

Fault Tolerant Topology Control for One-to-All Communications in Symmetric Wireless Networks

F. Wang*

Department of Computer Science and Engineering,
University of Minnesota, USA
E-mail: fwang@cs.umn.edu

*Corresponding author

K. Xu

Department of Computer Science and Engineering,
University of Minnesota, USA
E-mail: kxu@cs.umn.edu

M.T. Thai

Department of Computer and Information Science and Engineering,
University of Florida, USA
E-mail: mythai@cise.ufl.edu

D.Z. Du

Department of Computer Science,
University of Texas at Dallas, USA
E-mail: dzdu@utdallas.edu

Abstract: This paper introduces the problem of fault tolerant topology control for one-to-all communications in symmetric wireless networks. We investigate two algorithms to address the problem, namely Minimum Weight Based Algorithm (MWBA) and Nearest Neighbor Augmentation Algorithm (NNAA), and prove that the former is a $4k$ approximation and the latter is a $(k+4)$ approximation. Through simulations, we evaluate the average performance of these two algorithms and find that MWBA is slightly better than NNAA in terms of the total power consumption.

Keywords: topology control; fault tolerance; k -outconnected graph; wireless networks.

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Biographical notes: Feng Wang received her Ph.D. degree in Computer Science from the University of Minnesota in 2005. Her research interests include wireless networks and applied algorithms. She is currently working at Seagate Technology.

Kuai Xu is a Ph.D student in computer science at the University of Minnesota. His research interests include network security, and Internet measurement.

My T. Thai received her Ph.D. degree in Computer Science from the University of Minnesota in 2005. She is an assistant professor at the University of Florida. Her research interests include wireless networks, computational biology and applied algorithms.

Ding-Zhu Du received his Ph.D. degree in 1985 from the University of California at Santa Barbara. He is a professor at Department of Computer Science, University of Texas at Dallas. His research interests include combinatorial optimization, communication networks, and theory of computation.

1 INTRODUCTION

Fault tolerant topology control is an important and challenging problem in wireless networks. First, wireless networks are usually deployed under harsh environments, thus wireless nodes and links could experience frequent failures. Therefore, fault tolerance must be considered for upper layer applications to function. Second, topology control in wireless network has been proved effective in saving node power and reducing the MAC layer contention. The main idea of topology control is that instead of using its maximal transmission power, each node sets its power to a certain level such that the global topology satisfies a certain constraint. To increase fault tolerance, nodes in the network will consume more power, while the goal of topology control is to save power, thus how to add fault tolerance while using as little power as possible is very interesting and challenging.

In wireless networks, the communication models can be categorized into four classes: i) all-to-all, which represents end-to-end communication of every pair of nodes in the network; ii) one-to-one, which represents the communication from a given source node to a given destination node; iii) all-to-one, which indicates the communication from all nodes to a given (root) node; and iv) one-to-all, which indicates the communication from the root to all the other nodes. In literature, Hajiaghayi et al. (2003), Jia et al. (2005), and Li and Hou (2004) proposed approximations for constructing minimum power k -node disjoint paths between any two nodes, i.e., for all-to-all communication model. Srinivas and Modiano (2003) gave an optimal solution to construct k -node disjoint paths between the given source and destination, i.e., for one-to-one communication model. Wang et al. (2006) proposed two approaches for all-to-one and one-to-all communication model assuming asymmetric wireless links. In this paper, we focus on symmetric links, thus the all-to-one and one-to-all fault tolerant topology control are the same. To be specific, the problem we study in this paper is as follows: Given a network with symmetric links, find a topology that has k -node disjoint paths¹ between one node and all the other nodes (called k -outconnected) and consume as little power as possible. We call it Minimum Power k -Outconnectivity Problem (MPOP). MPOP is NP-hard because when $k=1$, this problem becomes the minimum power assignment in symmetric networks which has been proved to be NP-hard.

