Backtracking And Branch And Bound




Subset & Permutation Problems

- Subset problem of size n.
  - Nonsystematic search of the space for the answer takes $O(p2^n)$ time, where p is the time needed to evaluate each member of the solution space.
- Permutation problem of size n.
  - Nonsystematic search of the space for the answer takes O(pn!) time, where p is the time needed to evaluate each member of the solution space.
- Backtracking and branch and bound perform a systematic search; often taking much less time than taken by a nonsystematic search.

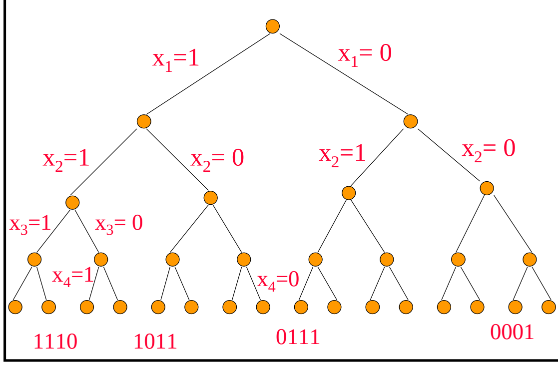
Tree Organization Of Solution Space

- Set up a tree structure such that the leaves represent members of the solution space.
- For a size n subset problem, this tree structure has 2^n leaves.
- For a size n permutation problem, this tree structure has n! leaves.
- The tree structure is too big to store in memory; it also takes too much time to create the tree structure.
- Portions of the tree structure are created by the backtracking and branch and bound algorithms as needed.

Subset Problem

- Use a full binary tree that has 2^n leaves.
- At level i the members of the solution space are partitioned by their x_i values.
- Members with $x_i = 1$ are in the left subtree.
- Members with $x_i = 0$ are in the right subtree.
- Could exchange roles of left and right subtree.

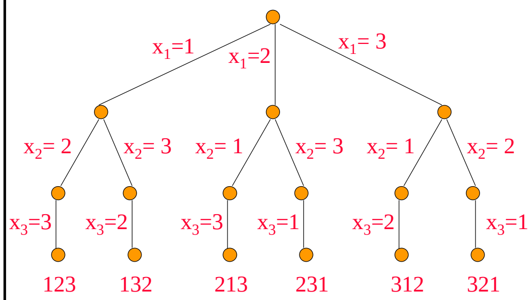
Subset Tree For n = 4



Permutation Problem

- Use a tree that has n! leaves.
- At level i the members of the solution space are partitioned by their x_i values.
- Members (if any) with $x_i = 1$ are in the first subtree.
- Members (if any) with $x_i = 2$ are in the next subtree.
- And so on.

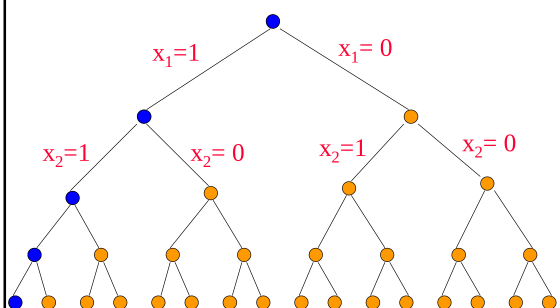
Permutation Tree For n = 3



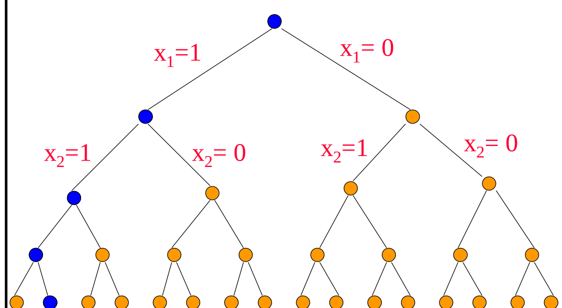
Backtracking

- Search the solution space tree in a depth-first manner.
- May be done recursively or use a stack to retain the path from the root to the current node in the tree.
- The solution space tree exists only in your mind, not in the computer.

