures LMPBOB (longest matching-prefix BOB) permits longest-prefix matching in $O(W)$ time; rule insertion and deletion each take $O(\log n)$ time. On practical rule tables, BOB and PBOB perform each of the three dynamic-table operations in $O(\log n)$ time and with $O(\log n)$ cache misses. The number of cache misses incurred by LMPBOB is also $O(\log n)$. Experimental results also are presented.

Keywords: Interval trees, packet classification, packet routing, router tables, highest-priority matching, longest-prefix matching, dynamic rule-tables, rule insertion and deletion.

1 Introduction

An Internet router classifies incoming packets into flows[1] utilizing information contained in packet headers and a table of (classification) rules. This table is called the *rule table* (equivalently, *router table*). In this paper, we assume that packet classification is done using only the destination address of a packet. Each rule-table rule is a pair of the form (F, A), where F is a filter and A is an action. The action component of a rule specifies what is to be done when a packet that satisfies the rule filter is received. Sample actions are drop the packet, forward the packet along a certain output link, and reserve a specified amount of bandwidth. We also assume that each rule filter is a range $[u, v]$ of destination addresses. A filter *matches*

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[1]A *flow* is a set of packets that are to be treated similarly for routing purposes.

the destination address d iff $u \leq d \leq v$. Since an Internet rule-table may contain several rules that match a given destination address d, a tie breaker is used to select a rule from the set of rules that match d. For purposes of this tie breaker, each rule is assigned a priority, and the highest-priority rule that matches d determines the action for all packets whose destination address is d (We may assume either that all priorities are distinct or that selection among rules that have the same priority may be done in an arbitrary fashion). Rule tables in which the filters are ranges and in which the highest-priority matching filter tie breaker is used are referred to as highest-priority range-tables (HPRT). When the filters of no two rules of an HPRT intersect, the HPRT is a nonintersecting HPRT (NHPRT).

In a *static* rule table, the rule set does not vary in time. For these tables, we are concerned primarily with the following metrics:

1. *Time required to process an incoming packet.* This is the time required to search the rule table for the rule to use. We refer to this operation as a **lookup**.

2. *Preprocessing time.* This is the time to create the rule-table data structure.

3. *Storage requirement.* That is, how much memory is required by the rule-table data structure?

In practice, rule tables are seldom truly static. At best, rules may be added to or deleted from the rule table infrequently. Typically, in a "static" rule table, inserts/deletes are batched and the rule-table data structure reconstructed as needed.

In a *dynamic* rule table, rules are added/deleted with some frequency and the action component of the rule may also be changed. For such tables, inserts/deletes are not batched. Rather, they are performed in real time. For such tables, we are concerned additionally with the time required to insert/delete a rule. For a dynamic rule table, the initial rule-table data structure is constructed by starting with an empty data structure and then inserting the initial set of rules into the data structure one by one. So, typically, in the case of dynamic tables, the preprocessing metric, mentioned above, is very closely related to the insert time.

Data structures for rule tables in which each filter is a destination address prefix and the rule priority is the length of this prefix[2] have been intensely researched in recent years. We refer to rule tables of this type as longest-matching prefix-tables (LMPT). Although every LMPT is also an NHPRT, an NHPRT may not be an LMPT. We use W to denote the maximum possible length of a prefix. In IPv4, $W = 32$ and in IPv6, $W = 128$.

[2]For example, the filter 10* matches all destination addresses that begin with the bit sequence 10; the length of this prefix is 2.

Although much of the research in the router-table area has focused on static prefix-tables, our focus here is dynamic prefix- and range-tables. We are motivated to study such tables for the following reasons. First, in a prefix-table, aggregation of prefixes is limited to pairs of prefixes that have the same length and match contiguous addresses. In a range-table, we may aggregate prefixes and ranges that match contiguous addresses regardless of the lengths of the prefixes and ranges being aggregated. So, range aggregation is expected to result in router tables that have fewer rules. Second, with the move to QoS services, router-table rules include ranges for port numbers (for example). Although ternary content addressable memories (TCAMs), the most popular hardware solution for prefix tables, can handle prefixes naturally, they are unable to handle ranges directly. Rather, ranges are decomposed into prefixes. Since each range takes up to $2W - 2$ prefixes to represent, decomposing ranges into prefixes may result in a large increase in router-table size. Since data structures for multidimensional classifiers are built on top of data structures for one-dimensional classifiers, it is necessary to develop good data structures for one-dimensional range router-tables (as we do in this paper). Third, in firewall filter-tables, the highest-priority matching tie breaker, which is a generalization of the first matching-rule tie breaker, is usually used. Fourth, dynamic tables that permit high-speed inserts and deletes are essential in QoS applications [1]. For example, edge routers that do stateful filtering require high-speed updates [2]. For forwarding tables at backbone routers, Labovitz et al. [3] found that the update rate could reach as high as 1000 per second. These updates stem from route failures, route repair and route fail-over. With the number of autonomous systems continuously increasing, it's reasonable to expect an increase in the required update rate.

There are basically two strategies to handle router-table updates. In one, we employ two copies–working and shadow–of the router table. Lookups are done using the working table. Updates are performed, in the background (either in real time on the shadow table or by batching updates and reconstructing an updated shadow at suitable intervals); periodically, the shadow replaces the working table. In this mode of operation, static schemes that optimize lookup time are suitable. However, in this mode of operation, many packets may be misclassified, because the working copy isn't immediately updated. The number of misclassified packets depends on the periodicity with which the working table can be replaced by an updated shadow. Further, additional memory is required for the shadow table and/or for periodic reconstruction of the working table. In the second mode of operation, there is only a working table and updates are made directly to the working table. In this mode, no packet is improperly classified. However, packet classification may be delayed while a preceding update completes.

To minimize this delay, it is essential that updates be done as fast as possible.

In this paper, we focus on data structures for dynamic NHPRTs, HPPTs and LMPTs. The case of general ranges (i.e., possibly intersecting ranges) is briefly considered in Section 6. We begin, in Section 3, by developing the terminology used in this paper. In Section 4, we review two popular but different interval tree structures. One of these is refined to obtain our proposed enhanced interval tree structure. In Section 5, we develop the data structure binary tree on binary tree (BOB), which, itself, is a refinement of our proposed enhanced interval tree structure. This data structure is proposed for the representation of dynamic NHPRTs. Using BOB, a lookup takes $O(\log^2 n)$ time and cache misses; a new rule may be inserted and an old one deleted in $O(\log n)$ time and cache misses. For HPPTs, we propose a modified version of BOB–PBOB (prefix BOB)–in Section 7. Using PBOB, a lookup, rule insertion and deletion each take $O(W)$ time and cache misses. In Section 8, we develop the data structures LMPBOB (longest matching-prefix BOB) for LMPTs. Using LMPBOB, the longest matching-prefix may be found in $O(W)$ time and $O(\log n)$ cache misses; rule insertion and deletion each take $O(\log n)$ time and cache misses. On practical rule tables, BOB and PBOB perform each of the three dynamic-table operations in $O(\log n)$ time and with $O(\log n)$ cache misses. Note that since an LMPT is also an HPPT and an NPHRT, BOB and PBOB may be used for LMPTs also. Experimental results are presented in Section 9.

2 Related Work

Data structures for longest-matching prefix-tables (LMPT) have been intensely researched in recent years. Ruiz-Sanchez et al. [4] review data structures for static LMPTs and Sahni et al. [5] review data structures for both static and dynamic LMPTs.

Ternary content-addressible memories, TCAMs, use parallelism to achieve $O(1)$ lookup [6]. A prefix may be inserted or deleted in $O(W)$ time, where W is the length of the longest prefix [7][3]. Although TCAMs provide a simple and efficient solution for static and dynamic router tables, this solution requires special hardware, costs more, and uses more power and board space than solutions that employ SDRAMs [8]. EZchip Technologies, for example, claim that classifiers can forgo TCAMs in favor of commodity memory solutions [2,8]. Algorithmic approaches that have lower power consumption and are conservative on board space at the price of slightly increased search latency are sought. "System vendors are willing to accept some latency in their searches if it means lowering the power of a line card" [8].

Several trie-based data structures for LMPTs have been proposed [9–15]. Structures such as that of [9]

[3]More precisely, W may be defined to be the number of different prefix lengths in the table.

perform each of the dynamic router-table operations (lookup, insert, delete) in $O(W)$ time. Others (e.g., [10–16]) attempt to optimize lookup time and memory requirement through an expensive preprocessing step. These structures, while providing very fast lookup capability, have a prohibitive insert/delete time and so, they are suitable only for static router-tables (i.e., tables into/from which no inserts and deletes take place).

Gupta et al. [16], for example, propose the DIR-24-8 scheme. This scheme uses a 32 MB table with 2^{24} 16-bit entries together with a potentially much larger table that has t 256-entry blocks, where t is the number of distinct 24-bit sequences x such x is the first 24 bits of some length 25 or greater prefix of the router table. Using these two tables, it is possible to find the longest matching-prefix in at most 2 memory accesses. Although the scheme using an excessive amount of memory, Gupta et al. [16] propose alternatives that use less memory but require more memory accesses.

Waldvogel et al. [17] have proposed a scheme that performs a binary search on hash tables organized by prefix length. This binary search scheme has an expected complexity of $O(\log W)$ for lookup. An alternative adaptation of binary search to longest-prefix matching is developed in [18]. Using this adaptation, a lookup in a table that has n prefixes takes $O(W + \log n) = O(W)$ time. Because the schemes of [17] and [18] use expensive precomputation, they are not suited for a dynamic router-tables.