We investigate two algorithms, namely Minimum Weight based Algorithm (MWBA) and Nearest Neighbor Augmentation Algorithm (NNAA). The main idea of MWBA is to utilize the minimum weight k -outconnectivity algorithm, while NNAA is to first construct a topology using small power, then augment it to be k -outconnected².

We prove that the MWBA is a $4k$ -approximation and NNAA is a $(k + 4)$ -approximation for minimum power k -outconnectivity problem.³

We also compare the average performance of the MWBA and NNAA through extensive simulations. The results show that MWBA is slightly better than NNAA in terms of total power consumption. For example, when node size is 16, MWBA outperforms NNAA 40 times over totally 100 runs, NNAA outperforms MWBA 33 times, and they perform equally 27 times. In most cases, NNAA generates k -outconnected graphs with smaller weight. In both algorithms, the total power over total weight ratio is around 1.2 for various node sizes.

The rest of this paper is organized as follows. Section 2 describes the related work. Section 3 discusses the network model we use and gives the formal problem definition. In section 4, we present the MWBA and NNAA algorithms and analyze their approximation ratios. The simulation results are illustrated in section 5. Section 6 concludes this paper.

2 RELATED WORK

Previous work related to fault tolerant topology control in static wireless networks can be categorized into four classes: 1) finding all-to-all k -node connected subgraph consuming minimum total power, 2) finding all-to-all k -node connected subgraph consuming minimum maximal power, i.e., the maximal possible power used by each node is minimized, 3) finding one-to-one (also called source-destination) k -node connected subgraph consuming minimum total power, and 4) finding all-to-one and/or one-to-all k -node connected subgraph consuming minimum total power for asymmetric networks. This work is most similar to the last one, while we focus on the symmetric networks. Below we briefly discuss each of these classes.

Minimum total power for all-to-all k -fault tolerance in symmetric networks: Hajiaghayi et al. (2003) analyzed a linear programming approach and proved that its approximation ratio is at least $O(\frac{n}{k})$. In the same paper, the authors also analyzed the k -approximation minimum weight k -connectivity algorithm and proved its approximation ratio of $8k$ for all-to-all k -fault tolerant problem. Jia et al. (2005) proposed a $3k$ -approximation algorithm that first constructs $(k - 1)$ th nearest neighbor graph and then augments it to be k -connected using existing minimum weight k -connected subgraph algorithm.

The only localized algorithm is Fault-Tolerant Cone-Based Topology Control (FCBTC) Bahramgiri et al. (2002) which generalized the well-known Cone-Based

¹There exists k -node disjoint paths between two nodes u and v means that there are k paths to reach from u to v and every pair of paths do not share any common nodes except the endpoints u and v .

²Minimum weight based approach and nearest neighbor augmentation approach have been proposed by Hajiaghayi et al. (2003) Jia

et al. (2005) for all-to-all communication respectively. In this paper, we show how to apply them to construct k -outconnected subgraph and compare their performance under average case.

³An algorithm is an x -approximation for MPOP means the total power consumption of the k -outconnected subgraph constructed by the algorithm is at most $4k$ times the optimal total power consumption of a k -outconnected subgraph

Table 1: Comparison of Fault Tolerant Topology Control Algorithms

Problem	Reference	Scheme	Approximation Ratio
all-to-all	Centralized	Hajiaghayi et al. (2003)(LP)	$O(\frac{n}{k})$
		Hajiaghayi et al. (2003)	$8(k-1)$
		Jia et al. (2005)	$3k$
	Localized	Bahramgiri et al. (2002)(FCBTC)	$\geq O(\frac{n}{k})$
one-to-one	Centralized	Srinivas and Modiano (2003)	Optimal
all-to-one	Centralized	Wang et al. (2006)	k
one-to-all	Centralized	Wang et al. (2006)	Δ^-

Topology Control (CBTC)It's approximation ratio is proved by Hajiaghayi et al. (2003) to be $O(\frac{n}{k})$.

