How to Determine All Candidate Keys for a Given Relation and a Set of FDs (II)

**Step 1:** *Determine the attributes that are neither on the left side nor on the right side of any FD.*
*Reason:* These attributes cannot be generated by any FD. Therefore, they must belong to any candidate key to generate themselves by reflexivity.

**Step 2:** *Determine the attributes that are only on the left side of any FD.*
*Reason:* These attributes can never be reached by any FD since there is no FD that has them on their right side. Therefore, they must also belong to any candidate key in order to generate themselves by reflexivity.

**Step 3:** *Determine the attributes that are only on the right side of any FD.*
*Reason:* These attributes can only be reached by other attributes or attribute combinations. That is, they cannot be part of any candidate key.

**Step 4:** *Combine the attributes from steps 1 and 2.*
*Reason:* Any candidate key must contain them.
How to Determine All Candidate Keys for a Given Relation and a Set of FDs (III)

**Step 5:** Compute the closure of the attributes from Step 4. If the attribute closure contains all attributes of \( R \), then the attributes from Step 4 form the only candidate key, and the algorithm terminates.

**Reason:** No matter how many candidate keys there are, every one of them must contain these attributes, and they already reach all attributes in \( R \). Hence they form the only key.

**Step 6:** Otherwise, determine the attributes that are included neither in Step 3 nor in Step 4.

**Reason:** The attributes from Step 3 cannot be part of any candidate key. The attributes from Step 4 have already been identified as parts of any candidate key. The remaining attributes are those that appear on both sides of an FD.

**Step 7:** Compute the closure of the attributes from Step 4 (if there are any) and every possible combination of attributes from Step 6, and determine those minimal attribute closures that are equal to \( R \).

**Reason:** We have to find possible combinations of attributes or single missing attributes so that all attributes in \( R \) can be reached from the resulting attribute sets.
Back to the example: Let $R(A, B, C, D, E, F)$ be a relation schema, and let $S = \{DF \rightarrow C, BC \rightarrow F, E \rightarrow A, ABC \rightarrow E\}$ be a set of FDs. Determine all candidate keys.

Step 1: None.
Step 2: $BD$
Step 3: None.
Step 4: $BD$
Step 5: The closure of $BD$ is $BD$, that is, $BD^+ = BD$. This does not include all attributes from $R$, and therefore we have to continue with the next step.
Step 6: $ACEF$
Step 7: Consider the sets with three attributes: $BDA^+ = ABD$, $BDC^+ = BCDF$, $BDE^+ = ABDE$, $BDF^+ = BCDF$. No candidate keys found.

Consider the sets with four attributes: $BDAC^+ = ABCDEF$, $BDAE^+ = ABDE$, $BDAF^+ = ABCDEF$, $BDCE^+ = ABCDEF$, $BDCF^+ = BCDF$, $BDEF^+ = ABCDEF$. Candidate keys found: $ABCD, ABDF, BCDE, BDEF$

Consider the sets with five attributes for $BDAE$ and $BDCF$: $BDAEC^+ = BDAC^+$, $BDAEF^+ = BDEF^+$, $BDCFA^+ = BDAF^+$, $BDFCE^+ = BDCE^+$. No further candidate keys found. Done!
How to Determine All Candidate Keys for a Given Relation and a Set of FDs (V)

Same example but different presentation: Let \( R(A, B, C, D, E, F) \) be a relation schema, and let \( S = \{DF \rightarrow C, BC \rightarrow F, E \rightarrow A, ABC \rightarrow E\} \) be a set of FDs. Determine all candidate keys.

Draw table with three columns (this corresponds to steps 1, 2, 3, 4, 6): L = Attributes on left sides only, R = Attributes on right sides only, B = Attributes on both sides

<table>
<thead>
<tr>
<th>L</th>
<th>B</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B, D)</td>
<td>A, C, E, F</td>
<td>-</td>
</tr>
</tbody>
</table>

Perform steps 5 and 7.