Sparse Matrices

sparse … many elements are zero

dense … few elements are zero

Example Of Sparse Matrices

diagonal

tridiagonal

lower triangular (?)

These are structured sparse matrices.

May be mapped into a 1D array so that a mapping function can be used to locate an element.

Unstructured Sparse Matrices

Airline flight matrix.

- airports are numbered 1 through n
- flight(i,j) = list of nonstop flights from airport i to airport j
- n = 1000 (say)
- n x n array of list references => 4 million bytes
- total number of flights = 20,000 (say)
- need at most 20,000 list references => at most 80,000 bytes

Unstructured Sparse Matrices

Web page matrix.

web pages are numbered 1 through n

web(i,j) = number of links from page i to page j

Web analysis.

authority page … page that has many links to it

hub page … links to many authority pages

Web Page Matrix

- n = 2 billion (and growing by 1 million a day)
- n x n array of ints => $16 * 10^{18}$ bytes ($16 * 10^{9}$ GB)
- each page links to 10 (say) other pages on average
- on average there are 10 nonzero entries per row
- space needed for nonzero elements is approximately 20 billion x 4 bytes = 80 billion bytes (80 GB)

Representation Of Unstructured Sparse Matrices

Single linear list in row-major order.

scan the nonzero elements of the sparse matrix in row-major order

each nonzero element is represented by a triple

(row, column, value)

the list of triples may be an array list or a linked list (chain)

Single Linear List Example

0 0 3 0 4
0 0 5 7 0
0 0 0 0 0
0 2 6 0 0

list =

row	1	1	2	2	4	4
column	3	5	3	4	2	3
value	3	4	5	7	2	6

Array Linear List Representation

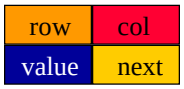
list =

row	1	1	2	2	4	4
column	3	5	3	4	2	3
value	3	4	5	7	2	6

element	0	1	2	3	4	5
row	1	1	2	2	4	4
column	3	5	3	4	2	3
value	3	4	5	7	2	6

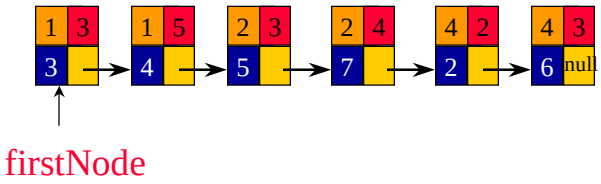
Chain Representation

Node structure.



Single Chain

$$\text{list} = \begin{matrix} \text{row} \\ \text{column} \\ \text{value} \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 2 & 4 & 4 \\ 3 & 5 & 3 & 4 & 2 & 3 \\ 3 & 4 & 5 & 7 & 2 & 6 \end{bmatrix}$$



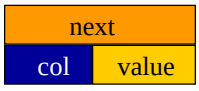
One Linear List Per Row

0 0 3 0 4
0 0 5 7 0
0 0 0 0 0
0 2 6 0 0

row1 = [(3, 3), (5,4)]
row2 = [(3,5), (4,7)]
row3 = []
row4 = [(2,2), (3,6)]

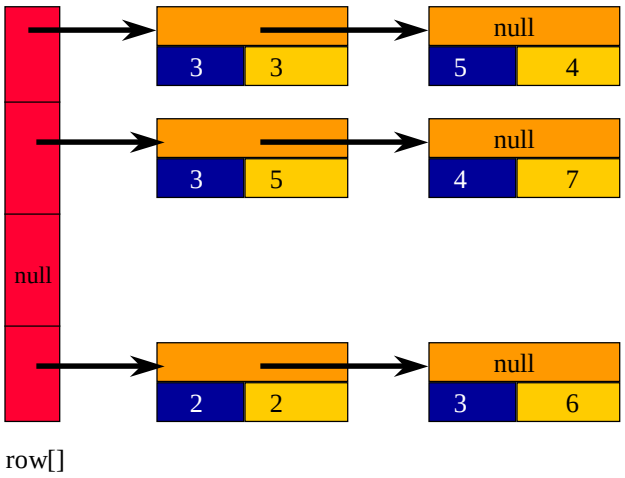
Array Of Row Chains

Node structure.



