Canonical cover

- In general, distinct equivalent sets of FDs exist. Two sets $F$ and $G$ of FDs are called \textbf{equivalent} iff $F^+ = G^+$ holds.

- Definition of equivalence is convincing, because the equality of the closures for $F$ and $G$ implies that the same FDs can be inferred from $F$ and $G$.

- For a given set $F$ of FDs there exists a unique closure $F^+$.

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  drawbacks of the closure $F^+$:
  - in general very many FDs in $F^+$ so that the handling with $F^+$ becomes difficult
  - large redundant set of FDs that has to be checked as consistency tests for database modifications

- goal: computation of a most possible small set of FDs which are equivalent to $F$
  $\rightarrow$ less effort for testing whether a new or updated tuple violates a FD
- $F_c$ is called **canonical cover** of a given set $F$ of FDs, if holds:
  - $F_c^+ = F^+$
  - In $F_c$ there are no FDs $A \rightarrow B$ where $A$ or $B$ contain *extraneous* attributes, i.e., they are reduced as much as possible.

  We cannot omit any attribute on the **left** sides of any FD, otherwise we would change the semantics:

  \[ \forall a \in A : (F_c - \{A \rightarrow B\} \cup \{(A - \{a\}) \rightarrow B\})^+ \neq F_c^+ \]

  **Example:** schema $\text{supplier}(\text{sname, saddr, product, price})$ and FDs $\{\text{sname, product}\} \rightarrow \{\text{saddr}\}$ and $\{\text{sname, product}\} \rightarrow \{\text{price}\}$. Can we omit one of the attributes on the left sides?

  We cannot omit any attribute on the **right** sides of any FD, otherwise we would change the semantics:

  \[ \forall b \in B : (F_c - \{A \rightarrow B\} \cup \{A \rightarrow (B - \{b\})\})^+ \neq F_c^+ \]

  - Each left side of the FDs in $F_c$ occurs only once, i.e.,
    
    if $A \rightarrow B$ and $A \rightarrow C$ hold, then in $F_c$ only the FD $A \rightarrow B \cup C$ is used.
algorithm for computing the canonical cover

- **step 1:** For each FD \( A \rightarrow B \in F \) perform a **left reduction**: check for all \( a \in A \) whether the attribute \( a \) is extraneous, i.e., whether
  \[
  B \subseteq \text{AttrClosure}(F, A \setminus \{a\})
  \]
  holds. If this is the case, replace \( A \rightarrow B \) by \((A \setminus \{a\}) \rightarrow B\).

- **step 2:** For each remaining FD \( A \rightarrow B \in F \) perform the **right reduction**: check for all \( b \in B \), whether the attribute \( b \) is extraneous, i.e., whether
  \[
  b \in \text{AttrClosure}(F \setminus \{A \rightarrow B\} \cup \{A \rightarrow (B \setminus \{b\})\}, A)
  \]
  holds. If this is the case, replace \( A \rightarrow B \) by \( A \rightarrow (B \setminus \{b\})\).

- **step 3:** Remove the FDs of the form \( A \rightarrow \emptyset \) which perhaps have been produced in the previous step.

- **step 4:** By using the union rule replace all FDs of the form \( A \rightarrow B_1, \ldots, A \rightarrow B_n \) by
  \[
  A \rightarrow B_1 \cup \ldots \cup B_n
  \]
example
- Given the set $F = \{A \rightarrow B, B \rightarrow C, A \cup B \rightarrow C\}$.
- step 1: $A \cup B \rightarrow C$ is replaced by $A \rightarrow C$, because $B$ on the left side is extraneous ($C$ is already functionally dependent from $A$ by the first two FDs).
- step 2: $A \rightarrow C$ is replaced by $A \rightarrow \emptyset$, because $C$ on the right side is extraneous. This results from the fact that $C \subseteq AttrClosure(\{A \rightarrow B, B \rightarrow C, A \rightarrow \emptyset\}, A)$.
- step 3: $A \rightarrow \emptyset$ is removed. We obtain: $F_c = \{A \rightarrow B, B \rightarrow C\}$.
- step 4: Nothing to be done.