Minimum maximal power for all-to-all k-fault tolerance in symmetric networks: The work by Ramanathan and Hain (2000) and Li and Hou (2004) fall in the second category, and they gave solutions for minimum maximal power consumption for 2-vertex and k-vertex connectivity, respectively. Both papers proposed greedy algorithms in the sense that at each iteration, the edge with minimum weight is chosen until the subgraph becomes 2 or k-connected. Li and Hou (2004) also proposed a localized implementation of the centralized algorithm and proved its optimality in aspect to minimize maximal power consumption among all localized algorithms.

Minimum total power for S-D k-fault tolerance in asymmetric networks: Srinivas and Modiano (2003) proposed an algorithm called Source Transmit Power Selection (STPS) based on the observation that each internal vertex on the k-vertex disjoint S-D paths has only one outgoing edge and only the source node has more than k outgoing edges. Thus their algorithm is to try every possible power setting for the source node and then apply minimum weight k-vertex disjoint S-D path algorithm J.W.Suurballe (1974) for each setting and pick the one with minimum power consumption.

Minimum total power for all-to-one and/or one-to-all k-fault tolerance in asymmetric networks and/or one-to-all k-fault tolerance in asymmetric networks: Wang et al. (2006) proposed six algorithms classified into two approaches: minimum weight approach and augmentation based approach. It is the first one to study the fault tolerance for all-to-one communication in asymmetric networks and proposed a k-approximation, and also the first one to study the fault tolerance for one-to-all communication, namely broadcast, in asymmetric networks and proposed a Δ^- -approximation, where Δ^- is the maximum out-degree in the minimum power k-one-to-all connected topology.

Table 2 summarizes these forementioned studies on fault tolerant topology control in wireless networks and their approximation ratios.

3 PROBLEM DEFINITION

In this paper, we use the following common network model. A wireless network consists of N nodes, each of which is equipped with an omni-directional antenna with a maximal transmission range of r_{max} . The power required for a node to attain a transmission range of r is at least Cr^α , where C is a constant, α is the *power attenuation exponent* and usually chosen between 2 and 4. For any two nodes u and v , there exists a link from u to v if the distance $d(u, v) \leq r_u$, where r_u is the transmission range for node u , determined by its power level. If the links are asymmetric, the existence of a link from u to v does not guarantee the existence of a link from v to u . In this paper, we consider *symmetric* links and assume the wireless network is static, i.e., the nodes in the network are stationary.

Given the coordination of the nodes in the plane and the transmission power of the nodes, the network can be mapped into a *cost graph* $G = (V, E, c)$, where V denotes the set of wireless nodes, E denotes the set of wireless links induced by the transmission power, and the weight c for a given edge (u, v) is computed as $Cd(u, v)^\alpha$, where d is the distance. By this mapping, a symmetric wireless network is represented by an undirected graph.

Wireless Networks have an important feature called *Wireless Multicast Advantage (WMA)* because of its broadcast media. WMA is often utilized to save power. For a node to send data to multiple nodes in its transmission range, instead of sending data multiple times, it only needs to send it once and all nodes in its transmission range can receive the same data. In light of WMA, the power and weight are different in wireless networks.⁴ Weight is link based, while power is node based. The power and weight are defined as follows: Given a cost graph $G = (V, E, c)$, let $p(u)$ be the power assignment of node u , $w(uv)$ be the weight of an edge uv , $c(G)$ be the weight of G , and $p(G)$ be the power of G , then we have

- $p(u) = \max_{uv \in E} w(uv)$,
- $c(G) = \sum_{e \in E} w(e)$
- $p(G) = \sum_{v \in V} p(v)$.

Traditional problems in graph theory are link based with the goal of minimizing total weight. However, wireless net-

⁴In this paper, we will use cost and weight, power and energy interchangeably.

works call for node-based algorithms to minimize the total power consumption.

In the following, we give formal definition for Minimum Power k -Outconnectivity Problem:

Definition 1. Minimum Power k -Outconnectivity Problem: *Given the cost graph of a network and a root node r , find the power assignment of each node such that there exist k -node disjoint paths between the root node r and every other node in the induced spanning subgraph, and total node power assignment is minimized.*

In this paper, we focus on investigating node connectivity. Our solutions for node connectivity can be applied directly to link connectivity. In the following, we will use *vertex* in the context of graph, and use *node* in the context of network.