For dynamic TCAM tables, Shah and Gupta [7] show how to insert and delete in $O(W)$ time. Continuing with the special-purpose hardware theme, Basu et al. [19] describe a dynamic router-table design for pipelined forwarding engines.

Suri et al. [20] have proposed an all-algorithmic design for dynamic LMPTs. Their design employs the B-tree data structure and permits one to find the longest matching-prefix, $LMP(d)$, in $O(\log n)$ time. However, inserts/deletes take $O(W \log n)$ time. When W bits fit in $O(1)$ words (as is the case for IPv4 and IPv6 prefixes) logical operations on W-bit vectors can be done in $O(1)$ time each. In this case, the scheme of [20] takes $O(W + \log n) = O(W)$ time for an update. The number of cache misses that occur when the data structure of [20] is used is $O(\log n)$ per operation.

Sahni and Kim [21, 22] develop data structures, called a collection of red-black trees (CRBT) and alternative collection of red-black trees (ACRBT), that support the three operations of a dynamic LMPT in $O(\log n)$ time each. The number of cache misses is also $O(\log n)$. In [22], Sahni and Kim show that their ACRBT structure is easily modified to extend the biased-skip-list structure of Ergun et al. [23] so as to obtain a biased-skip-list structure for dynamic LMPTs. Using this modified biased skip-list structure, lookup, insert, and delete can each be done in $O(\log n)$ expected time and $O(\log n)$ expected

cache misses. Like the original biased-skip list structure of [23], the modified structure of [22] adapts so as to perform lookups faster for bursty access patterns than for non-bursty patterns. The ACRBT structure may also be adapted to obtain a collection of splay trees structure [22], which performs the three dynamic LMPT operations in $O(\log n)$ amortized time and which adapts to provide faster lookups for bursty traffic. Lu and Sahni [24] show how priority search trees may be used to support the three dynamic router-table operations in $O(\log n)$ time each. Their work applies to both prefix filters as well as to the case of a set of conflict-free range filters.

When highest-priority prefix-table (HPPT) is represented as a binary trie [25], each of the three operations takes $O(W)$ time and cache misses. Gupta and McKeown [26] have developed two data structures for dynamic highest-priority range-table (HPRT)—heap on trie (HOT) and binary search tree on trie (BOT). The HOT structure takes $O(W)$ time for a lookup and $O(W \log n)$ time for an insert or delete. The BOT structure takes $O(W \log n)$ time for a lookup and $O(W)$ time for an insert/delete. The number of cache misses in a HOT and BOT is asymptotically the same as the time complexity of the corresponding operation.

3 Preliminaries

Definition 1 *A* **range** $r = [u, v]$ *is a pair of addresses* u *and* v, $u \leq v$. *The range* r *represents the addresses* $\{u, u+1, ..., v\}$. $\boldsymbol{start(r)} = u$ *is the start point of the range and* $\boldsymbol{finish(r)} = v$ *is the finish point of the range. The range* r **matches** *all addresses* d *such that* $u \leq d \leq v$.

The prefix 1101* matches all destination addresses that begin with 1101 and 10010* matches all destination addresses that begin with 10010. We use W to denote the maximum possible length of a prefix. For example, when $W = 5$, 1101* matches the addresses $\{11010, 11011\} = \{26, 27\}$, and when $W = 6$, 1101* matches $\{110100, 110101, 110110, 110111\} = \{52, 53, 54, 55\}$. Notice that every prefix of a prefix router-table is a range. For example, when $W = 6$, the prefix $p = 1101*$ may be represented as range $[52, 55]$. So, we say $p = 1101* = [52, 55]$, $start(p) = 52$, and $finish(p) = 55$. However, a range may not be represented as a prefix. For example, The range [3,9] does not correspond to any single prefix. When $W = 5$, we need three prefixes, 00011, $001*$ and $0100*$, to represent [3,9]. Actually each range takes up to $2W - 2$ prefixes to represent.

Since a range represents a set of (contiguous) points, we may use standard set operations and relations such as $\cap$ and $\subset$ when dealing with ranges. So, for example, $[2, 6] \cap [4, 8] = [4, 6]$. Note that some operations between ranges my not yield a range. For example, $[2, 6] \cup [8, 10] = \{2, 3, 4, 5, 6, 8, 9, 10\}$,

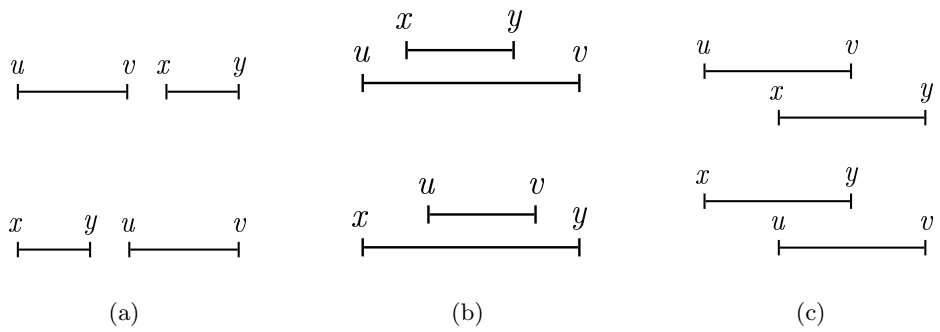


Figure 1: Relationships between pairs of ranges. (a). Disjoint ranges. (b). Nested ranges. (c). Intersecting ranges.

which is not a range.

Definition 2 *Let $r = [u, v]$ and $s = [x, y]$ be two ranges.*

(a) r and s are disjoint iff $r \cap s = \emptyset$

(b) r and s are nested iff one of the ranges is contained within the other, i.e., $r \subseteq s$ or $s \subseteq r$

(c) r and s are intersecting iff r and s have a nonempty intersection that is different from both r and s, i.e., $u < x \leq v < y$ or $x < u \leq y < v$.

Figure 1 shows the relationships between pairs of ranges. [2, 4] and [6, 9] are disjoint; [2,4] and [3,4] are nested; [2,4] and [4,6] intersect.

Definition 3 *Let R be a range set.* ***ranges(d, R)*** *(or simply $ranges(d)$ when R is implicit) is the subset of ranges of R that match the destination address d.*

When $R = \{[2, 4], [1, 6], [7, 10]\}$, $ranges(3) = \{[2, 4], [1, 6]\}$.

4 Enhanced Interval Tree

4.1 Interval Trees

Two fundamentally different interval tree data structures for range data are described in [27] and [28]. Although both employ a red-black tree and in both each range is stored in exactly one node of this red-black tree, the structures differ in the strategy used to allocate ranges to nodes. In the interval tree data structure of [27], each node of the red-black tree z stores exactly one range, $range(z)$. Hence, the

red-black tree has exactly n nodes, where n is the number of ranges. Additionally, each node z stores a value $maxFinish(z)$, which is the maximum of the finish points of all ranges stored in the subtree rooted at z. Each range r in the left subtree of z satisfies

$$(start(r) < start(range(z)) \text{ or } (start(r) = start(range(z)) \text{ and } finish(r) > finish(range(z)))$$

Each range r in the right subtree of z satisfies

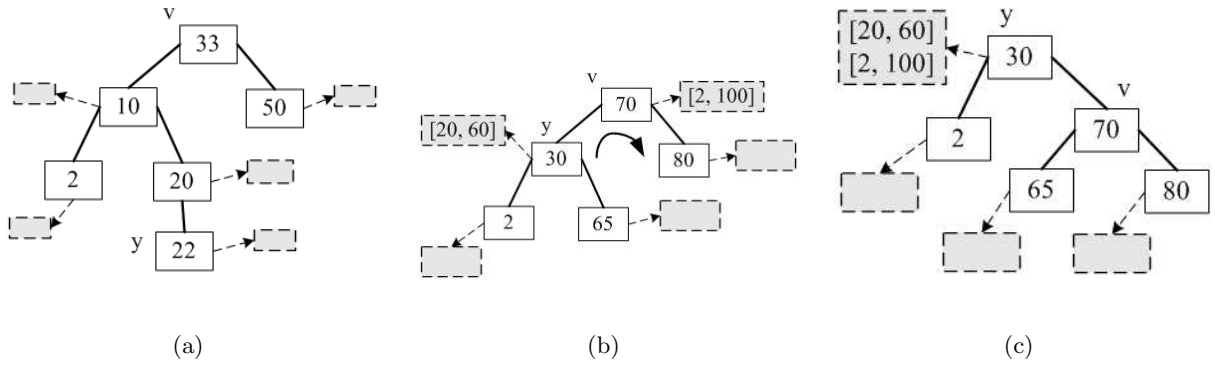
$$(start(r) > start(range(z)) \text{ or } (start(r) = start(range(z)) \text{ and } finish(r) < finish(range(z)))$$

When the prefixes of a router table are stored in the interval tree of [27], prefix insertion and deletion take $O(\log n)$ time. However, it takes $O(\min\{n, k \log n\})$ time to find the longest matching-prefix (as well as the highest-priority matching-prefix), where k is the number of prefixes that match the given destination address.

