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A Note on Optical Network with Nonsplitting Nodes

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Abstract. We study the problem of computing the minimum total weight multicast route in an optical network with both nonsplitting and splitting nodes, and present a simple approximation with performance ratio 3, which is better than existing one in the literature.

Keywords:

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1. Introduction

All-optical networks can provide high bandwidth and low end-to-end delay because the wavelength-routing eliminates the electronic bottleneck. Due to the requirement of the many applications running over optical networks, it is desirable that optical network layer provides multicast capability. General multicast problem is that given a network topology, source of the multicast session, multicast members, finds a multicast route that spans all the members.

Often, an optical network is formulated as a graph with switches as nodes. There are actually two types of switches, nonsplitting and splitting. Corresponding nodes are said to be *nonsplitting* and *splitting*, respectively. A nonsplitting switch cannot split an input signal into several outputs. Therefore, in a multicast route, a signal may pass a nonsplitting node several times (figure 1), but cannot be splited. If all nodes are nonsplitting, a multicast route becomes a path and a broadcast route becomes a Hamiltonian path. It is well-known that the minimum weight Hailtonian path has a 1.5-approximation Garey and Jonnson (1979).

If there is no nonsplitting node, then the minimum total weight multicast route is the minimum Steiner tree. The current best known polynomial-time approximation has performance ratio 1.55 (Robine and Zelikovsky, 2000).

The interesting problem is that when both nonspliting and splitting nodes exist, how to construct a good approximation. This problem is called the *minimum total weight multicast route in optical networks with nonsplitting nodes* (MWMT-ONNN). Clearly, the minimum

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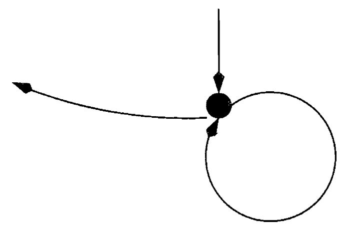

Figure 1. A nonsplitting node.

Steiner tree and minimum Hamiltonian path are special cases of this problem. Note that the minimum Hamiltonian path is usually studied through the traveling salesman problem. Therefore, the approximation for this problem should be constructed with techniques from studies in Steiner trees and traveling salesman problem.

The approximation given by Yan et al. (2003) consists of two steps. In the first step, a Steiner tree T is constructed to interconnect the source node and all multicast members under assumption that all nodes are splitting. In the second step, construct a tour starting from the source node along the Steiner tree to reach all multicast members in the depth-first-search rule. Note that the Steiner tree T constructed in the first step can be bounded by 1.55 *opt* where *opt* is the total weight of the minimum muticast route in the given optical network. Moreover, the tour constructed in the second step has total weight at most twice of the total weight of T and hence can be bounded by 3.1 *opt*.

In this note, we will present a simpler approximation with performance ratio 3, which is a little better. We also discuss the possibility of further improvement.

2. Main result

Let us consider the following construction:

Step 1. Construct a weighted complete graph G with the source node and all multicast members as nodes and the weight of edge (u, v) equals the total weight of the shortest path between u and v in the original optical network.

Step 2. Construct a traveling salesman tour Q in G with Christofides approximation (Christofides, 1976). This approximation consists of construction of a minimum spanning tree T and construction of a minimum perfect matching for all nodes with odd degree in T.

Step 3. Along tour Q, travel from the source node to all multicast members and turn this path in G to a path in the original optical network.

Theorem 1. *The above approximation is within a factor of* 3 *from optimal.*

Proof: It is well known that the Christofides approximation gives a tour within a factor of 1.5 from the optimal traveling salesman tour in graph G. We next prove that the optimal traveling salesman tour has total weight at most 2 *opt*. Consider a minimum total-weight multicast tree T^* in the given optical network. Starting from the source node, travel along tree T^* in the depth-first search way. Then we would obtain a tour passing through the source node and all multicast members in the given optical network. Turn this tour into a traveling salesman tour in graph G. The total weight of this tour is exactly $2opt$. Therefore, the optimal one is bounded by $2opt$. Hence, the above approximation has total weight at most $3opt$. □

Note that the previously known 3.1-approximation requires to construct a 1.55-approximation for the minimum Steiner tree. This requires to construct a k-restricted Steiner tree with quite high running time (Robins and Zelikovsky, 2000). However, the above 3-approximation has running time not exceed $O(n^3)$. Therefore, this 3-approximation has a better performance and can be constructed faster.

Could we do better? A naive idea is as follows:

Step 1. Construct a Steiner tree T for the source node and all multicast members.
Step 2. Construct a perfect matching M for all multicast members with odd degree, if the number of those members is even; or for the source node and all multicast members with odd degree, otherwise.
Step 3. Find a multicast route in the union of T and M.

If this algorithm produces a solution, then its total weight may be within $2.55opt$. In fact, the Steiner tree can be a 1.55-approximation and the matching can be within *opt*.

Unfortunately, this union sometimes does not give a multicast route. In figure 2(a), we see a Steiner tree with three Steiner nodes, a source node S and nine multicast members. Among them, A is a nonsplitting node. In figure 2(b), a perfect matching for odd is added. However, from the source node S, traveling through nonsplitting node A can reach only one branch.

Although this algorithm does not always work, it does produces a good approximation solution frequently. Therefore, we may take it as a heuristic in practice.

It is an interesting and challenge open problem to modify this heuristic into an approximation with performance ratio smaller than three.

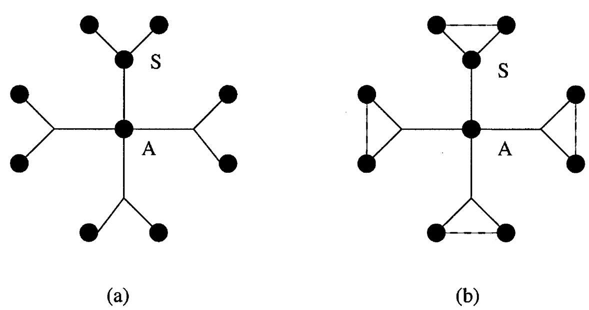

Figure 2. A counterexample.

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