4 TWO ALGORITHMS

In this section, we present two proposed algorithms, Minimum Weight Based Algorithm and Nearest Neighbor Augmentation Algorithm and give theoretical analysis.

4.1 Minimum Weight Based Algorithm

The main idea of Minimum Weight Based Algorithm (MWBA) is to construct a k -outconnected subgraph with the goal to minimize its weight, then analyze its performance for minimum power k -outconnectivity problem. Given $G = (V, E)$, k , and root node $r \in V$, MWBA utilizes an algorithm proposed by Frank *et al.* Frank and E.Tardos (1989) (let's call it FT) which constructs a minimum weight directed k -outconnected subgraph for a directed graph as follows:

1. Construct G' by replacing each edge in G with two opposite directed edges. The weight of each directed edge is the same weight as the original edge;
2. $D_{FT} = FT(G', k, r)$, D_{FT} is the directed k -outconnected subgraph with optimal weight;
3. Construct the undirected version of D_{FT} , called G_{FT} . An undirected graph is constructed from a directed graph as follows: if there is a directed edge uv in D_{FT} , then there exists an undirected edge uv in G_{FT} . It is obvious that G_{FT} is an undirected k -outconnected subgraph.

Note that Khuller and Raghavachari (1995) also gave an algorithm to construct undirected k -outconnected subgraph using the algorithm FT by adding a new root node. And they prove the weight of the subgraph constructed by their algorithm is less than twice the optimal weight of a k -connected subgraph. The difference of MWBA to Khuller and Raghavachari (1995) is that MWBA does not add a new vertex. Furthermore, we prove the relationship of the *power* of the output of MWBA to the optimal power k -outconnected subgraph.

In the following, we give theoretical analysis of the MWBA algorithm.

Lemma 1 (Hajiaghayi et al. (2003), Jia et al. (2005)). *For undirected graph G , $p(G) \leq 2c(G)$, and $c(G) \leq \Delta p(G)$, where Δ is the maximal degree of G .*

For any tree T , $c(T) \leq p(T)$.

For any forest F , $c(F) \leq p(F)$.

Theorem 1 (Cheriyani et al. (2001)). *Let S be a graph that is k -outconnected from a node r . In S , a cycle consisting of critical edges must be incident to a node $v \neq r$ such that $\deg(v) = k$. A critical edge is the edge that if it is deleted, the graph is not k -outconnected any more.*

Lemma 2 (Hajiaghayi et al. (2003)). *For any undirected graph G which can be written as a union of t forests, $c(G) \leq tp(G)$.*

Lemma 3. *Let G be any critically k -outconnected graph, we have $c(G) \leq kp(G)$, where critically k -outconnected means every edge is critical edge.*

Proof. We can split G into k forests in the same way as Hajiaghayi et al. (2003) except that Hajiaghayi et al. (2003) use Mader Theorem Mader (1972), while our construction utilize the result in Theorem 1. Then by Lemma 2, $c(G) \leq kp(G)$. \square

Lemma 4. *Let G_{MWBA} be the output of MWBA, let G_{wopt} be the undirected k -outconnected subgraph with minimum weight, we have $c(G_{MWBA}) \leq 2c(G_{wopt})$.*

Proof. Construct the bidirectional graph B_{wopt} of G_{wopt} by replacing each edge in G_{wopt} with two opposite directed edges with the same weight, $c(B_{wopt}) = 2c(G_{wopt})$ ⁵. Let D_{MWBA} be the directed version of G_{MWBA} which is constructed by the FT algorithm, we have $c(G_{MWBA}) \leq c(D_{MWBA})$. We know that D_{MWBA} has the optimal weight, thus $c(D_{MWBA}) \leq c(B_{wopt}) = 2c(G_{wopt})$. Thus we have $c(G_{MWBA}) \leq 2c(G_{wopt})$. \square