In the interval tree of [28], each node z of the red-black tree stores a non-empty subset, $ranges(z)$[4], of the ranges. Hence the number of nodes in the red-black tree is $O(n)$. There is a point, $point(z)$, associated with every node of the red-black tree. The points in the left subtree of z are $< point(z)$ and those in its right subtree are $> point(z)$. Hence, we call this red-black tree the point search tree (PTST). Let R be the set of ranges stored in the PTST and let $root$ be the root of the PTST. All ranges $r \in R$ such that $start(r) \leq point(z) \leq finish(r)$ are stored in the root (these ranges define $ranges(z)$); all $r \in R$ such that $finish(r) < point(z)$ are stored in the left subtree of z; and the remaining ranges of R are stored in the right subtree of z. This range allocation rule is recursively applied to the left and right subtrees of the PTST. In each node z of the PTST, two lists, $left(z)$ and $right(z)$, are maintained. $left(z)$ stores the ranges of $ranges(z)$ sorted by their start points while $right(z)$ stores these ranges sorted by their finish points. Using this interval tree organization for a prefix router-table, the longest matching-prefix (as well as the highest-priority matching-prefix) can be found in $O(\log n + k)$ time. Prefix insertion and deletion are very expensive.

The high complexity of the update operations stems from the requirement that $ranges(z)$ be non-empty for every node z. Consider the deletion operation. Suppose that following the deletion of a range, $ranges(v)$ becomes empty for some node v. When v is a degree-2 node, we replace $point(v)$ with either the largest point in the left subtree of v or the smallest point in the right subtree of v. Suppose we do the former. Further suppose that the largest point is in the node y. The ranges stored in all nodes on

[4]We have overloaded the function $ranges$. When u is a node, $ranges(u)$ refers to the ranges stored in node u; when u is a destination address, $ranges(u)$ refers to the ranges that match u



(a) (b) (c)

Figure 2: (a). Node v becomes empty. (b). Before rotation. (c). After rotation

the path from v to y need to be examined and possibly relocated to satisfy the PTST range allocation rule. This reallocation may cause one or more of the nodes on the path from v to y to become empty, and so the process needs to be repeated.

As an example, consider the PTST of Figure 2(a). The number inside a node is its point value; shaded boxes show $ranges(z)$. Suppose that $ranges(v)$ has become empty following the deletion of a range. Now $point(v)$ is replaced with the largest point, 22, in the left subtree of v. Now we need to search $ranges(z)$ for every node z on the path from v to y and move those ranges that include 22 to node v.

Rebalancing rotations performed during an insertion or deletion also complicate the update operation. Suppose that an LL rotation (see [25]) is to be performed in Figure 2(b) with v as the pivot. Figure 2(c) shows the tree after the rotation. Since node y becomes the parent of node v, it may be necessary to move some ranges from $ranges(v)$ to $ranges(y)$ to satisfy the PTST range allocation rule. In our example, the range [2, 100] is moved from $ranges(v)$ to $ranges(y)$. Notice that now node v becomes empty and needs to be removed. The removal of v may cause yet other nodes to become empty.

4.2 Our Enhanced Interval Tree

Our enhanced interval tree is the PTST of [28] with the following modifications/restrictions.

1. $ranges(z)$ may be empty for some nodes of the PTST.

2. The total number of nodes in the PTST is at most $2n$, where n is the number of ranges stored in the PTST. This restriction on the number of nodes in the PTST is referred to as the **PTST size constraint**.

Note that the number of nodes in the PTST of [28] is $\leq n$ and the PTST has no node z with an empty $ranges(z)$. Henceforth, we use PTST to refer to the PTST of our enhanced interval tree. Table 1 gives an example set of ranges, and Figure 3 shows a possible PTST for this set of ranges (we say possible, because we haven't specified how to select the $point(z)$ values and even with specified $point(z)$ values, the corresponding red-black tree isn't unique). The number inside each node is $point(z)$, and outside each node, we give $ranges(z)$. Since the PTST may have as many as $2n$ nodes and since each range of R is in exactly one of the sets $ranges(z)$, some of the $ranges(z)$ sets may be empty.

range	priority
[2, 100]	4
[2, 4]	33
[2, 3]	34
[8, 68]	10
[8, 50]	9
[10, 50]	20
[10, 35]	3
[15, 33]	5
[16, 30]	30
[54, 66]	18
[60, 65]	7
[69, 72]	10
[80, 80]	12

Table 1: A sample range set

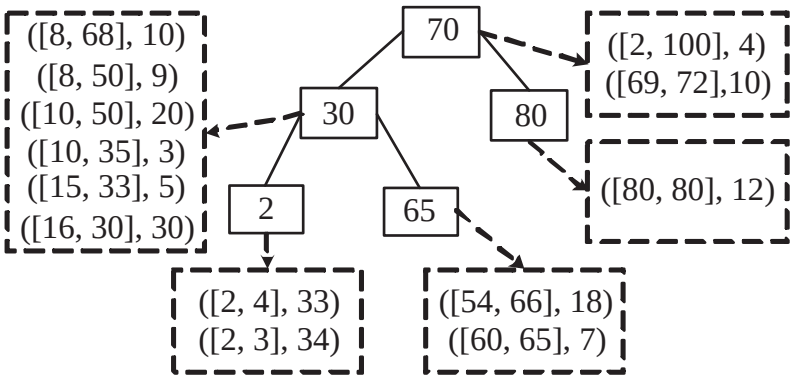


Figure 3: A possible PTST

4.2.1 Insert a Range

Figure 4 shows the algorithm to insert a range r. In the `while` loop, we find the node z nearest to the root such that r matches $point(z)$ (i.e., $start(r) \leq point(z) \leq finish(r)$). If such a z exists, the range r is inserted into $ranges(z)$. In case the PTST has no z such that r matches $point(z)$, we insert a new node into the PTST. This insertion is done using the method $insertNewNode$.

```
Algorithm insert(r) {
  //insert the range r
  z = root; //root of PTST
  while (z ≠ null){
    if (finish(r) < point(z))
      z = leftChild(z);
    else if (start(r) > point(z))
      z = rightChild(z);
    else//r matches point(z)
      {ranges(z).insert(r); return; }
  }
  //there is no node z such that r matches point(z)
  //insert a new node into PTST
  insertNewNode(r);
}
```

Figure 4: Algorithm to insert a range r

To insert a new node into the PTST, we first create a new PTST node y and define $point(y)$ and $ranges(y)$ ($ranges(y)$ is empty initially). $point(y)$ may be set to be any value matched by r (i.e., any value such that $start(r) \leq point(y) \leq finish(r)$) may be used. In our implementation, we use $point(y) = start(r)$. If the PTST is currently empty, y becomes the new root and we are done. Otherwise, the new node y may be inserted where the search conducted in the `while` loop of Figure 4 terminated. That is, as a child of the last non-null value of z. Following this insertion, the traditional bottom-up red-black rebalancing pass is made [25]. This rebalancing pass may require color changes and at most one rotation. Color changes do not affect the tree structure. However, a rebalancing rotation, if performed, affects the tree structure and may lead to a violation of the range allocation rule. Rebalancing rotations are investigated in the next section.

We note that if the number of nodes in the PTST was at most $2|R|$, where $|R|$ is the number of ranges prior to the insertion of a new range r, then following the insertion, $|PTST| \leq 2|R| + 1 < 2(|R| + 1)$, where $|PTST|$ is the number of nodes in the PTST and $|R| + 1$ is the number of ranges following the

insertion of r. Hence an insert does not violate the PTST size constraint.

Exclusive of the time required to perform the tasks associated with a rebalancing rotation (notice that at most one rebalancing rotation is needed following an insert), the time required to insert a range is $O(height(PTST)) = O(\log n)$.

4.2.2 Red-Black-Tree Rotations

Figures 5 and 6, respectively, show the red-black LL and RR rotations used to rebalance a red-black tree following an insert or delete (see [25]). In these figures, $pt()$ is an abbreviation for $point()$. Since the remaining rotation types, LR and RL, may, respectively, be viewed as a RR rotation followed by an LL rotation and an LL rotation followed by an RR rotation, it suffices to examine LL and RR rotations alone.

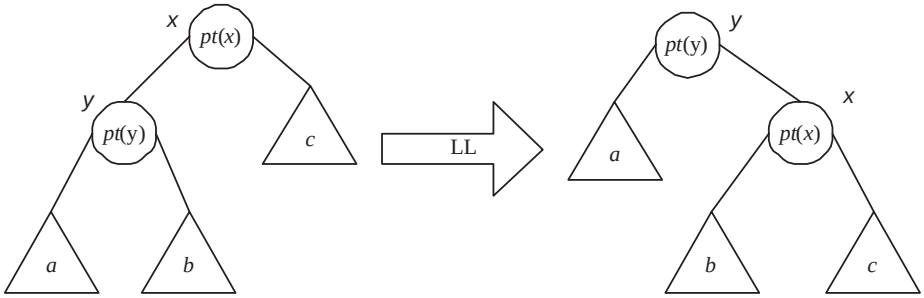


Figure 5: LL Rotation

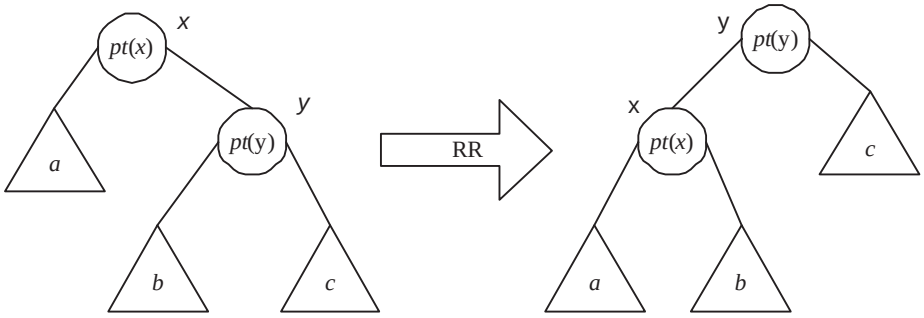


Figure 6: RR Rotation

Lemma 1 *Let R be a set of ranges. Let $ranges(z) \subseteq R$ be the ranges allocated by the range allocation rule to node z of the PTST prior to an LL or RR rotation. Let $ranges'(z)$ be this subset for the PTST node z following the rotation. Then $ranges(z) = ranges'(z)$ for all nodes z in the subtrees a, b, and c of*

Figures 5 and 6.

