Example for Computing the Attribute Closure

_Notation:_ For the set \{A, C, E\} we permit to write ACE (juxtaposition) to be able to omit braces. In particular, \{D\} is written as D.

_Example:_ Let \(R(A, B, C, G, H, I)\) be a relation schema, and let \(F = \{A \rightarrow B, A \rightarrow C, CG \rightarrow H, CI \rightarrow G\}\) be a set of FDs.

Task: Compute \(AG^+\).

Solution: We use the algorithm AttrClosure and initialize \(AG^+\) with \(AG\).

In the loop we set \(Old\_AG^+ := AG\) and check all FDs whether they can contribute to \(AG^+\).

First we take \(A \rightarrow B\) and check whether \(A \subseteq AG^+\) holds. This is the case. Therefore, we set \(AG^+ := AG^+ \cup B = ABG\) (due to transitivity).

Next, we take \(A \rightarrow C\). Using the same argument as before, we obtain \(AG^+ := AG^+ \cup C = ABCG\).

Next, we take \(CG \rightarrow H\). We find that \(CG \subseteq AG^+\) holds. We get \(AG^+ := AG^+ \cup H = ABCGH\).

Next we take \(CI \rightarrow G\). We find that \(CI \not\subseteq AG^+\) holds.

Since \(Old\_AG^+ \neq AG^+\) holds, we perform a second loop. We set \(Old\_AG^+\) to \(AG^+\), that is, \(Old\_AG^+ := ABCGH\). We see soon that no FD from \(F\) can increase \(AG^+\). This means that \(Old\_AG^+ = AG^+\) holds, the algorithm terminates, and we get \(AG^+ := ABCGH\).
Canonical cover

- In general, distinct equivalent sets of FDs exist. Two sets $F$ and $G$ of FDs are called **equivalent** iff $F^+ = G^+$ holds.

- Definition of equivalence is convincing, because the equality of the closures for $F$ and $G$ implies that the same FDs can be inferred from $F$ and $G$.

- For a given set $F$ of FDs there exists a unique closure $F^+$.

- draw backs of the closure $F^+$:
  - in general very many FDs in $F^+$ so that the handling with $F^+$ becomes difficult
  - large redundant set of FDs that has to be checked as consistency tests for database modifications

- goal: computation of a most possible small set of FDs which are equivalent to $F$
  - less effort for testing whether a new or updated tuple violates a FD
$F_c$ is called **canonical cover** of a given set $F$ of FDs, if holds:

- $F_c^+ = F^+$
- In $F_c$ there are no FDs $A \rightarrow B$ where $A$ or $B$ contain *extraneous* attributes, i.e., they are reduced as much as possible.

We cannot omit any attribute on the **left** sides of any FD, otherwise we would change the semantics:

$$\forall a \in A : (F_c - \{A \rightarrow B\} \cup \{(A - \{a\}) \rightarrow B\})^+ \neq F_c^+$$

*Example*: schema `supplier(sname, saddr, product, price)` and FDs \{sname, product\} $\rightarrow$ \{saddr\} and \{sname, product\} $\rightarrow$ \{price\}. Can we omit one of the attributes on the left sides?

We cannot omit any attribute on the **right** sides of any FD, otherwise we would change the semantics:

$$\forall b \in B : (F_c - \{A \rightarrow B\} \cup \{A \rightarrow (B - \{b\})\})^+ \neq F_c^+$$

- Each left side of the FDs in $F_c$ occurs only once, i.e.,

  if $A \rightarrow B$ and $A \rightarrow C$ hold, then in $F_c$ only the FD $A \rightarrow B \cup C$ is used.
algorithm for computing the canonical cover

- **step 1:** For each FD $A \rightarrow B \in F$ perform a **left reduction**: check for all $a \in A$ whether the attribute $a$ is extraneous, i.e., whether
  \[ B \subseteq AttrClosure(F, A - \{a\}) \]
  holds. If this is the case, replace $A \rightarrow B$ by $(A - \{a\}) \rightarrow B$.

- **step 2:** For each remaining FD $A \rightarrow B \in F$ perform the **right reduction**: check for all $b \in B$, whether the attribute $b$ is extraneous, i.e., whether
  \[ b \in AttrClosure(F - \{A \rightarrow B\} \cup \{A \rightarrow (B - \{b\})\}, A) \]
  holds. If this is the case, replace $A \rightarrow B$ by $A \rightarrow (B - \{b\})$.

- **step 3:** Remove the FDs of the form $A \rightarrow \emptyset$ which perhaps have been produced in the previous step.

- **step 4:** By using the union rule replace all FDs of the form $A \rightarrow B_1, \ldots, A \rightarrow B_n$ by
  \[ A \rightarrow B_1 \cup \ldots \cup B_n \]
example
- Given the set \( F = \{A \rightarrow B, B \rightarrow C, A \cup B \rightarrow C\} \).
- step 1: \( A \cup B \rightarrow C \) is replaced by \( A \rightarrow C \), because \( B \) on the left side is extraneous (\( C \) is already functionally dependent from \( A \) by the first two FDs).
- step 2: \( A \rightarrow C \) is replaced by \( A \rightarrow \emptyset \), because \( C \) on the right side is extraneous. This results from the fact that \( C \subseteq \text{AttrClosure} \{A \rightarrow B, B \rightarrow C, A \rightarrow \emptyset\}, A\).
- step 3: \( A \rightarrow \emptyset \) is removed. We obtain: \( F_c = \{A \rightarrow B, B \rightarrow C\} \).
- step 4: Nothing to be done.