Theorem 2. *Let G_{popt} be an k -outconnected undirected subgraph consuming minimum power, we have $p(G_{MWBA}) \leq 4kp(G_{popt})$*

Proof. Let $G_{critical}$ be a subgraph of G_{popt} that is critically k -outconnected. From Lemma 3, we have $c(G_{critical}) \leq kp(G_{critical})$. Also $p(G_{critical}) \leq p(G_{popt})$ since $G_{critical}$ is a subgraph of G_{popt} . Thus, we have

$$\begin{aligned} p(G_{MWBA}) &\leq 2c(G_{MWBA}) \\ &\leq 4c(G_{wopt}) \\ &\leq 4c(G_{critical}) \\ &\leq 4kp(G_{critical}) \\ &\leq 4kp(G_{popt}) \end{aligned}$$

\square

To summarize, we present the MWBA algorithm and analyze its approximation ratio for minimum power k -outconnectivity problem as $4k$. Next, we introduce the NNAA algorithm.

⁵It is believed that minimum weight k -outconnectivity problem in undirected graph is NP-hard.

4.2 Nearest Neighbor Augmentation Algorithm

Before we present NNAA, first we introduce the definitions of i th nearest neighbor graph and k -outconnected augmentation.

Definition 2. i th nearest neighbor graph G_i : Given $G = (V, E)$, $G_i = (V, E')$ is its i th nearest neighbor graph, where for each $v \in G$, the first i edges incident to v with smaller weight are in E' .

We can define the i th nearest outgoing neighbor graph D_i of the bidirectional version of G_i as containing the first i outgoing edges incident to each v with smaller weight.

Definition 3. k -outconnected augmentation: Given $G = (V, E)$ and a subgraph of G called H , F is called k -outconnected augmentation to H if $H \cup F$ is k -outconnected spanning subgraph of G .

The main idea of Nearest Neighbor Augmentation Algorithm (NNAA) is to first construct the G_{k-1} , then find a k -outconnected augmentation to G_{k-1} by setting the weight of edges in G_{k-1} to zero, then apply the MWBA to calculate the augmentation. This algorithm is the application of the algorithm in Jia et al. (2005) to k -outconnectivity problem.

NNAA is illustrated as follows: Given an undirected graph $G = (V, E)$,

1. Construct the $(k-1)$ th nearest neighbor graph G_{k-1} .
2. Set the weight of edges of G in G_{k-1} to zero
3. Construct G' by replacing each edge in G with two opposite directed edges with the same weight as the original edge;
4. output MWBA(G')

In the following, we give theoretical analysis for NNAA.

Lemma 5 (Jia et al. (2005)). Let G_{k-1} be the undirected $(k-1)$ th nearest neighbor graph, D_{k-1} be the directed $(k-1)$ th nearest outgoing neighbor graph, $p(G_{k-1}) \leq kp(D_{k-1})$.

Lemma 6. Let G_{popt} be an undirected k -outconnected subgraph consuming optimal power, $p(G_{k-1}) \leq kp(G_{popt})$

Proof. First, by definitions of D_{k-1} and G_{popt} , the minimal degree of G_{popt} is at least k , while the outgoing degree of each node in D_{k-1} is $k-1$, thus $p(D_{k-1}) \leq p(G_{popt})$. By Lemma 5, $p(G_{k-1}) \leq kp(G_{popt})$. \square

Lemma 7. For undirected graph, let $F_{popt} \subseteq G_{popt}$ be the minimum weight k -outconnected augmentation to G_{k-1} among all augmentations which are subgraphs of G_{popt} , $c(F_{popt}) \leq p(G_{popt})$.

