Algorithm Design Methods

- Greedy method.
- Divide and conquer.
- Dynamic Programming.
- Backtracking.
- Branch and bound.

Some Methods Not Covered

- Linear Programming.
- Integer Programming.
- Simulated Annealing.
- Neural Networks.
- Genetic Algorithms.
- Tabu Search.

Optimization Problem

A problem in which some function (called the optimization or objective function) is to be optimized (usually minimized or maximized) subject to some constraints.

Machine Scheduling

Find a schedule that minimizes the finish time.

- optimization function … finish time
- constraints
  - each job is scheduled continuously on a single machine for an amount of time equal to its processing requirement
  - no machine processes more than one job at a time

Bin Packing

Pack items into bins using the fewest number of bins.

- optimization function … number of bins
- constraints
  - each item is packed into a single bin
  - the capacity of no bin is exceeded

Min Cost Spanning Tree

Find a spanning tree that has minimum cost.

- optimization function … sum of edge costs
- constraints
  - must select n-1edges of the given n vertex graph
  - the selected edges must form a tree

Feasible And Optimal Solutions

A feasible solution is a solution that satisfies the constraints.

An optimal solution is a feasible solution that optimizes the objective/optimization function.

Greedy Method

- Solve problem by making a sequence of decisions.
- Decisions are made one by one in some order.
- Each decision is made using a greedy criterion.
- A decision, once made, is (usually) not changed later.

Machine Scheduling

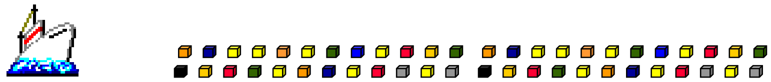
LPT Scheduling.

- Schedule jobs one by one and in decreasing order of processing time.
- Each job is scheduled on the machine on which it finishes earliest.
- Scheduling decisions are made serially using a greedy criterion (minimize finish time of this job).
- LPT scheduling is an application of the greedy method.

LPT Schedule

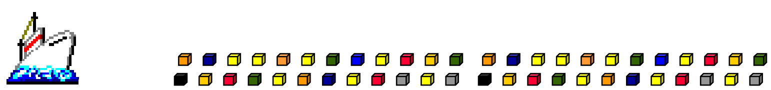
- LPT rule does not guarantee minimum finish time schedules.
- (LPT Finish Time)/(Minimum Finish Time) <= 4/3 - 1/(3m) where m is number of machines
- Minimum finish time scheduling is NP-hard.
- In this case, the greedy method does not work.
- The greedy method does, however, give us a good heuristic for machine scheduling.

Container Loading



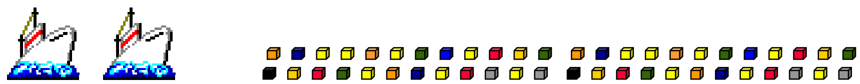
- Ship has capacity c.
- m containers are available for loading.
- Weight of container i is w_i.
- Each weight is a positive number.
- Sum of container weights $> c$.
- Load as many containers as is possible without sinking the ship.

Greedy Solution



- Load containers in increasing order of weight until we get to a container that doesn’t fit.
- Does this greedy algorithm always load the maximum number of containers?
- Yes. May be proved using a proof by induction (see text).

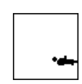
Container Loading With 2 Ships



Can all containers be loaded into 2 ships whose capacity is c (each)?

- Same as bin packing with 2 bins.
  - Are 2 bins sufficient for all items?
- Same as machine scheduling with 2 machines.
  - Can all jobs be completed by 2 machines in c time units?
- NP-hard.

0/1 Knapsack Problem



0/1 Knapsack Problem

- Hiker wishes to take n items on a trip.
- The weight of item i is w_i.
- The items are to be carried in a knapsack whose weight capacity is c.
- When sum of item weights $<= c$, all n items can be carried in the knapsack.
- When sum of item weights $> c$, some items must be left behind.
- Which items should be taken/left?

0/1 Knapsack Problem

- Hiker assigns a profit/value p_i to item i.
- All weights and profits are positive numbers.
- Hiker wants to select a subset of the n items to take.
  - The weight of the subset should not exceed the capacity of the knapsack. (constraint)
  - Cannot select a fraction of an item. (constraint)
  - The profit/value of the subset is the sum of the profits of the selected items. (optimization function)
  - The profit/value of the selected subset should be maximum. (optimization criterion)

0/1 Knapsack Problem

Let $x_i = 1$ when item i is selected and let $x_i = 0$ when item i is not selected.

$$\text{maximize } \sum_{i=1}^{n} p_i x_i$$

$$\text{subject to } \sum_{i=1}^{n} w_i x_i <= c$$

and $x_i = 0$ or 1 for all i

Greedy Attempt 1

Be greedy on capacity utilization.

- Select items in increasing order of weight.

n = 2, c = 7

w = [3, 6]

p = [2, 10]

only item 1 is selected

profit (value) of selection is 2

not best selection!

Greedy Attempt 2

Be greedy on profit earned.

- Select items in decreasing order of profit.

n = 3, c = 7

w = [7, 3, 2]

p = [10, 8, 6]

only item 1 is selected

profit (value) of selection is 10

not best selection!

Greedy Attempt 3

Be greedy on profit density (p/w).

- Select items in decreasing order of profit density.

n = 2, c = 7

w = [1, 7]

p = [10, 20]

only item 1 is selected

profit (value) of selection is 10

not best selection!

Greedy Attempt 3

Be greedy on profit density (p/w).

- Works when selecting a fraction of an item is permitted
- Select items in decreasing order of profit density, if next item doesn't fit take a fraction so as to fill knapsack.

n = 2, c = 7

w = [1, 7]

p = [10, 20]

item 1 and 6/7 of item 2 are selected

0/1 Knapsack Greedy Heuristics

- Select a subset with <= k items.
- If the weight of this subset is > c, discard the subset.
- If the subset weight is <= c, fill as much of the remaining capacity as possible by being greedy on profit density.
- Try all subsets with <= k items and select the one that yields maximum profit.

0/1 Knapsack Greedy Heuristics

- (best value - greedy value)/(best value) <= 1/(k+1)

k	0%	1%	5%	10%	25%
0	239	390	528	583	600
1	360	527	598	600	
2	483	581	600		

Number of solutions (out of 600) within x% of best.

0/1 Knapsack Greedy Heuristics

- First sort into decreasing order of profit density.
- There are $O(n^k)$ subsets with at most k items.
- Trying a subset takes $O(n)$ time.
- Total time is $O(n^{k+1})$ when $k > 0$.
- (best value - greedy value)/(best value) <= 1/(k+1)