

Final Examination

COT 6315 Formal Languages and Theory of Computation
May 2, 1995

Instructions

1. This is a closed-book examination.
2. You may refer to three 8.5 by 11 inch sheet of handwritten notes for this exam.
3. You have two (2) hours to complete this examination.
4. **Answer three (3) questions, and no more, including at least one from the last three (3) questions.**
5. Start the answer to each question on a new page (i.e., do **not** put the answer to more than one question on the same page).
6. Assemble your answers in numerical order of the questions when you submit them.
7. Leave a one inch square of blank space in the upper left-hand corner of each page for the staple.

1. Consider the NFSA below.
 - (a) Give a regular expression for the language it recognizes.
 - (b) Produce a minimal (deterministic) FSA for the language this NFSA accepts.

2. For the following languages, prove that they are regular or not.
 - (a) $L = \{x \mid x \text{ is a substring of some string } w \in \{a^i b^i \mid i \geq 0\}\}$
 - (b) $L = \{a^i b^j c^k \mid \text{either } i = 0 \text{ or } i > 0 \text{ and } j = k\}$
 - (c) $L_c = \{a^i b^j \mid i = \lceil \log_c(j) \rceil\}$

3. Define $\Pi_a(L) = \{x \in a^* \mid \exists y \in L, |x| = \#_a(y)\}$.
 - (a) What form must $\Pi_a(L)$ have for L context-free?
 - (b) Prove it.
 - (c) What can you say about a context-free language over a one-symbol alphabet?

4. For the following languages over $\{a, b\}$, prove that they are context-free or not.
 - (a) $L = \{x \mid \#_b(x) \leq \#_a(x) \leq 2\#_b(x)\}$
 - (b) $L = \{x^R y x y^R \mid x, y \in \{a, b\}^*\}$

5. Show that the union of an r.e. set of r.e. languages is an r.e. language. (Hint: Consider an enumerator for a set of encodings for TMs that accept the given set of languages.)

6. For each question below, show that the question is undecidable, or give a decision algorithm for it.
 - (a) For an arbitrary TM, M , and non-halting state q of M , does M enter state q when started on input w ?
 - (b) For an arbitrary TM, M , input string w , and integer k , does M read at least k tape cells when started on input w ?