Proof. Because G_{popt} is k -outconnected from r , thus it contains a subgraph which is a k -outconnected augmentation to G_{k-1} . Next we prove F_{popt} is a forest by contradiction. Suppose F_{popt} is not a forest, then there exists a cycle C in F_{popt} . Every edge in F_{popt} is critical for

$G_{k-1} \cup F_{popt}$ to be k -outconnected because F_{popt} has the minimum weight among all augmentations which are subgraphs of G_{popt} . Applying Theorem 1, $C \subset F_{popt}$ must contain a node $v \neq r$ and the degree of v is k . On the other hand, since F_{popt} is the k -outconnected augmentation to G_{k-1} in which each vertex degree is at least $k-1$, and F_{popt} contains a cycle including v , the degree of v must be at least $k-1+2 = k+1$, which contradicts that the degree of v is k . Thus F_{popt} is a forest. By Lemma 1, $c(F_{popt}) \leq p(F_{popt}) \leq p(G_{popt})$. \square

Lemma 8. Let F_{NNAA} be the k -outconnected augmentation constructed in algorithm NNAA, $p(F_{NNAA}) \leq 4p(G_{popt})$

Proof. Let F_{wopt} be the optimal weight k -outconnected augmentation to G_{k-1} . We have

$$\begin{aligned} p(F_{NNAA}) &\leq 2c(F_{NNAA}) \\ &\leq 4c(F_{wopt}) \\ &\leq 4c(F_{popt}) \\ &\leq 4p(G_{popt}) \end{aligned}$$

The first two inequalities follow Lemma 1 and Lemma 4 respectively. The third inequality is true because F_{wopt} has the optimal weight. The fourth inequality follows Lemma 7. \square

Theorem 3. $p(G_{NNAA}) \leq (k+4)p(G_{popt})$, i.e., the algorithm NNAA has approximation ratio $k+4$.

Proof. From Lemma 6 and 8, we have $p(G_{NNAA}) = p(G_{k-1} \cup F_{NNAA}) \leq p(G_{k-1}) + p(F_{NNAA}) \leq kp(G_{popt}) + 4p(G_{popt}) = (k+4)p(G_{popt})$. \square

In this section, we propose a $4k$ -approximation called MWBA and $(k+4)$ -approximation called NNAA for minimum power k -outconnectivity problem. Although NNAA performs much better than MWBA in worst case, it is hard to predict their performance under average cases. We evaluate their average performance by simulation in the next section.

5 PERFORMANCE EVALUATION

In this section, we evaluate and compare the performance of MWBA and NNAA in terms of total power and total weight required for constructing undirected k -outconnected spanning subgraphs.

To set up the simulation environment, we randomly generate various number of nodes in a fixed area and construct a complete cost graph from these nodes by setting the weight of each edge uv as $Cd^2(uv)$ and C is randomly chosen between $(0.5, 1.5)$. For each node size, we run the simulation for 100 times.

Fig.1.(a) shows the comparison of average power consumption. We observe that MWBA always outperforms NNAA slightly for different node size. Moreover, the average power of both schemes tends to decrease as the node size increase. This reflects the effect of node density to