Proof Consider an LL rotation. Let $ranges(subtree(x))$ be the union of the range sets allocated to the nodes in the subtree whose root is x. Since the range allocation rule allocates each range r to the node z nearest the root such that r matches $point(z)$, $ranges(subtree(x)) = ranges'(subtree(y))$. Further, $r \in ranges(a)$ iff $r \in ranges(subtree(x))$ and $finish(r) < point(y)$. Consequently, $r \in ranges'(a)$. From this and the fact that the LL rotation doesn't change the positioning of nodes in a, it follows that for every node z in the subtree a, $ranges(a) = ranges'(a)$. The proof for the nodes in b and c as well as for the RR rotation is similar. ■

Let x and y be as in Figures 5 and 6. From Lemma 1, it follows that $ranges(z) = ranges'(z)$ for all z in the PTST except possibly for $z \in \{x, y\}$. It is not too difficult to see that $ranges'(y) = ranges(y) \cup S$ and $ranges'(x) = ranges(x) - S$, where $S = \{r | r \in ranges(x)$ **and** $start(r) \leq point(y) \leq finish(r)\}$.

The time required to do an LL or RR rotation depends on the time taken to determine S, remove S from $ranges(x)$, and add S to $ranges(y)$. This depends on the data structure used to represent $ranges()$. The time for LR and RL rotations is roughly twice that for LL and RR rotations.

4.2.3 Delete a Range

Figure 7 gives our algorithm to delete a range r. Note that if r is one of the ranges in the PTST, then r must be in the $ranges(z)$ of the node z that is closest to the root and such that r matches $point(z)$. The `while` loop of Figure 7 finds this z and deletes r from $ranges(z)$.

```
Algorithm delete(r) {
  // delete the range r
  z = root; // root of PTST
  while (z ≠ null)
    if (finish(r) < point(z))
      z = leftChild(z);
    else if (start(r) > point(z))
      z = rightChild(z);
    else {// r matches point(z)
      ranges(z).delete(r);
      cleanup(z);
      return;
    }
}
```

Figure 7: Algorithm to delete a range

Assume that r is, in fact, one of the ranges in our PTST. We delete r from $ranges(z)$. Then we perform a cleanup operation that is necessary to maintain the size constraint of the PTST. Figure 8 gives the steps in the method *cleanup*.

```
Algorithm cleanup(z) {
  // maintain size constraint
  if (ranges(z) is empty and the degree of z is 0 or 1)
    delete node z from the PTST and rebalance;

  while (|PTST| > 2|R|)
    delete a degree 0 or degree 1 node z with empty
    ranges(z) from the PTST and rebalance;
}
```

Figure 8: Algorithm to maintain size constraint following a delete

Notice that following the deletion of r from $ranges(z)$, $ranges(z)$ may or may not be empty. If $ranges(z)$ becomes empty and the degree of node z is either 0 or 1, node z is deleted from the PTST using the standard red-black node deletion algorithm [25]. If this deletion requires a rotation (at most one rotation may be required) the rotation is done as described in Section 4.2.2. Since the number of ranges and nodes has each decreased by 1, the size constraint may be violated (this happens if $|PTST| = 2|R|$ prior to the delete). Hence, it may be necessary to remove a node from the PTST to restore the size constraint.

If $ranges(z)$ becomes empty and the degree of z is 2 or if $ranges(z)$ does not become empty, z is not deleted from the PTST. Now, $|PTST|$ is unchanged by the deletion of r and $|R|$ reduces by 1. Again, it is possible that we have a size constraint violation. If so, up to two nodes may have to be removed from the PTST to restore the size constraint.

The size constraint, if violated, is restored in the `while` loop of Figure 8. This restoration is done by removing one or two (as needed) degree 0 or degree 1 nodes that have an empty $ranges()$. Lemma 2 shows that whenever the size constraint is violated, the PTST has at least one degree 0 or degree 1 node with an empty $ranges()$. So, the node z needed for deletion in each iteration of the `while` loop always exists.

Lemma 2 *When the PTST has $> 2n$ nodes, where $n = |R|$, the PTST has at least one degree 0 or degree 1 node that has an empty $ranges()$.*

Proof Suppose not. Then the degree of every node that has an empty $ranges()$ is 2. Let n_2 be the

total number of degree 2 nodes, n_1 the total number of degree 1 nodes, n_0 the total number of degree 0 nodes, n_e the total number of nodes that have an empty $ranges()$, and n_n the total number of nodes that have a nonempty $ranges()$. Since all PTST nodes that have an empty $ranges()$ are degree 2 nodes, $n_2 \geq n_e$. Further, since there are only n ranges and each range is stored in exactly one $ranges()$, there are at most n nodes that have a nonempty $ranges()$, i.e., $n \geq n_n$. Thus $n_2 + n \geq n_e + n_n = |PTST|$, i.e., $n_2 \geq |PTST| - n$. From [25] (Lemma 5.3), we know that $n_0 = n_2 + 1$. Hence, $n_0 + n_1 + n_2 = n_2 + 1 + n_1 + n_2 > n_2 + n_2 \geq 2|PTST| - 2n > |PTST|$. This contradicts $n_0 + n_1 + n_2 = |PTST|$. ■

To find the degree 0 and degree 1 nodes that have an empty $ranges()$ efficiently, we maintain a doubly-linked list of these nodes. Also, a doubly-linked list of degree 2 nodes that have an empty $ranges()$ is maintained. When a range is inserted or deleted, PTST nodes may be added/removed from these doubly-linked lists and nodes may move from one list to another. The required operations can be done in $O(1)$ time each.

Notice that we only remove degree 0 or degree 1 nodes from the PTST. The cleanup step removes up to 2 nodes from the PTST. At most one rebalancing rotation is needed following the removal of a node from the PTST. Thus $O(1)$ number of $ranges()$s need to be adjusted. The overall delete time is the $O(\log n)$ time needed to find the PTST node z that contains the range r that is to be deleted, plus the time to delete r from $ranges(z)$, plus the time needed to adjust $O(1)$ number of $ranges()$s. The complexity of deleting r from $ranges(z)$ and adjusting $ranges()$s depends on the data structure used for $ranges()$s.

5 BOB for Nonintersecting Ranges

In this section, we refine the enhanced interval tree structure of Section 4.2 to obtain a data structure, BOB (binary tree on binary tree), for nonintersecting ranges. BOB comprises a red-black PTST as in Section 4.2. This tree is called the top-level tree. For every node z of the PTST, $ranges(z)$ is represented as a balanced binary search tree called the range search tree (RST). The RST in each node is called a second-level tree. Using BOB, we can find the highest-priority matching range in $O((\log n)(\log maxR))$ time, where n is the number of ranges and $maxR$ is the maximum number of ranges that match any destination address (so $|ranges(z)| \leq maxR$ for every node z of the PTST). Inserting or deleting a range can be done in $O(\log n)$ time. The data structure uses $O(n)$ space.

Sahni and Kim [21] have analyzed the prefixes in several real IPv4 prefix router-tables. They report that a destination address is matched, on average, by about 1 prefix; the maximum number of prefixes

that match a destination address is at most 6. Assuming that this analysis holds true even for real range router-tables (no data is available for us to perform such an analysis), we conclude that $maxR \leq 6$. So, the expected complexity of BOB on real router-tables is $O(\log n)$ per operation. Note that every prefix set is a set of nonintersecting ranges.

5.1 Nonintersecting Ranges

Definition 4 *The range set R is **nonintersecting** iff no pair of ranges intersect (Figure 1(c)).*

Definition 5 *Let R be a range set. **hpr(d)** is the highest-priority range in $ranges(d)$. We assume that ranges are assigned priorities in such a way that $hpr(d)$ is uniquely defined for every d.*

Now we define the $<$ relation between two ranges. Range r is less than range s iff the start point of r is smaller than the start point of s, or the finish point of r is larger than the finish point of s when they have the same start point. Note that for every pair, r and s, of different ranges, either $r < s$ or $s < r$.

Definition 6 *Let r and s be two ranges. $r < s \Leftrightarrow$*
*$start(r) < start(s)$ **or** $(start(r) = start(s)$ **and** $finish(r) > finish(s))$.*

Lemma 3 *Let R be a nonintersecting range set. If $r \cap s \neq \emptyset$ for r , $s \in R$, then the following are true:*

1. $start(r) < start(s) \Rightarrow finish(r) \geq finish(s)$.
2. $finish(r) > finish(s) \Rightarrow start(r) \leq start(s)$.