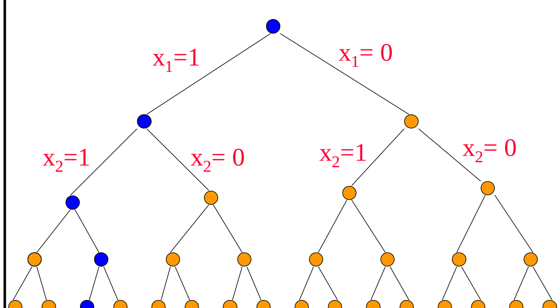
Backtracking Depth-First Search



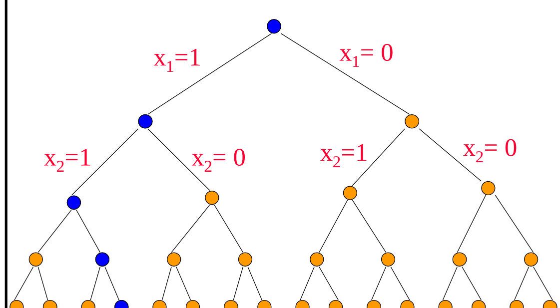
Backtracking Depth-First Search



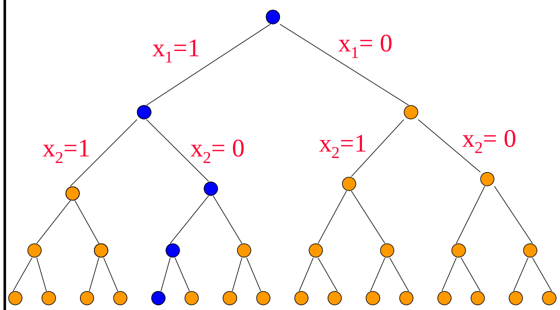
Backtracking Depth-First Search



Backtracking Depth-First Search

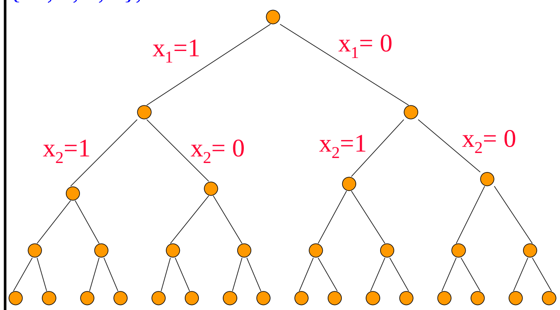


Backtracking Depth-First Search



$O(2^n)$ Subet Sum & Bounding Functions

{10, 5, 2, 1}, c = 14



Each forward and backward move takes O(1) time.

Bounding Functions

- When a node that represents a subset whose sum equals the desired sum c, terminate.
- When a node that represents a subset whose sum exceeds the desired sum c, backtrack. I.e., do not enter its subtrees, go back to parent node.
- Keep a variable r that gives you the sum of the numbers not yet considered. When you move to a right child, check if current subset sum + r >= c. If not, backtrack.

Backtracking

- Space required is O(tree height).
- With effective bounding functions, large instances can often be solved.
- For some problems (e.g., 0/1 knapsack), the answer (or a very good solution) may be found quickly but a lot of additional time is needed to complete the search of the tree.
- Run backtracking for as much time as is feasible and use best solution found up to that time.

Branch And Bound

- Search the tree using a breadth-first search (FIFO branch and bound).
- Search the tree as in a bfs, but replace the FIFO queue with a stack (LIFO branch and bound).
- Replace the FIFO queue with a priority queue (least-cost (or max priority) branch and bound). The priority of a node p in the queue is based on an estimate of the likelihood that the answer node is in the subtree whose root is p.

Branch And Bound

- Space required is O(number of leaves).
- For some problems, solutions are at different levels of the tree (e.g., 16 puzzle).

4		14	1
13	2	3	12
6	11	5	10
9	8	7	15

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Branch And Bound

- FIFO branch and bound finds solution closest to root.
- Backtracking may never find a solution because tree depth is infinite (unless repeating configurations are eliminated).

- Least-cost branch and bound directs the search to parts of the space most likely to contain the answer. So it could perform better than backtracking.