Array Of Row Chains

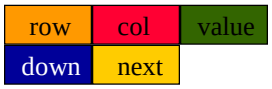
0 0 3 0 4
0 0 5 7 0
0 0 0 0 0
0 2 6 0 0



Orthogonal List Representation

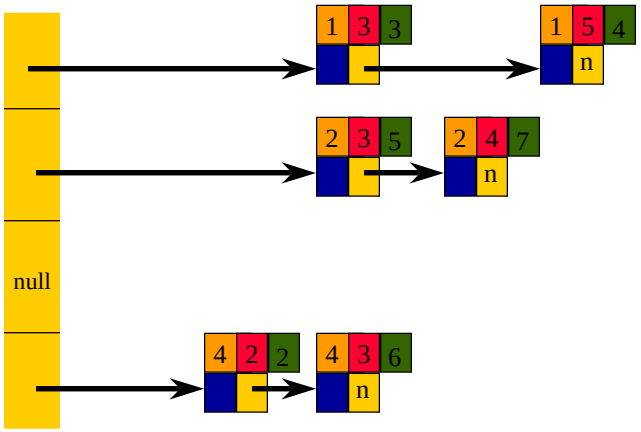
Both row and column lists.

Node structure.



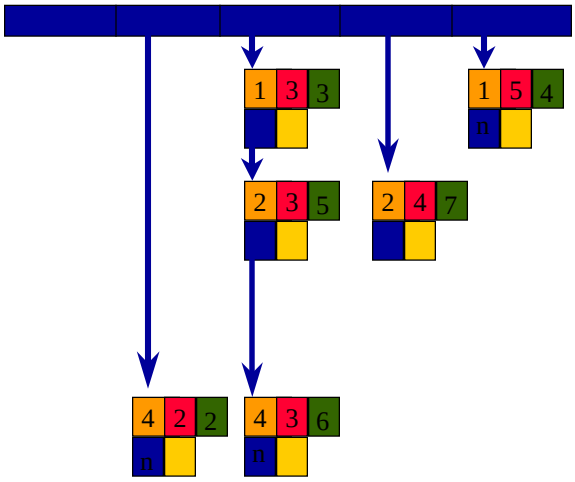
Row Lists

0 0 3 0 4
0 0 5 7 0
0 0 0 0 0
0 2 6 0 0



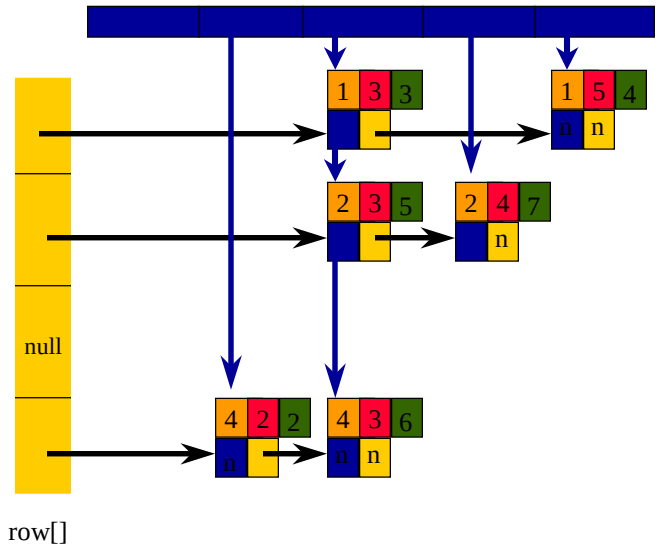
Column Lists

0 0 3 0 4
0 0 5 7 0
0 0 0 0 0
0 2 6 0 0



Orthogonal Lists

0 0 3 0 4
0 0 5 7 0
0 0 0 0 0
0 2 6 0 0



Variations

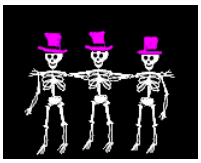
May use circular lists instead of chains.

Approximate Memory Requirements

500 x 500 matrix with 1994 nonzero elements

2D array	500 x 500 x 4 = 1million bytes
Single Array List	3 x 1994 x 4 = 23,928 bytes
One Chain Per Row	23928 + 500 x 4 = 25,928

Runtime Performance

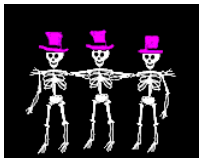


Matrix Transpose

500 x 500 matrix with 1994 nonzero elements

2D array	210 ms
Single Array List	6 ms
One Chain Per Row	12 ms

Performance



Matrix Addition.

500 x 500 matrices with 1994 and 999 nonzero elements

2D array	880 ms
Single Array List	18 ms
One Chain Per Row	29 ms