Proof Straightforward. ■

5.2 The Data Structure for $ranges()$

The set of ranges in Table 1 is a nonintersecting range set. For such range set, the ranges in $ranges(z)$ may be ordered using the $<$ relation of Definition 6. Using this $<$ relation, we put the ranges of $ranges(z)$ into a red-black tree (any balanced binary search tree structure that supports efficient search, insert, delete, join, and split may be used) called the range search-tree or $RST(z)$. Each node x of $RST(z)$ stores exactly one range of $ranges(z)$. We refer to this range as $range(x)$. Every node y in the left (right) subtree of node x of $RST(z)$ has $range(y) < range(x)$ $(range(y) > range(x))$. In addition, each node x stores the quantity $mp(x)$, which is the maximum of the priorities of the ranges associated with the nodes in the subtree rooted at x. $mp(x)$ may be defined recursively as below.

$$mp(x) = \begin{cases} p(x) & \text{if } x \text{ is leaf} \\ \max\{mp(leftChild(x)), mp(rightChild(x)), p(x)\} & \text{otherwise} \end{cases}$$

where $p(x) = priority(range(x))$. Figure 9 gives a possible RST structure for $ranges(node30)$ of Figure 3.

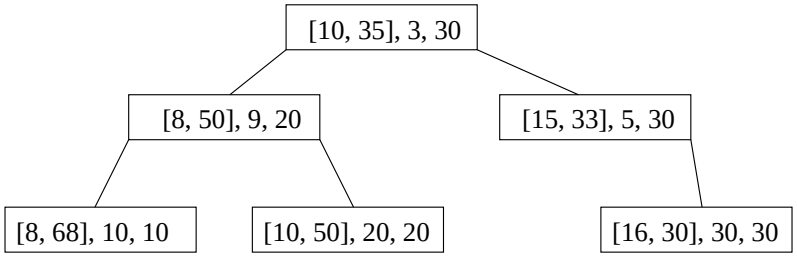


Figure 9: An example RST for $ranges(node30)$ of Figure 3. Each node shows $(range(x), p(x), mp(x))$

Lemma 4 *Let z be a node in a PTST and let x be a node in $RST(z)$. Let $st(x) = start(range(x))$ and $fn(x) = finish(range(x))$.*

1. *For every node y in the right subtree of x, $st(y) \geq st(x)$ and $fn(y) \leq fn(x)$.*

2. *For every node y in the left subtree of x, $st(y) \leq st(x)$ and $fn(y) \geq fn(x)$.*

Proof For 1, we see that when y is in the right subtree of x, $range(y) > range(x)$. From Definition 6, it follows that $st(y) \geq st(x)$. Further, since $range(y) \cap range(x) \neq \emptyset$, if $st(y) > st(x)$, then $fn(y) \leq fn(x)$ (Lemma 3); if $st(y) = st(x)$, $fn(y) < fn(x)$ (Definition 6). The proof for 2 is similar. ■

5.3 Search for $hpr(d)$

The highest-priority range that matches the destination address d may be found by following a path from the root of the PTST toward a leaf of the PTST. Figure 10 gives the algorithm. For simplicity, this algorithm finds $hp = priority(hpr(d))$ rather than $hpr(d)$. The algorithm is easily modified to return $hpr(d)$ instead.

We begin by initializing $hp = -1$ and z is set to the root of the PTST. This initialization assumes that all priorities are ≥ 0. The variable z is used to follow a path from the root toward a leaf. When $d > point(z)$, d may be matched only by ranges in $RST(z)$ and those in the right subtree of z. The method `RST(z).hpRight(d,hp)` (Figure 11) updates hp to reflect any matching ranges in $RST(z)$. This method makes use of the fact that $d > point(z)$. Consider a node x of $RST(z)$. If $d > fn(x)$, then d is to the right (i.e., $d > finish(range(x))$) of $range(x)$ and also to the right of all ranges in the right subtree of x. Hence, we may proceed to examine the ranges in the left subtree of x. When $d \leq fn(x)$, $range(x)$

```
Algorithm hp(d) {
  // return the priority of hpr(d)
  // easily extended to return hpr(d)
  hp = −1; // assuming 0 is the smallest priority value
  z = root; // root of PTST
  while (z ≠ null) {
    if (d > point(z)) {
      RST(z).hpRight(d, hp);
      z = rightChild(z);
    }else if (d < point(z)) {
      RST(z).hpLeft(d, hp);
      z = leftChild(z);
    }else // d == point(z)
      return max{hp, mp(RST(z).root)};
  }
  return hp;
}
```

Figure 10: Algorithm to find $priority(hpr(d))$

```
Algorithm hpRight(d, hp) {
  // update hp to account for any ranges in RST(z) that match d
  // d > point(z)
  x = root; // root of RST(z)
  while (x ≠ null)
    if (d > fn(x))
      x = leftChild(x);
    else {
      hp = max{hp, p(x), mp(leftChild(x))};
      x = rightChild(x);
    }
}
```

Figure 11: Algorithm $hpRight(d, hp)$

as well as all ranges in the left subtree of x match d. Additional matching ranges may be present in the right subtree of x. $hpLeft(d, hp)$ is the analogous method for the case when $d < point(z)$.

The complexity of the invocation $RST(z).hpRight(d, hp)$ is readily seen to be $O(height(RST(z)) = O(\log maxR)$. Consequently, the complexity of $hp(d)$ is $O((\log n)(\log maxR))$. To determine $hpr(d)$ we need only add code to the methods $hp(d)$, $hpRight(d, hp)$, and $hpLeft(d, hp)$ so as to keep track of the range whose priority is the current value of hp. So, $hpr(d)$ may be found in $O((\log n)(\log maxR))$ time also.

5.4 Adjust $RST()$s During Red-Black-Tree Rotations

Since we are dealing with a set of nonintersecting ranges, all ranges in $ranges(y)$ are nested within the ranges of S (see Section 4.2.2). Figure 12 shows the ranges of $ranges(x)$ using solid lines and those of $ranges(y)$ using broken lines. S is the set of ranges drawn above $ranges(y)$ (i.e., the solid lines above the broken lines).

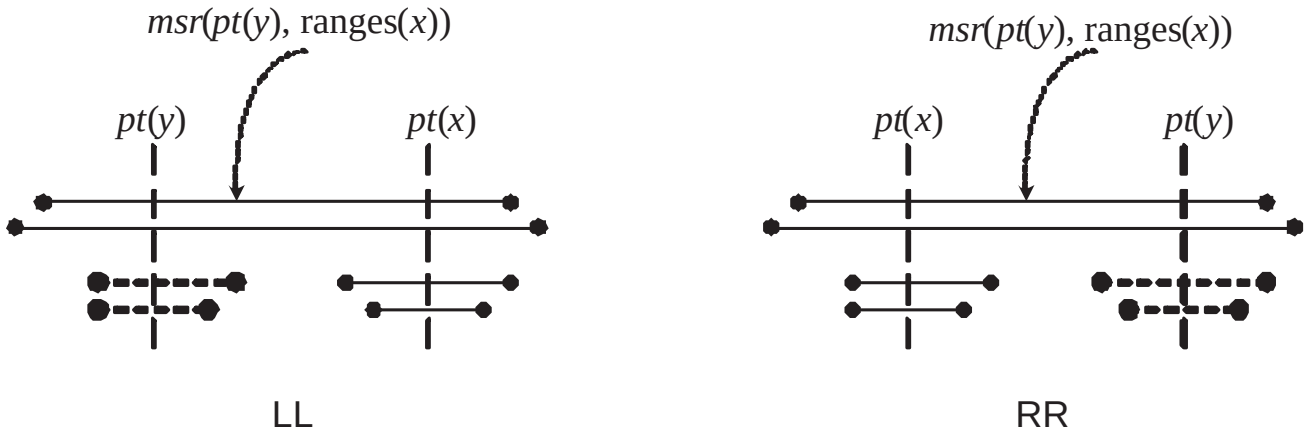


Figure 12: $ranges(x)$ and $ranges(y)$ for LL and RR rotations. Nodes x and y are as in Figures 5 and 6

The range $rMax$ of S with largest $start()$ value may be found by searching $RST(x)$ for the range with largest $start()$ value that matches $point(y)$. (Note that $rMax = msr(point(y), ranges(x))$.) Since $RST(x)$ is a binary search tree of an ordered set of ranges (Definition 6), $rMax$ may be found in $O(height(RST(x))$ time by following a path from the root downward. If $rMax$ doesn't exist, $S = \emptyset$, $ranges'(x) = ranges(x)$ and $ranges'(y) = ranges(y)$.

Assume that $rMax$ exists. We may use the *split* operation [25] to extract from $RST(x)$ the ranges that belong to S. The operation

$$RST(x) \rightarrow split(small, rMax, big)$$

separates $RST(x)$ into an RST $small$ of ranges $<$ (Definition 6) than $rMax$ and an RST big of ranges $>$ than $rMax$. We see that $RST'(x) = big$ and $RST'(y) = join(small, rMax, RST(y))$, where $join$ [25] combines the red-black tree $small$ with ranges $< rMax$, the range $rMax$, and the red-black tree $RST(y)$ with ranges $> rMax$ into a single red-black tree.

The standard *split* and *join* operations of [25] need to be modified slightly so as to update the mp values of affected nodes. This modification doesn't affect the asymptotic complexity, which is logarithmic

in the number of nodes in the tree being split or logarithmic in the sum of the number of nodes in the two trees being joined, of the *split* and *join* operations. So, the complexity of performing an LL or RR rotation (and hence of performing an LR or RL rotation) in the PTST is $O(\log maxR)$.

5.5 Insert/Delete a Range

A range r that is known to have no intersection with any of the existing ranges in the router table, may be inserted using the algorithm of Figure 4. When a new PTST node y is created, $RST(y)$ has only a root node and this root contains r; its mp value is $priority(r)$.

