Examination 3

COT 6315 Formal Languages and Theory of Computation

April 26, 1995

Instructions

1. This is a closed-book examination.
2. You may refer to one 8.5 by 11 inch sheet of handwritten notes for this exam.
3. You have two (2) hours to complete this examination.
4. **Answer three (3) questions, and no more, including at least one from the last three (3) questions.**
5. Start the answer to each question on a new page (i.e., do **not** put the answer to more than one question on the same page).
6. Assemble your answers in numerical order of the questions when you submit them.
7. Leave a one inch square of blank space in the upper left-hand corner of each page for the staple.

1. (a) Given a TM, M, that enumerates all strings in a language L, provide a construction (and argue its correctness) for a new TM, M', that uses M to accept strings in L.
   (b) Given a TM, M', that accepts strings in L, provide a construction (and argue its correctness) for a new TM, M, that enumerates all strings in L using M'.

2. Show that a multi-head TM with the special head moves, "Move head i to the same cell as head j," is no more powerful in terms of language recognition than a standard multihead TM (i.e., simulate the former using the latter).

3. Give a TM that takes as input a binary string of 0's and 1's, interprets it as a binary number (with MSB nearest the head initially) and converts it to unary (in 1's). Show that your machine properly performs this conversion.

Answer at least one question from those below.

4. For each question below, show that the question is undecidable, or give a decision algorithm for it.
   (a) For an arbitrary TM, M, input string w, and integer k, does M enter at least k states when started on input w?
   (b) Given two TM's, M_1 and M_2, do they accept the same language?

5. For each language below, show that there exists a TM that recognizes it, or show that there can be no such TM.
   (a) $L = \{< M > | L(M) = \emptyset\}$
   (b) $L = \{< M > | L(M) \neq \emptyset\}$

6. Is the Post Correspondence Problem solvable if $|\Sigma| = 1$? If not, prove, it; if so, give a solution algorithm.