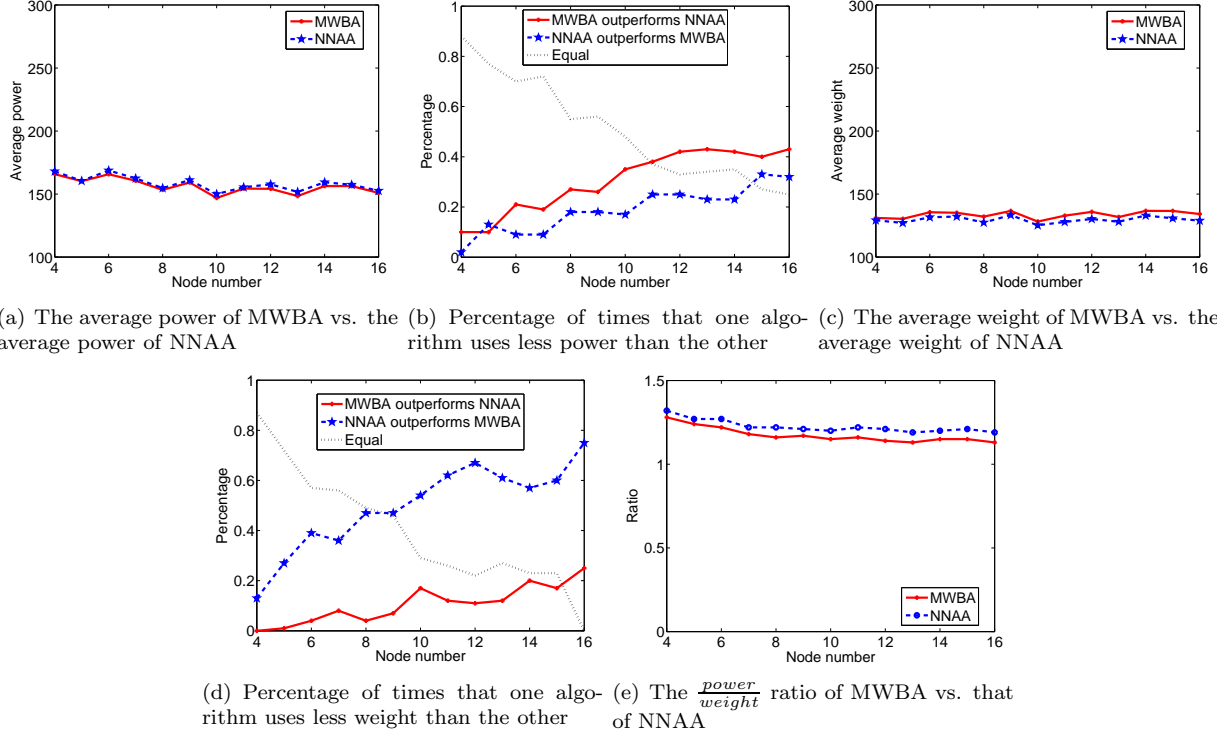


Figure 1: Comparison of MWBA and NNAA

total power. As the network gets denser, the total power to satisfy the connectivity requirement decreases because the power assignment of each node is reduced.

Besides total power consumption, we also show the number of times one algorithm outperforms the other in terms of power in Fig.1.(b). The solid line illustrates the percentage of times when MWBA outperforms NNAA, the dashed line with star illustrates the percentage of times when NNAA outperforms MWBA, and the dotted line illustrates the percentage of times when both algorithms use the same power. MWBA achieves better performance in general than NNAA. This suggests that algorithms for minimum weight k -outconnectivity is also a good approximation for minimum power k -outconnectivity.

The comparison of total weight is shown in Fig.1.(c) and (d). Fig.1.(c) illustrates that NNAA has less weight than MWBA and Fig.1.(d), which uses the same legend as in Fig.1.(b), shows that NNAA uses less weight than MWBA for most of the runs. This conveys that although minimum weight directed k -outconnectivity has an optimal solution, its undirected version for undirected graphs does not guarantee small weight.

We also compare the ratio of power and weight of both algorithms. We define the ratio as follows:

$$Ratio = \frac{\sum_{i=1}^r \frac{Power_i}{Weight_i}}{r}$$

where r is the number of runs.

As illustrated in Fig. 1.(e), MWBA and NNAA both show similar $\frac{power}{weight}$ ratio over various node size. MWBA has slightly smaller ratio of around 1.16 than that of

NNAA, which is around 1.2. The reason is that output of MWBA has greater total weight than NNAA while their total power are almost the same.

To conclude, MWBA is slightly better than NNAA in terms of total power consumption and NNAA is moderately better than MWBA in terms of total weight.

6 CONCLUSIONS

In this paper, we introduced the minimum power k -outconnectivity problem in symmetric networks and studied its hardness. We presented a $4k$ -approximation called minimum weight based algorithm and a $(k+4)$ -approximation called nearest neighbor augmentation algorithm for this problem. In addition, we compare their performance under average case using extensive simulation. The simulation results show that although the NNAA outperforms MWBA in the worst case, under average case, MWBA is slightly better than NNAA in terms of total power consumption.

Our future work will focus on the distributed and localized algorithms since if the networks are dynamic, distributed and localized algorithms are necessary.

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