A rebalancing rotation can be done in $O(\log maxR)$ time. Since at most one rebalancing rotation is needed following an insert, the time to insert a range is $O(\log n + \log maxR) = O(\log n)$. In case it is necessary for us to verify that the range to be inserted does not intersect an existing range, we can augment the PTST with priority search trees as in [24] and use these trees for intersection detection. The overall complexity of an insert remains $O(\log n)$.

Assume that r is, in fact, one of the ranges $RST(z)$ of our PTST node z. To delete r from $RST(z)$, we use the standard red-black deletion algorithm [25] modified to update mp values as necessary.

It takes $O(\log n)$ time to find the PTST node z that contains the range r that is to be deleted. Another $O(\log maxR)$ time is needed to delete r from $RST(z)$. The cleanup step removes up to 2 nodes from the PTST. This takes another $O(\log n + \log maxR)$ time. So, the overall delete time is $O(\log n)$.

6 Compact BOB (CBOB) for General Range-Tables

In this section, we deal with general ranges that may intersect. We assume that $maxR$, the maximum number of ranges that match any destination address is a small constant. Hence, $|ranges(z)|$ is small for every node z of the PTST. Because of this assumption, it suffices to use an array linear list (ALL, [29]) to represent $ranges(z)$. In each node z of CBOB, we maintain 2 ALLs. $ALLleft(z)$ comprises pairs of the form $(range, priority)$ sorted by the start points of the ranges in $ranges(z)$. $ALLright(z)$ comprises pairs of the form $(range, priority)$ sorted by the finish points of the ranges in $ranges(z)$. Notice that these two lists are the same as the two lists stored in each node of the interval tree of [28].

Figure 13 gives the algorithm to find the priority of the highest-priority range that matches the destination address d. The method $ALLleft(z).maxp()$ returns the highest priority of any range in $ALLleft(z)$ (note that all ranges in $ALLleft(z)$ match $point(z)$). The method $ALLleft(z).searchALL(d, hp)$ examines the ranges in $ALLleft(z)$ and updates hp taking into account the priorities of those ranges that

match d.

```
Algorithm hp(d) {
  // return the priority of hpp(d)
  // easily extended to return hpp(d)
  hp = −1; // assuming 0 is the smallest priority value
  z = root; // root of PTST
  while (z ≠ null) {
    if (d == point(z))
      return max{hp, ALLleft(z).maxp()};
    if (d < point(z)){
      ALLleft(z).searchALL(d, hp);
      z = leftChild(z);
    }
    else{
      ALLright(z).searchALL(d, hp);
      z = rightChild(z);
    }
  }
  return hp;
}
```

Figure 13: Algorithm to find $priority(hpp(d))$

The number of PTST nodes reached in the `while` loop of Figure 13 is $O(\log n)$ and the time spent at each node z that is reached is $|ranges(z)| \leq maxR$. So, the complexity of our lookup algorithm is $O(\log n + maxR)$. The CBOB algorithms to insert/delete a range follow the strategy described in Section 4.2 for an enhanced interval tree. Since adjusting $ranges()$, following a rotation, takes $O(maxR)$ for an array linear list, the CBOB algorithms to insert/delete a range take $O(\log n + maxR)$ time. As noted earlier, $maxR$ is a small constant; so, in practice, CBOB takes $O(\log n)$ time and makes this many cache misses per operation (lookup, insert and delete).

7 Highest-Priority Prefix-Tables (HPPTs)—PBOB

7.1 The Data Structure

When all rule filters are prefixes, $maxR \leq \min\{n, W\}$. Hence, if BOB is used to represent an HPPT, the search complexity is $O(\log n * \min\{\log n, \log W\})$; the insert and delete complexities are $O(\log n)$ each.

Since $maxR \leq 6$ for real prefix router-tables, we may expect to see better performance using a simpler structure (i.e., a structure with smaller overhead and possibly worse asymptotic complexity) for $ranges(z)$ than the RST structure described in Section 5. In PBOB, we replace the RST in each node,

z, of the BOB PTST with an array linear list [29], $ALL(z)$, of pairs of the form $(pLength, priority)$, where $pLength$ is a prefix length (i.e., number of bits) and $priority$ is the prefix priority. $ALL(z)$ has one pair for each range $r \in ranges(z)$. The $pLength$ value of this pair is the length of the prefix that corresponds to the range r and the $priority$ value is the priority of the range r. The pairs in $ALL(z)$ are in ascending order of $pLength$. Note that since the ranges in $ranges(z)$ are nested and match $point(z)$, the corresponding prefixes have different length.

7.2 Lookup

Figure 14 gives the algorithm to find the priority of the highest-priority prefix that matches the destination address d. The method `maxp()` returns the highest priority of any prefix in $ALL(z)$ (note that all prefixes in $ALL(z)$ match $point(z)$). The method `searchALL(d,hp)` examines the prefixes in $ALL(z)$ and updates hp taking into account the priorities of those prefixes in $ALL(z)$ that match d.

```
Algorithm hp(d) {
  // return the priority of hpp(d)
  // easily extended to return hpp(d)
  hp = -1; // assuming 0 is the smallest priority value
  z = root; // root of PTST
  while (z ≠ null) {
    if (d == point(z))
      return max{hp, ALL(z).maxp()};
    ALL(z).searchALL(d, hp);
    if (d < point(z))
      z = leftChild(z);
    else
      z = rightChild(z);
  }
  return hp;
}
```

Figure 14: Algorithm to find $priority(hpp(d))$

The method `searchALL(d,hp)` utilizes the following lemma. Consequently, it examines prefixes of $ALL(z)$ in increasing order of length until either all prefixes have been examined or until the first (i.e., shortest) prefix that doesn't match d is examined.

Lemma 5 *If a prefix in $ALL(z)$ doesn't match a destination address d, then no longer-length prefix in $ALL(z)$ matches d.*

Proof Let p_1 and p_2 be prefixes in $ALL(z)$. Let l_i be the length of p_i. Assume that $l_1 < l_2$ and that p_1

doesn't match d. Since both p_1 and p_2 match $point(z)$, p_2 is nested within p_1. Therefore, all destination addresses that are matched by p_2 are also matched by p_1. So, p_2 doesn't match d. ■

One way to determine whether a length l prefix of $ALL(z)$ matches d is to use the following lemma. The check of this lemma may be implemented using a mask to extract the most-significant bits of $point(z)$ and d.

Lemma 6 *A length l prefix p of $ALL(z)$ matches d iff the most-significant l bits of $point(z)$ and d are the same.*

Proof Straightforward. ■

Complexity We assume that the masking operations can be done in $O(1)$ time each. (In IPv4, for example, each mask is 32 bits long and we may extract any subset of bits from a 32-bit integer by taking the logical and of the appropriate mask and the integer.) The number of PTST nodes reached in the `while` loop of Figure 14 is $O(\log n)$ and the time spent at each node z that is reached is linear in the number of prefixes in $ALL(z)$ that match d. Since the PTST has at most $maxR$ prefixes that match d, the complexity of our lookup algorithm is $O(\log n + maxR) = O(W)$ (note that $\log_2 n \leq W$ and $maxR \leq W$).

7.3 Insertion and Deletion

The PBOB algorithms to insert/delete a prefix are simple adaptations of the corresponding algorithms for BOB. $rMax$ is found by examining the prefixes in $ALL(x)$ in increasing order of length. $ALL'(y)$ is obtained by prepending the prefixes in $ALL(x)$ whose length is $\leq$ the length of $rMax$ to $ALL(y)$, and $ALL'(x)$ is obtained from $ALL(x)$ by removing the prefixes whose length is $\leq$ the length of $rMax$. The time require to find $rMax$ is $O(maxR)$. This is also the time required to compute $ALL'(y)$ and $ALL'(x)$. The overall complexity of an insert/delete operation is $O(\log n + maxR) = O(W)$.

As noted earlier, $maxR \leq 6$ in practice. So, in practice, PBOB takes $O(\log n)$ time and makes $O(\log n)$ cache misses per operation.

8 Longest-Matching Prefix-Tables (LMPTs)—LMPBOB

8.1 The Data Structure

Using $priority = pLength$, a PBOB may be used to represent an LMPT obtaining the same performance as for an HPPT. However, we may achieve some reduction in the memory required by the data structure

if we replace the array linear list that is stored in each node of the PTST by a W-bit vector, *bit*. $bit(z)[i]$ denotes the ith bit of the bit vector stored in node z of the PTST, $bit(z)[i] = 1$ iff $ALL(z)$ has a prefix whose length is i. We note that Suri et al. [20] use W-bit vectors to keep track of prefix lengths in their data structure also.

8.2 Lookup

Figure 15 gives the algorithm to find the length of the longest matching-prefix, $lmp(d)$, for destination d. The method `longest()` returns the largest i such that $bit(z)[i] = 1$ (i.e., it returns the length of the longest prefix stored in node z). The method `searchBitVector(d,hp,k)` examines $bit(z)$ and updates hp taking into account the lengths of those prefixes in this bit vector that match d. The method `same(k+1, point(z), d)` returns `true` iff $point(z)$ and d agree on their $k+1$ most significant bits.

```
Algorithm lmp(d) {
  // return the length of lmp(d)
  // easily extended to return lmp(d)
  hp = 0; // length of lmp
  k = 0; // next bit position to examine is k + 1
  z = root; // root of PTST
  while (z ≠ null) {
    if (d == point(z))
      return max{k, z.longest()};
    bit(z).searchBitVector(d, hp, k);
    if (d < point(z))
      z = leftChild(z);
    else
      z = rightChild(z);
  }
  return hp;
}
```

Figure 15: Algorithm to find $length(lmp(d))$

The method `searchBitVector(d,hp,k)` (Figure 16) utilizes the next two lemmas.

Lemma 7 *If $bit(z)[i]$ corresponds to a prefix that doesn't match the destination address d, then $bit(z)[j]$, $j > i$ corresponds to a prefix that doesn't match d.*

Proof $bit(z)[q]$ corresponds to the prefix p_q whose length is q and which equals the q most significant bits of $point(z)$. So, p_i matches all points that are matched by p_j. Hence, if p_i doesn't match d, p_j doesn't match d either. ■

```
Algorithm searchBitVector(d, hp, k) {
  // update hp and k
  while (k < W and same(k + 1, point(z), d)) {
    if (bit(z)[k + 1] == 1)
      hp = k + 1;
    k++;
  }
}
```

Figure 16: Algorithm to search a bit vector for prefixes tht match d

Lemma 8 *Let w and z be two nodes in a PTST such that w is a descendent of z. Suppose that $z.bit(q)$ corresponds to a prefix p_q that matches d. $w.bit(j)$, $j \leq q$ cannot correspond to a prefix that matches d.*

Proof Suppose that $w.bit(j)$ corresponds to the prefix p_j, p_j matches d, and $j \leq q$. So, p_j equals the j most significant bits of d. Since p_q matches d and also $point(z)$, d and $point(z)$ have the same q most significant bits. Therefore, p_j matches $point(z)$. So, by the range allocation rule, p_j should be stored in node z and not in node w, a contradiction. ■

Complexity We assume that the method `same` can be implemented using masks and Boolean operations so as to have complexity $O(1)$. Sine a bit vector has the same number of bits as does a destination address, this assumption is consistent with the implicit assumption that arithmetic on destination addresses takes $O(1)$ time. The total time spent in all invocations of `searchBitVector` is $O(W + \log n)$. The time spent in the remaining steps of `lmp(d)` is $O(\log n)$. So, the overall complexity of `lmp(d)` is $O(W + \log n) = O(W)$. Even though the time complexity is $O(W)$, the number of cache misses is $O(\log n)$ (note that each bit vector takes the same amount of space as needed to store a destination address).

8.3 Insertion and Deletion

The insert and delete algorithms are similar to the corresponding algorithms for PBOB. The essential difference are as below.

1. Rather than insert or delete a prefix from an $ALL(z)$, we set $bit(z)[l]$, where l is the length of the prefix being inserted or deleted, to 1 or 0, respectively.

2. For a rotation, we do not look for $rMax$ in $bit(x)$. Instead, we find the largest integer $iMax$ such that the prefix that corresponds to $bit(x)[iMax]$ matches $point(y)$. The first (bit 0 comes before bit 1) $iMax$ bits of $bit'(y)$ are the first $iMax$ bits of $bit(x)$ and the remaining bits of $bit'(y)$ are the

same as the corresponding bits of $bit(y)$. $bit'(x)$ is obtained from $bit(x)$ by setting its first $iMax$ bits to 0.

Complexity $iMax$ may be determined in $O(\log W)$ time using binary search; $bit'(x)$ and $bit'(y)$ may be computed in $O(1)$ time using masks and boolean operations. The remaining tasks performed during an insert or delete take $O(\log n)$ time. So, the overall complexity of an insert or delete operation is $O(\log n + \log W) = O(\log(Wn))$. The number of cache misses is $O(\log n)$.

9 Experimental Results

Test Data and Memory Requirement

We implemented the BOB, PBOB, and LMPBOB data structures and associated algorithms in C++ and measured their performance on a 1.4 GHz PC. In our implementation, each node is aligned to a 4-byte memory boundary and we use a byte as the basic unit for each field in a node. For example, we use 1 byte for the `color` field of a red-black node even though 1 bit suffices. We use 4 bytes for child pointer fields.

To assess the performance of these data structures, we used six IPv4 prefix databases obtained from [30][5]. We assigned each prefix a priority equal to its length. Hence, BOB, PBOB, and LMPBOB were all used in a longest matching-prefix mode. For dynamic router-tables that use the longest matching-prefix tie breaker, the PST structure of Lu and Sahni [24] provides $O(\log n)$ lookup, insert, and delete. So, we included the PST in our experimental evaluation of BOB, PBOB, and LMPBOB.

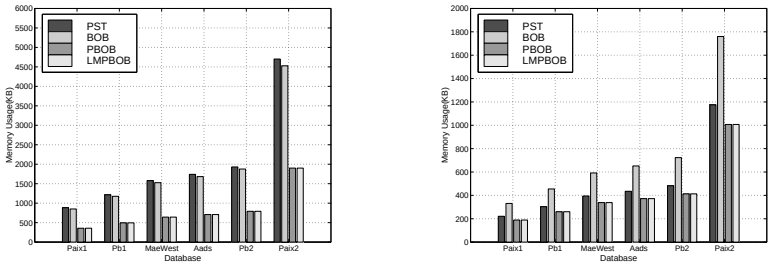
The number of prefixes in each of our 6 databases as well as the memory requirement for each database of prefixes are shown in Table 2. For the memory requirement, we performed two measurements. Measure1 gives the memory used by a data structure that is the result of a series of insertions made into an initially empty instance of the data structure. For Measure1, less than 1% of the PTST-nodes in the constructed BOB, PBOB, and LMPBOB instances are empty. So, these data structures use close to the minimum amount of memory they could use. Measure2 gives the memory used after 75% of the prefixes in the data structure constructed for Measure1 are deleted. In the resulting BOB, PBOB, and LMPBOB instances, almost half the PTST nodes are empty. The databases Paix1, Pb1, MaeWest and Aads were obtained on Nov 22, 2001, while Pb2 and Paix2 were obtained Sep 13, 2000. Figure 17 histograms the data of Table 2. The memory required by PBOB and LMPBOB is the same when rounded to the

[5]Our experiments are limited to prefix databases because range databases are not available for benchmarking. Although, we could randomly generate a database of ranges, this database is unlikely to have nesting properties similar to that of real databases [2].

nearest KB. In Measure1, the memory required by BOB is about 2.38 times that required by PBOB and LMPBOB. However, in Measure2, this ratio is about 1.75. Also, note that, in Measure1, PST takes slightly more memory than does BOB, whereas, in Measure2, BOB takes about 50% more memory than does PST. We note also that the memory requirement of PST may be reduced by about 50% using a priority-search-tree implementation different from that used in [24]. Of course, using this more memory efficient implementation would increase the run-time of PST.

Database		Paix1	Pb1	MaeWest	Aads	Pb2	Paix2
Num of Prefixes		16172	22225	28889	31827	35303	85988
Measure1 (KB)	PST	884	1215	1579	1740	1930	4702
	BOB	851	1176	1526	1682	1876	4527
	PBOB	357	495	642	708	790	1901
	LMPBOB	357	495	642	708	790	1901
Measure2 (KB)	PST	221	303	395	435	482	1175
	BOB	331	455	592	652	723	1760
	PBOB	189	260	338	372	413	1007
	LMPBOB	189	260	338	372	413	1007

Table 2: Memory Usage. The nodes are aligned to 4-byte memory-boundaries.



(a) Measure1 (b) Measure2

Figure 17: Memory usage

Run-Time Experiments

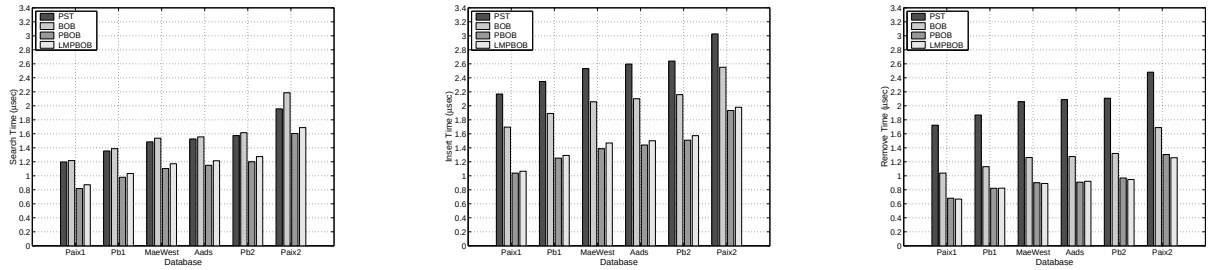
To obtain the mean lookup-time, we started with a BOB, PBOB, or LMPBOB that contained all prefixes of a prefix database. Next, we created a list of the start points of the ranges corresponding to the prefixes in a database and then added one to each of these start points. Call this list L. A random permutation of L was generated and this permutation determined the order in which we searched for the longest matching-prefix for each of addresses in L. The time required to determine all of these longest-matching

prefixes was measured and averaged over the number of addresses in L (actually, since the time to perform all these lookups was too small to measure accurately, we repeated the lookup for all addresses in L several times and then averaged). The experiment was repeated ten times, each time using different random permutation of L, and the mean and standard deviation of these average times computed. These mean times and standard deviations are reported in Table 3. The mean times are also histogrammed in Figure 18(a). It is interesting to note that PBOB, which can handle prefix tables with arbitrary priority assignments is actually 20% to 30% faster than PST, which is limited to prefix tables that employ the longest matching-prefix tie breaker. Further, lookups in BOB, which can handle range tables with arbitrary priorities are slightly slower than in PST. LMPBOB, which, like PST, is designed specifically for longest-matching-prefix lookups is slightly inferior to the more general PBOB.

Database			Paix1	Pb1	MaeWest	Aads	Pb2	Paix2
Search (μsec)	PST	Mean	1.20	1.35	1.49	1.53	1.57	1.96
		Std	0.01	0.01	0.04	0.01	0.00	0.01
	BOB	Mean	1.22	1.39	1.54	1.56	1.62	2.19
		Std	0.01	0.02	0.02	0.02	0.02	0.01
	PBOB	Mean	0.82	0.98	1.10	1.15	1.20	1.60
		Std	0.01	0.01	0.01	0.01	0.01	0.01
	LMPBOB	Mean	0.87	1.03	1.17	1.21	1.27	1.69
		Std	0.01	0.01	0.01	0.01	0.01	0.01
Insert (μsec)	PST	Mean	2.17	2.35	2.53	2.60	2.64	3.03
		Std	0.07	0.04	0.03	0.01	0.05	0.01
	BOB	Mean	1.70	1.89	2.06	2.10	2.16	2.55
		Std	0.06	0.06	0.05	0.05	0.05	0.03
	PBOB	Mean	1.04	1.25	1.39	1.44	1.51	1.93
		Std	0.06	0.05	0.00	0.05	0.05	0.06
	LMPBOB	Mean	1.06	1.29	1.47	1.50	1.57	1.98
		Std	0.07	0.07	0.06	0.06	0.04	0.01
Delete (μsec)	PST	Mean	1.72	1.87	2.06	2.09	2.11	2.48
		Std	0.04	0.05	0.05	0.06	0.04	0.06
	BOB	Mean	1.04	1.13	1.26	1.27	1.32	1.69
		Std	0.06	0.05	0.04	0.05	0.06	0.06
	PBOB	Mean	0.68	0.82	0.90	0.91	0.97	1.30
		Std	0.07	0.06	0.05	0.06	0.03	0.05
	LMPBOB	Mean	0.67	0.82	0.89	0.92	0.95	1.26
		Std	0.06	0.06	0.05	0.05	0.03	0.05
Num of Copies			15	11	9	8	8	3

Table 3: Prefix times on a 1.4GHz Pentium 4 PC, with an 8K L1 data cache and a 256K L2 cache

To obtain the mean insert-time, we started with a random permutation of the prefixes in a database,



(a) Search (b) Insert (c) Delete

Figure 18: Execution Time

inserted the first 67% of the prefixes into an initially empty data structure, measured the time to insert the remaining 33%, and computed the mean insert time by dividing by the number of prefixes in 33% of the database. (Once again, since the time to insert the remaining 33% of the prefixes was too small to measure accurately, we started with several copies of the data structure and inserted the 33% prefixes into each copy; measured the time to insert in all copies; and divided by the number of copies and number of prefixes inserted). This experiment was repeated ten times, each time starting with a different permutation of the database prefixes, and the mean of the mean as well as the standard deviation in the mean computed. These latter two quantities as well as the number of copies of each data structure we used for the inserts are given in Table 3. Figure 18(b) histograms the mean insert-time. As can be seen, insertions into PBOB take between 40% and 60% less time than do insertions into PST; insertions into LMPBOB take slightly more time than do insertions into PBOB; and insertions into PST take 20% to 25% more time than do insertions into BOB.

The mean and standard deviation data reported for the delete operation in Table 3 and Figure 18(c) was obtained in a similar fashion by starting with a data structure that had 100% of the prefixes in the database and measuring the time to delete a randomly selected 33% of these prefixes. Deletion from PBOB takes less than 50% the time required to delete from an PST. For the delete operation, however, LMPBOB is slightly faster than PBOB. Deletions from BOB take about 40% less time than do deletions from PST.

The actual, in-field, performance of each of the tested structures is expected to be somewhat better than indicated by our experiments, because the use of test data pollutes the cache causing more cache misses than would occur in a real application.

10 Conclusion

We have proposed an enhancement of the interval tree structure of [28]. Our enhancement permits nodes that are empty (i.e., contain no range) but requires there be at most $2n$ nodes in the interval tree. The enhanced structure supports efficient insertion and deletion of ranges. Several data structures, based on this enhanced interval tree, have been proposed for the representation of dynamic IP router-tables.

Our experiments show that PBOB is to be preferred over PST and LMPBOB for the representation of dynamic longest-matching prefix-router-tables. This is somewhat surprising because PBOB may be used for highest-priority prefix-router-tables, not just longest-matching prefix-router-tables. A possible reason why PBOB is faster than LMPBOB is that in LMPBOB one has to check $O(W)$ prefix lengths, whereas in PBOB $O(maxR)$ lengths are checked (note that in our test databases, $W = 32$ and $maxR \leq 6$). BOB is slower than and requires more memory than PBOB when tested with longest-matching prefix-router tables. The same relative performance between BOB and PBOB is expected when filters are prefixes with arbitrary priority. Of the data structures considered in this paper, BOB or CBOB may be used when the filters are ranges that have an associated priority.

References

[1] C. Macian and R. Finthammer. An evaluation of the key design criteria to achieve high update rates in packet classifiers. *IEEE Network*, pages 24–29, Nov./Dec. 2001.

[2] F. Baboescu, S. Singh, and G. Varghese. Packet classification for core routers: is there an alternative to cams? In *IEEE INFOCOM*, 2003.

[3] C. Labovitz, G. Malan, and F. Jahanian. Internet routing instability. *IEEE/ACM Transactions on Networking*, 1997.

[4] M. Ruiz-Sanchez, E. Biersack, and W. Dabbous. Survey and taxonomy of ip address lookup algorithms. *IEEE Network*, 15(2):8–23, March/April 2001.

[5] S. Sahni, K. Kim, and H. Lu. Data structures for one-dimensional packet classification using most-specific-rule matching. In *International Symposium on Parallel Architectures, Algorithms, and Networks (ISPAN)*, May 2002.

[6] A. McAuley and P. Francis. Fast routing table lookups using cams. In *IEEE INFOCOM*, pages 1382–1391, 1993.

[7] D. Shah and P. Gupta. Fast updating algorithms for tcams. *IEEE MICRO*, 21(1):36–47, 2001.

[8] C. Matsumoto. Cam vendors consider algorithmic alternatives. *EETimes*, May 2002.

[9] K. Sklower. A tree-based routing table for berkeley unix. Technical report, University of California - Berkeley, 1993.

[10] M. Degermark, A. Brodnik, S. Carlsson, and S. Pink. Small forwarding tables for fast routing lookups. In *ACM SIGCOMM*, pages 3–14, 1997.

[11] W. Doeringer, G. Karjoth, and M. Nassehi. Routing on longest-matching prefixes. *IEEE/ACM Transactions on Networking*, 4(1):86–97, 1996.

[12] S. Nilsson and G. Karlsson. Fast address look-up for internet routers. *IEEE Broadband Communications*, 1998.

[13] V. Srinivasan and G. Varghese. Faster ip lookups using controlled prefix expansion. *ACM Transactions on Computer Systems*, pages 1–40, Feb 1999.

[14] S. Sahni and K. Kim. Efficient construction of fixed-stride multibit tries for ip lookup. In *Proceedings 8th IEEE Workshop on Future Trends of Distributed Computing Systems*, 2001.

[15] S. Sahni and K. Kim. Efficient construction of variable-stride multibit tries for ip lookup. In *Proceedings IEEE Symposium on Applications and the Internet (SAINT)*, 2002.

[16] P. Gupta, S. Lin, and N. McKeown. Routing lookups in hardware at memory access speeds. In *IEEE INFOCOM*, 1998.

[17] M. Waldvogel, G. Varghese, J. Turner, and B. Plattner. Scalable high speed ip routing lookups. In *ACM SIGCOMM*, pages 25–36, 1997.

[18] B. Lampson, V. Srinivasan, and G. Varghese. Ip lookup using multi-way and multicolumn search. In *IEEE INFOCOM*, 1998.

[19] A. Basu and G. Narlika. Fast incremental updates for pipelined forwarding engines. In *IEEE INFOCOM*, 2003.

[20] S. Suri, G. Varghese, and P. Warkhede. Multiway range trees: Scalable ip lookup with fast updates. In *GLOBECOM*, 2001.

[21] S. Sahni and K. Kim. $o(\log n)$ dynamic packet routing. In *IEEE Symposium on Computers and Communications*, 2002.

[22] S. Sahni and K. Kim. Efficient dynamic lookup for bursty access patterns. In *http://www.cise.ufl.edu/∼sahni*, 2003.

[23] F. Ergun, S. Mittra, S. Sahinalp, J. Sharp, and R. Sinha. A dynamic lookup scheme for bursty access patterns. In *IEEE INFOCOM*, 2001.

[24] H. Lu and S. Sahni. $o(\log n)$ dynamic router-tables for prefixes and ranges. In *IEEE Symposium on Computers and Communications*, 2003.

[25] E. Horowitz, S. Sahni, and D. Mehta. *Fundamentals of Data Structures in C++*. W.H. Freeman, New York, 1995.

[26] P. Gupta and N. McKeown. Dynamic algorithms with worst-case performance for packet classification. In *IFIP Networking*, 2000.

[27] T. Cormen, C. Lieserson, R. Rivest, and C. Stein. *Introduction to Algorithms*. MIT Press, second edition edition, 2001.

[28] M. D. Berg, M. V. Kreveld, M. Overmars, and O. Schwarzkopf. *Computational Geometry: Algorithms and Applications*. Springer Verlag, 1997.

[29] S. Sahni. *Data structures, algorithms, and applications in Java*. McGraw Hill, New York, 2000.

[30] Merit. Ipma statistics. In *http://nic.merit.edu/ipma*, 2000, 2001.